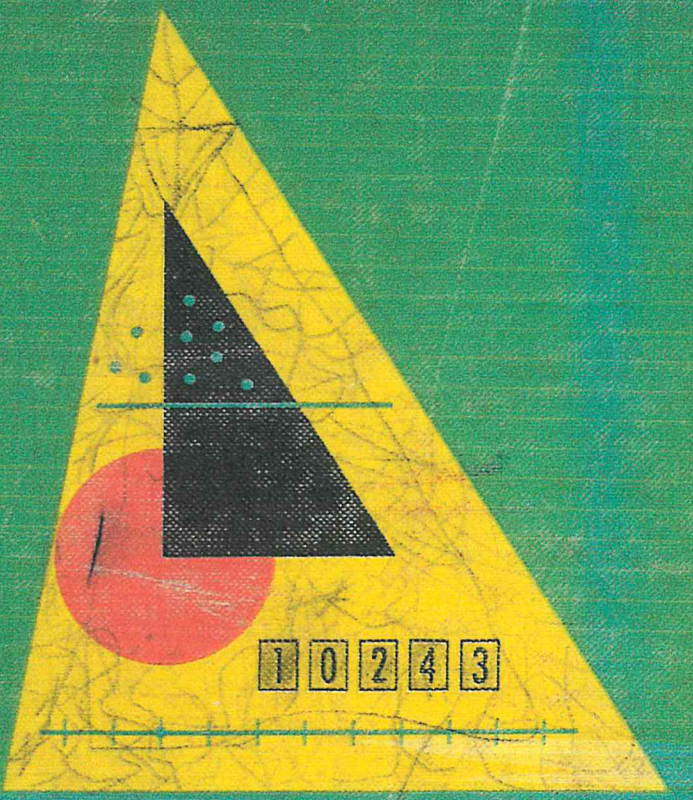
Modern Arithmetic

THROUGH DISCOVERY



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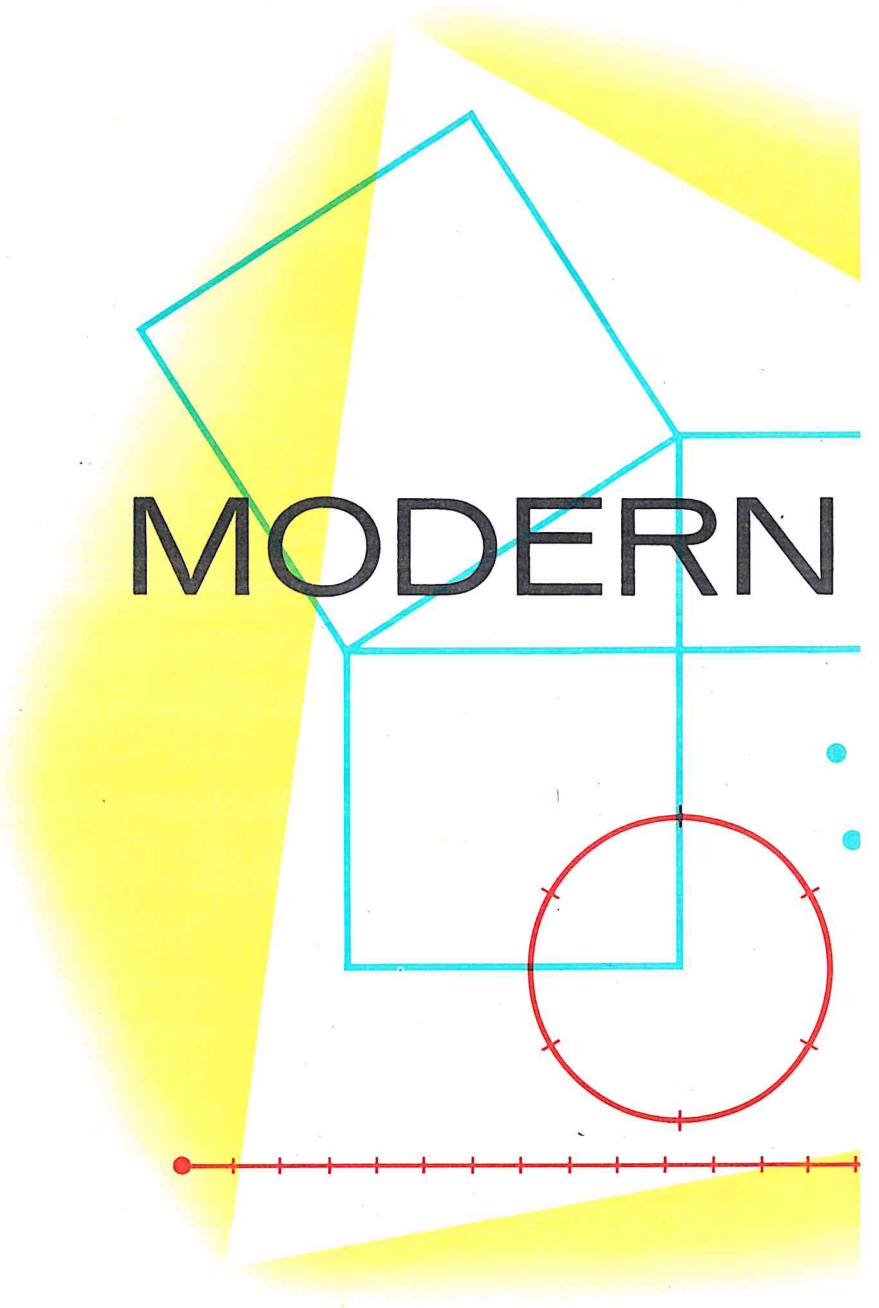
MODERN ARITHMETIC *Through Discovery*

6 MODERN



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ROBERT LEE MORTON
MYRON F. ROSSKOPF
H. STEWART MOREDOCK
MERLE GRAY
EDWARD E. SAGE
WAGNER G. COLLINS

ARITHMETIC
THROUGH DISCOVERY

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Preface

IN SCIENCE, in industry, in commerce—indeed in every area of our complex society—mathematics plays a role of ever-increasing importance. To provide our children with the necessary foundation for their future, leaders in the field of elementary arithmetic have long seen the need to revitalize the elementary school arithmetic curriculum; they have sought a new program that clarifies mathematical ideas, offers stimulating content, and helps pupils prepare for intelligent participation in a society that will be even more technological than our own.

MODERN ARITHMETIC THROUGH DISCOVERY is such a program. Modern in viewpoint, in content, and in spirit, it is a sequential learning program in which new approaches and new content are always utilized for a valid mathematical and pedagogical purpose.

The authors have had one goal through every stage in the preparation of the material for these texts. That goal was the development of a program that would help establish new standards for the teaching of arithmetic at the same time that it provided pupils with a series of challenging, meaningful, and—perhaps most important—*appropriate* learning experiences.

The authors have examined carefully several exploratory programs in elementary mathematics. These include programs of the following: the School Mathematics Study Group, the Syracuse University "Madison Project," the University of Illinois Committee on School Mathematics, the University of Maryland Mathematics Project, the Greater Cleveland Mathematics Program, and others. Each of these sources has supplied useful ideas.

From all the topics that might be taught, the authors have selected those topics which, in their judgment, can be taught most appropriately at each grade level. This material is then

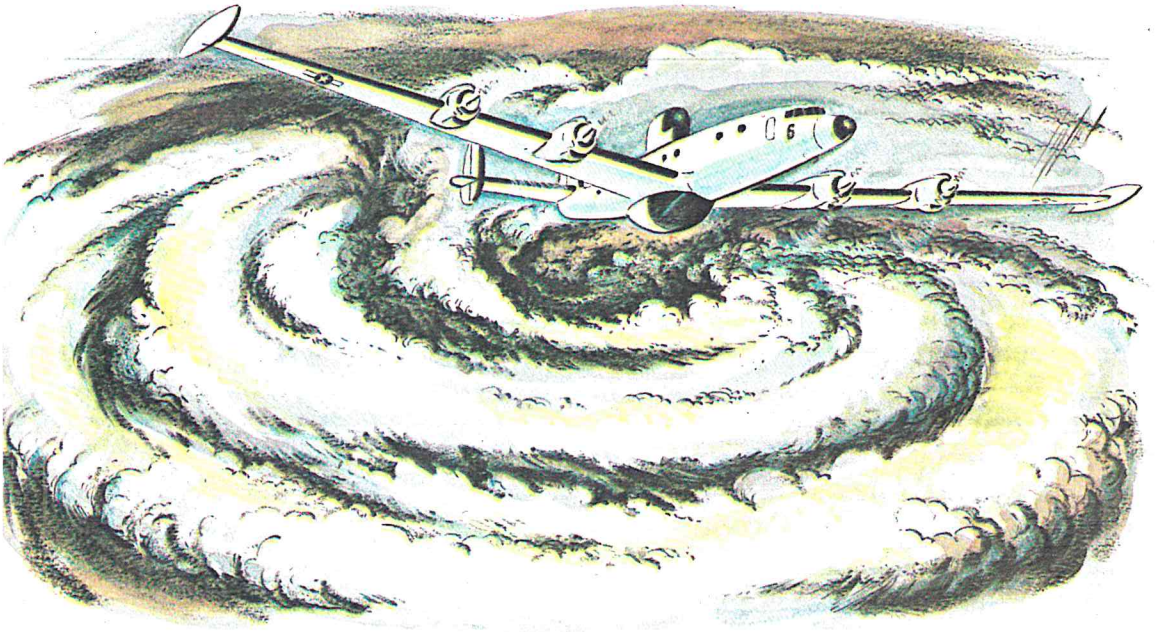
presented with a concern for the learner, a consciousness for the teacher's problems, and a careful craftsmanship based on years of authorship and teaching experience.

One significant aspect of the series is its consistent, sustained use of number sentences to illuminate ideas about numbers and operations. The number line is also utilized, both as a graphic tool and conceptually, to build important basic understandings. The text carefully introduces and maintains the idea of addition and subtraction, and multiplication and division, as inverse operations. Strong emphasis is given to ideas of place value in our decimal system, and the pupils' understanding of the system is broadened—as early as the fifth and sixth grades—by work with other place-value numeration systems. Informally and simply, ideas of structure are introduced. Throughout the grades, the pupils are encouraged to look for patterns and relationships among numbers.

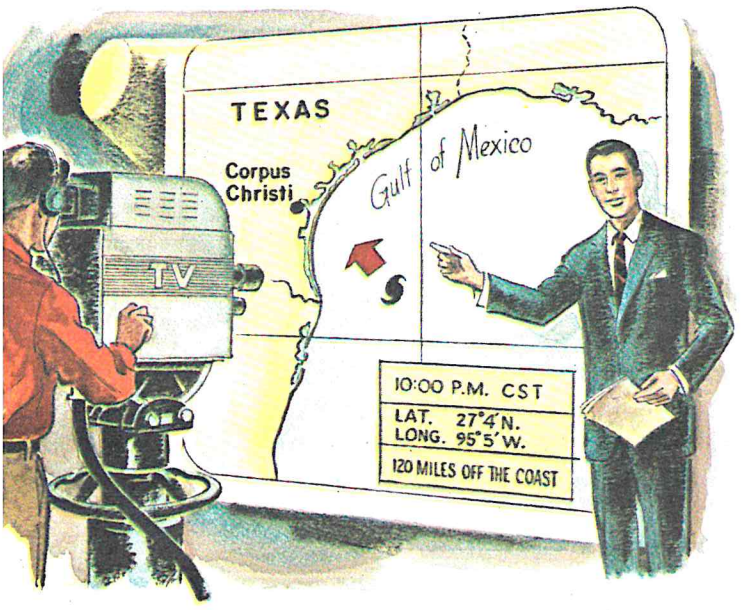
To be truly modern, an arithmetic program must be modern in spirit. It must be a program that emphasizes inquiry, exploration, and discovery. MODERN ARITHMETIC THROUGH DISCOVERY is constructed to stimulate this inquiry and this discovery. The texts carefully help pupils to move—step by step—from one idea to another; and throughout this procedure, the pupils are encouraged to explore and discover relationships. The classroom teacher has, of course, always encouraged exploration of ideas by the pupil. Now, with the aid of texts that consciously promote this exploration, deeper understandings will result.

The teachers' editions for the series are an invaluable accompaniment to the text. They are designed so that maximum help is offered. The major part of each teacher's edition consists of the complete pupil's textbook, each page reproduced in full size, with answers and marginal teaching suggestions in color. All new content and approaches are explained in these marginal notes. In a section at the back of the teacher's edition, supplementary background information is supplied.

MODERN ARITHMETIC THROUGH DISCOVERY offers—in effective, practical textbook form—an elementary school program that will prepare the pupils for their later study of mathematics and will communicate the excitement of ideas.



This Navy weather plane is flying through the eye of a hurricane. The pilot reported by radio that the speed of the wind at times reached 125 miles an hour.



The TV announcer is giving the position of the eye of the hurricane in two ways; (1) latitude and longitude, and (2) the approximate distance from the coast.

1000 FAMILIES HOMELESS
STORM DAMAGE
ESTIMATE: $200,000,000

Texas authorities today estimated that Hurricane Carla had destroyed at least 1,000 homes. Floodwaters raged through Corpus Christi streets. The damage from wind, flood, and floating debris may total $200 million or more.

State troopers and National guardsmen are alerted for round-the-clock duty as the disaster continued to spread.

This is part of the report of a New York newspaper on the Gulf Coast storms.

Numbers in News Reports

1

Three kinds of reports on a hurricane are illustrated on the opposite page.

1. The pilot reported that the speed of the wind reached 125 miles an hour. Do you think this was the exact top wind speed? Do you think the highest wind speed was closer to 125 miles an hour than to 100 or to 150 miles an hour?

The pilot used his instruments carefully. But the measurement he gave was an approximation of the actual speed of the wind. Whenever a measurement of any kind is made, it is considered approximate; it is as correct as the measuring instrument used will allow. No measuring instrument can give a measurement that may be considered exact.

2. The broadcaster used two ways of locating the eye of the hurricane: (1) *latitude 27°4′ N., longitude 95°5′ W.*, and (2) *120 miles off the coast.* Which way do you think locates the storm more precisely? Explain your answer.

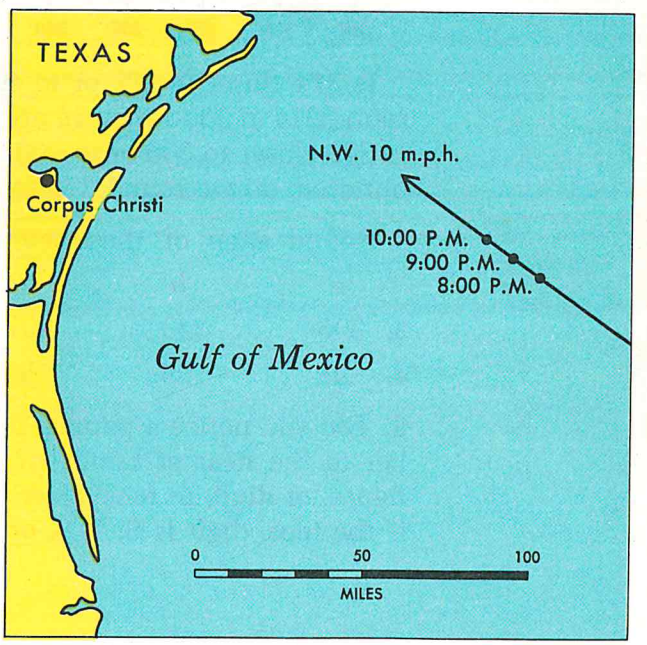
3. Do you think the number of houses lost was exactly 1,000, as the newspaper said? Do you think the amount of storm damage reported in the newspaper tells to the nearest dollar what the damage was? to the nearest hundred dollars? to the nearest thousand dollars?

4. In a weather station the position of a hurricane is shown on a map, from hour to hour, with different colored pins.

a. About how far did the storm move between 8:00 and 9:00 P.M.? between 9:00 and 10:00 P.M.?

b. Is the speed of this hurricane steady, or increasing?

c. About how far do you think the storm will travel between 10:00 P.M. and midnight?



2 Rounding to the Nearest Hundred

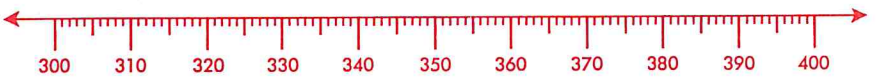
A trail called the "Long Path" once led along the Hudson River to a mountain near the Canadian border. Recently, this overgrown and almost forgotten trail was cleared and reopened by hiking clubs.

1. A newspaper reporter who wrote about the reopening of the Long Path knew that it is about 400 miles from New York City to Canada. So his story gave the length of the Long Path as 400 miles.

Which would you choose as the most likely interpretation of the reported distance—400 miles to the nearest hundred miles, 400 miles to the nearest ten miles, or 400 miles to the nearest mile?

When we give a measurement "to the nearest ten," "the nearest hundred," etc., we are **rounding** the number.

2. The part of the number line below shows certain numerals corresponding to points on the line. The space between 300 and 400 is divided into one hundred smaller spaces.



Is 314 closer to 300 or to 400 on the number line? Then we round 314 to 300 when we are rounding to the nearest hundred. Is 378 closer to 300 or to 400? Then in rounding to the nearest hundred, do we round 378 to 300 or to 400?

Round each of these numbers to the nearest hundred.

	a	b	c	d	e	f
3.	306	343	407	412	299	256
4.	98	125	559	940	1,435	2,899

5. Did you notice a pattern in the way you rounded each number to the nearest hundred? Which way do you round if the figure, or **digit,** in tens' place of the numeral is 0, 1, 2, 3, or 4? if the tens' digit is 6, 7, 8, or 9?

1. Find 283 on the number line. To round 283 to the nearest ten, would you round it to 280 or to 290? Use the number line to help you round each of the following numbers to the nearest ten.

a. 288 **b.** 281 **c.** 286 **d.** 289

2. Should 285 be rounded to 280 or to 290 when rounding to the nearest ten? By agreement, in such cases we "round up." So 285 would be rounded to 290.

3. Round each of these numbers to the nearest hundred.

a. 40,678 **b.** 27,659 **c.** 1,784 **d.** 961,534 **e.** 995

4. Round each of the numbers in Exercise **3** to the nearest thousand; to the nearest ten.

5. In a recent World Series, each share for members of the winning ball club was $7,189.12. Round the share to the nearest dollar; to the nearest ten dollars; to the nearest fifty dollars; to the nearest hundred dollars.

6. In the same Series, the share for the second-place club was $5,356.36. Round this share to the nearest dollar; to the nearest ten dollars; to the nearest fifty dollars.

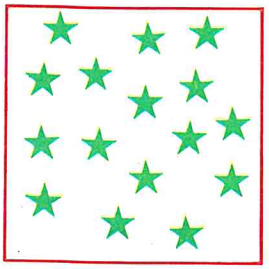
7. In Exercise **5,** would it be correct to say that each share for the winning club was about $7,000? The amount is rounded to the nearest thousand dollars. About what was each share for the losing club?

8. Round each of the following to the nearest hundred dollars.

a. $1,832 **b.** $3,478 **c.** $15,623 **d.** $11,154
e. $23,550 **f.** $18,748 **g.** $12,058 **h.** $12,041

9. Round each of the following to the nearest thousand dollars.

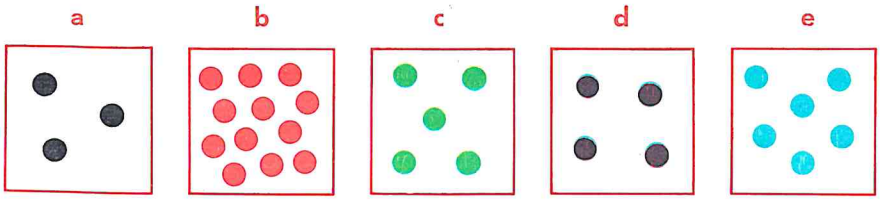
a. $1,976 **b.** $34,034 **c.** $5,085 **d.** $22,821 **e.** $50,398



You know that a numeral is a name for a number, or a way to express a number. For example, to express the number of objects in the set at the left, in our system, we write "16" or "sixteen" or "1 ten and 6 ones."

1. Our system is called a *decimal* numeral system because we group by tens in order to express numbers. (The word *decimal* comes from a Latin word meaning "ten.") How many symbols, or digits, do we need to write numerals in our system?

2. The **cardinal number** of a set tells how many objects there are in the set. What is the cardinal number of the set above? Write the numerals for the cardinal numbers of these sets.



3. Decide what order the numbers of the sets would be in if the smallest number were first and the largest number last. Write the numerals in that order.

4. Numbers that tell the order of things in a sequence, such as book pages, pupils in line, or days of the month, are called **ordinal numbers.** These numbers have names, too, such as *first, second, third,* etc. Write the ordinal names for the numbers in the sequence from 1 to 21.

5. The decimal numeral system is a *place-value* numeral system. The place of a digit in a numeral has meaning—a value is assigned to each place. The numeral 3,487 has four places. The 7 is in ones' place, the 8 in tens' place, the 4 in hundreds' place, and the 3 in thousands' place. The names of the places suggest the values given to the places. In 9,256, in what place is the 6? the 5? the 2? the 9?

The paid attendance at a baseball game was 48,976. This numeral tells the total number of tickets collected.

1. As the diagram below shows, each digit in the numeral 48,976 has a certain place value.

4	8	9	7	6
				6
			7	0
		9	0	0
	8	0	0	0
4	0	0	0	0
4	8	9	7	6

The 6 stands for 6 ones.
The 7 stands for 7 tens, or 70.
The 9 stands for 9 hundreds, or 900.
The 8 stands for 8 thousands, or 8,000.
The 4 stands for 4 ten thousands, or 40,000. Read the sum. Compare the sum with the numeral at the top.

2. Make a similar diagram to explain this numeral: 51,632.

3. Tell what each digit stands for in the following numerals.

a. 7,921 **b.** 34,583 **c.** 123,604 **d.** 4,394,540

4. To make the reading of large numerals easy, the digits are grouped in sets of three, called **periods,** beginning at the right. Commas are used to separate the periods. The words at the top of this chart name some of the periods.

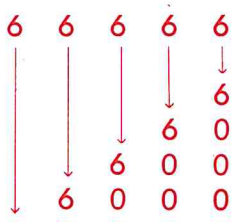
billions				millions				thousands				units		
	hundred	ten	one		hundred	ten	one		hundred	ten	one	hundreds	tens	ones
8	1	5	,	7	9	2	,	0	4	3	,	6	5	3

When we express the number above in words, we use the names of all the periods except the units' period: "eight hundred fifteen *billion,* seven hundred ninety-two *million,* forty-three *thousand,* six hundred fifty-three."

Read the following.

a. 56,783 **b.** 120,079 **c.** 33,576,854

d. 1,456,975,008 **e.** 492,116,320 **f.** 781,003,958,362



1. This diagram shows the value represented by each 6 in the numeral 66,666. What does the first 6 at the right mean? the second 6 from the right? The second 6 represents how many times the value of the first? The third 6 represents how many times the value of the second? of the first? The fourth 6 represents how many times the value of the third? the second? the first? Answer similar questions for the fifth 6.

Did you find that each place in the numeral 66,666 represents ten times the value of the place at its right? Do you think that each place in any numeral has ten times the value of the place at its right?

2. In the numeral 3,452, the 4 in hundreds' place means 400. What does the 3 in thousands' place mean? What does the 5 mean? the 2? Does the 4 in hundreds' place name the same number as 4×100? Do "3,000" and "$3 \times 1{,}000$" name the same number? Do "50" and "5×10" name the same number?

3. What is 10×10? Do "$4 \times 10 \times 10$" and "4×100" name the same number? Do "$4 \times 10 \times 10$" and "400" name the same number?

4. Beginning with 7, tell the value of each digit in 935,627.

The value of 7 in ones' place is: 7×1, or 7
The value of 2 in tens' place is: 2×10, or 20
The value of 6 in hundreds' place is: $6 \times 10 \times 10$, or ?
The value of 5 in thousands' place is: $5 \times 10 \times 10 \times 10$, or ?
The value of 3 in ten-thousands' place is: $3 \times 10 \times 10 \times 10 \times 10$, or ?
The value of 9 in hundred-thousands' place is: $9 \times 10 \times 10 \times 10 \times 10 \times 10$, or ?

Copy the numeral 935,627 on your paper. Write the value of each digit and find the sum of the values. What do you find?

5. Express the value of each digit as in Exercise **4.**

a. 8,967 **b.** 49,715 **c.** 571,839 **d.** 3,892,567

1. You have seen that 100 can be written as $1 \times 10 \times 10$, or simply as 10×10. What is a similar way to write 1,000? Even 1,000,000,000 can be written this way: $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$.

2. There is a short way to write products of 10's. We can write 10^2 for 10×10; 10^3 for $10 \times 10 \times 10$. What could you write for $10 \times 10 \times 10 \times 10$? What would 10^6 mean?

3. In an expression such as 10^4, the "4" is called an **exponent.** The "10" is called the **base.** What is the exponent in 10^2? in 10^3? in 10^6? in 10^8? What is the base in each of these?

▶ The exponent tells how many times the base is used in multiplying. We read an expression in exponent form as follows:

10^2 is read as "ten to the second power," or "ten squared."
10^3 is read as "ten to the third power," or "ten cubed."
10^4 is read as "ten to the fourth power."
10^7 is read as "ten to the seventh power."

4. In the table at the right, do you notice a pattern? Do you see a relation between the exponent and the number of zeros in the numeral at the right? What is the exponent in 10^4? How many zeros are there in 10,000? Using the pattern, write a numeral for each of the following.

10	$= 10$
10^2	$= 100$
10^3	$= 1{,}000$
10^4	$= 10{,}000$
10^5	$= 100{,}000$

a. 10^8 **b.** 10^6 **c.** 10^9 **d.** 10^7

5. You know that 4×100, or $4 \times 10 \times 10$, is 400. Then what is 4×10^2? How would you express 2×10^3 as a single numeral? Copy each of these number sentences, replacing N with the numeral that will make a true sentence.

a. $5 \times 10^2 = N$ **b.** $3 \times 10^3 = N$ **c.** $7 \times 10^3 = N$

6. In 800, how many zeros are there? Then what is the missing exponent in this sentence: $800 = 8 \times 10^{?}$.

In each of these sentences, replace N, using 10 as the base with the correct exponent.

a. $600 = 6 \times N$ **b.** $9{,}000 = 9 \times N$ **c.** $2{,}000 = 2 \times N$

1. Express each of the following as a single digit times 10 or a power of 10.

a. 70 **b.** 30,000 **c.** 300 **d.** 9,000 **e.** 10,000

2. Study the meaning of 3,856 as shown below.

$$\begin{aligned} 3{,}856 &= 3{,}000 + 800 + 50 + 6 \\ &= (3 \times 1{,}000) + (8 \times 100) + (5 \times 10) + (6 \times 1) \\ &= (3 \times 10^3) + (8 \times 10^2) + (5 \times 10) + (6 \times 1) \end{aligned}$$

Is the place value of each digit shown? Is the meaning of the number expressed by using 1, 10, and powers of 10?

3. Copy the following, replacing each ? with a power of 10, with 10, or with 1.

a. $743 = (7 \times ?) + (4 \times ?) + (3 \times ?)$

b. $2{,}639 = (2 \times ?) + (6 \times ?) + (3 \times ?) + (9 \times ?)$

Since 1 multiplied by any number is that number, we can write the digit only, omitting the "× 1." Do this in **c.**

c. $65{,}083 = (6 \times 10^4) + (5 \times 10^3) + (0 \times 10^2) + (8 \times 10) + ?$

4. Express each of the following as in Exercise **3c.**

a. 894 **b.** 1,328 **c.** 47,823 **d.** 1,056,794

5. Express each of the following as a power of 10: one million; one billion; one hundred thousand; ten million.

6. Exponents can be used with bases other than 10. For example, 3^3 means $3 \times 3 \times 3$. What does 4^2 mean? Are "4^2", "4×4," and "16" all names for the same number?

7. Shown at the right is a model example expressing a product of 4's as a power. Express each of the following as a power.

$4 \times 4 \times 4 \times 4 = 4^4$

a. 2×2 **b.** $9 \times 9 \times 9$ **c.** $10 \times 10 \times 10 \times 10 \times 10$

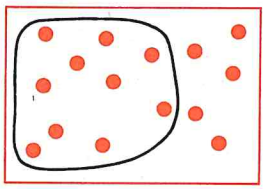
d. 13×13 **e.** $5 \times 5 \times 5 \times 5$ **f.** $11 \times 11 \times 11 \times 11 \times 11$

8. We can express a power as a product as shown at the right. Follow this form for these examples.

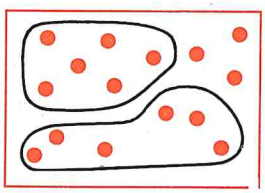
$5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

a. 5^3 **b.** 2^4 **c.** 3^7 **d.** 7^2 **e.** 2^7 **f.** 10^5

1. The diagram shows how we group objects in the decimal, or base ten, system. A ring has been drawn around ten of the dots. How many groups of ten dots are there? How many dots are left over? What numeral stands for 1 ten, 5 ones?



2. In a numeral system based on six, we would group objects in sixes, as shown in the diagram. Here we have the same number of dots as in the first diagram, but this time rings are drawn around groups of six dots.



How many groups of six dots are there? How many dots are left over? In a base six system, the numeral would be written 23_{six}. It would be read "two, three, base six."

3. Do the two diagrams contain the same number of dots? Then do "15" in the base ten system and "23_{six}" name the same number?

4. Draw diagrams similar to the diagrams in Exercises **1–2.** Each diagram should contain a set of dots as follows:

a. 16 **b.** 13 **c.** 25 **d.** 24 **e.** 35 **f.** 30

In each diagram, draw rings around as many groups of six dots as you can and notice how many extra dots there are. In Set **a,** for instance, there are 2 groups of six and 4 extra dots. For this diagram of 16 dots, the base six numeral is 24_{six}. The *six* means "base six." The numeral means 2 sixes and 4 ones. What is $(2 \times 6) + 4$?

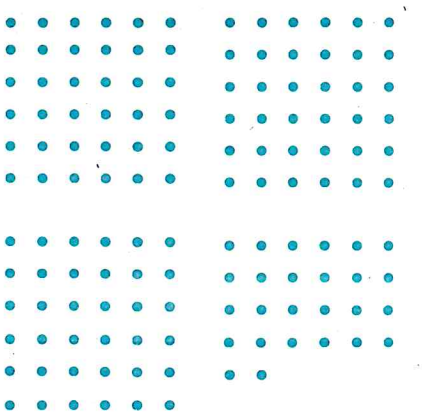
5. In the base ten system, we use the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. When we have 9 ones and 1 more, what numeral do we write in base ten? How many places does it have? What does the 1 mean? the 0?

6. In the base six system, we use the digits 1, 2, 3, 4, 5, 0. When we have 5 ones and 1 more in the base six system, what numeral do you think we write? How many places does it have? What does the 1 in 10_{six} mean? What does the 0 mean?

1. You know that the numeral 10_{six} means 1 six and 0 ones. What does 20_{six} mean? 30_{six}? 40_{six}? 50_{six}?

2. Does the numeral 6 in the base ten system name the same number as 10_{six}? What is the base ten numeral that names the same number as 20_{six}? 30_{six}? 40_{six}? 50_{six}?

3. How many dots are there in the diagram below? Express your answer using a base ten numeral.



4. In a base ten numeral, the first place at the right is in ones' place; the place at the left of ones' place is tens' place: the next place is hundreds' place (10^2). Does the numeral 134 mean $(1 \times 10^2) + (3 \times 10) + 4$?

5. In a base six numeral, the first place at the right is ones' place; but the place at the left of ones' place is sixes' place; and the next place is thirty-sixes' place (6^2).

In the set of dots, there are groups of thirty-six, groups of six, and some extra dots. How many groups of thirty-six are there? How many groups of six? How many extra dots?

Then, to express the number of dots as a base six numeral, we write: 342_{six}. The numeral 342_{six} is read, "three, four, two, base six," and it means $(3 \times 6^2) + (4 \times 6) + 2$.

6. Suppose there had been exactly 4 groups of thirty-six. Then could the number of dots be expressed as this base six numeral: 400_{six}? How would you read this numeral?

7. Write a base six numeral for each of the following. Remember to write the **subscript** "six" below and to the right of the numeral to show that the numeral is in base six.

a. 2 thirty-sixes, 3 sixes, 4 ones
b. 3 thirty-sixes, 5 sixes, 3 ones
c. 1 thirty-six, 2 sixes, 0 ones
d. 5 thirty-sixes, 0 sixes, 0 ones

8. To express 95 as a base six numeral, you need to know how many groups of thirty-six there are in 95; how many groups of six; and how many ones. To find this grouping, you can divide.

How many 36's in 95? (2, remainder 23) Write "2" in thirty-sixes' place as shown.

How many 6's in 23? (3, remainder 5) Write "3" in sixes' place.

How many ones are left? (5) Write "5" in ones' place.

Add. Then 95, expressed as a base six numeral, is 235_{six}.

Are 95 and 235_{six} names for the same number?

thirty-sixes	sixes	ones
2	0	0
	3	0
		5
2	3	5_{six}

9. To express 49 as a base six numeral, you can use the same procedure.

First, fill thirty-sixes' place.
(49 ÷ 36 = ?, remainder ?)

Next fill sixes' place.
(13 ÷ 6 = ?, remainder ?)

How many ones will there be in ones' place?

Add, and express the sum as a base six numeral. Remember to use the subscript "six."

thirty-sixes	sixes	ones
?	0	0
	?	0
		?

10. Express each of the following base 10 numerals as a base six numeral.

a. 45 **b.** 77 **c.** 118 **d.** 188

11. If you express 35 as a base six numeral, will there be only two places in the numeral? Why?

Express 35 as a base six numeral; then express 36 as a base six numeral.

If you are counting in base six, is 100_{six} the next numeral after 55_{six}?

12. What is 72 expressed as a base six numeral? 108? 180?

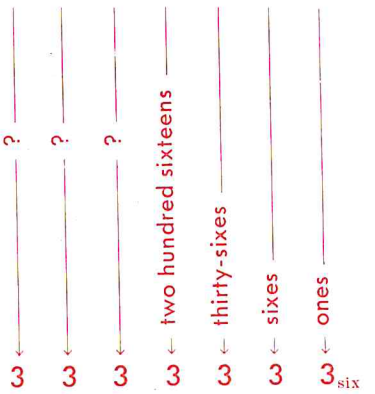
13. Express 215 as a base six numeral.

1. In the base six system, we express numbers using one, six, and powers of six. Copy and complete this table of powers of six. It will be useful in working with base six numerals.

$6^2 = 6 \times 6$, or 36
$6^3 = 6 \times 6 \times 6$, or 216
$6^4 = ?$ or ?
$6^5 = ?$ or ?
$6^6 = ?$ or ?

2. The diagram at the right shows place value in the base six system. In this system, each place has 6 times the value of the place at its right.

Copy and complete the chart to seven places.



3. Use the table and chart to help you change 1,761 to a base six numeral.

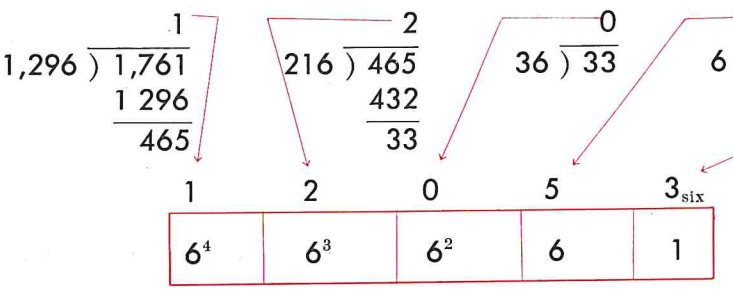
What is the highest power of six that can be used to divide 1,761? Begin to divide, using this number.

a Divide 1,761 by 1,296 (6^4).

b Divide 465 by 216 (6^3).

c Divide 33 by 36 (6^2).

d Divide 33 by 6.



4. Write the following decimal numerals as base six numerals. Be sure to divide first by the highest power of six you can. Then decide how many places the base six numeral will have. Put a digit in each place; sometimes it will be zero.

a. 493 **b.** 986 **c.** 1,925 **d.** 2,682

1. To change base ten numerals to base six, you divided by powers of six. How do you think you could change a base six numeral back to base ten?

2. Change this numeral to a base ten numeral by multiplying the value of each place by the digit in that place, as shown. When you add the products, the sum is the equivalent base ten numeral.

$5\ 3\ 0\ 4_{six}$

	Base Ten
$(5 \times 6^3) =$	1,080
$(3 \times 6^2) =$	108
$(0 \times 6) =$	0
$(4 \times 1) =$	4
	1,192

Check by converting 1,192 to a base six numeral.

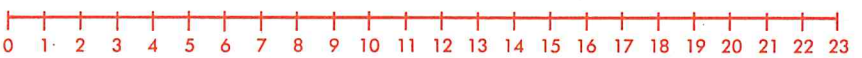
Are 5304_{six} and 1,192 two names for the same number? One is a base ? name, and the other is a base ? name.

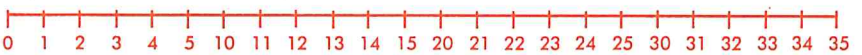
3. Express each of the following as a base ten numeral.

a. 45_{six} **b.** 324_{six} **c.** 1043_{six} **d.** 24034_{six}

4.a. Base ten numerals use ten digits. What are they?
b. Base six numerals use only six digits. What are they?

5. Which number line below has base ten numerals for the points? Which has base six numerals?





a. Which number is greater, 7 or 10_{six}? 15_{six} or 12? 24_{six} or 16?
b. Count from 1_{six} to 101_{six}.

6. You know that with base ten numerals $2 + 4 = 4 + 2$. You also know that $2 \times 3 = 3 \times 2$. The order in which a pair of numbers is added or multiplied does not change the result.

Is this true with base six numerals? Try it.

a. $2_{six} + 4_{six} = \underline{?}$; $4_{six} + 2_{six} = \underline{?}$
b. $2_{six} \times 3_{six} = \underline{?}$; $3_{six} \times 2_{six} = \underline{?}$

The Roman numeral system is a "group additive system" rather than a place-value system. For example, the Roman numeral DCCC stands for 500 + 100 + 100 + 100. The three C's are grouped together and then added to the D.

The table at the right shows Roman numerals and the corresponding Hindu-Arabic numerals which we use.

Roman	Hindu-Arabic
I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

To write other Roman numerals, we follow these rules:

1. When a numeral is followed by a numeral for a smaller number, the smaller number is added to the larger one. For example: XV = 10 + 5; MC = 1,000 + 100.
2. When a numeral comes before a numeral that represents a larger number, the smaller number is subtracted from the larger number. For example: XL = 50 − 10; IX = 10 − 1. (Only I, X, and C are used in this way.)
3. In a numeral such as MCD, the subtractive principle is applied first. We interpret CD as 400, then add it to 1,000.

There are many ways of naming the same number, as you know. Express the following numbers, using Roman numerals.

	a	b	c	d	e
1.	9	3	26	44	38
2.	32	87	90	57	105
3.	116	69	135	3,447	1,963

Express the following numbers, using Hindu-Arabic numerals.

4.	XXV	LXXXVI	CXV	CX	LII
5.	XL	MCDXCII	CDV	XC	MCMLXV
6.	MD	MDCXX	MCM	LXX	DC

7. What differences have you noticed between the place-value systems you have worked with and the Roman system? In the Roman system, does the value represented by a symbol depend upon the place the symbol occupies? Did the Romans have a symbol for zero? These are the two important differences.

1. Addition and subtraction are ways of thinking about pairs of numbers.

a. Suppose we start with this pair of numbers: 5, 7. If we think, "5 and 7" and "12" are both names for the same number, are we adding, or subtracting?

▶We can express this idea as a number sentence: $5 + 7 = 12$. Can we also express this idea as $7 + 5 = 12$? Why?

▶Suppose we have added 7 to 5: $5 + 7 = 12$. Now let us "undo" what we have just done; that is, subtract from the sum, 12, the 7 we added: $12 - 7 = 5$. Do you see that subtracting is the **inverse** (opposite) of adding?

b. Write four related number sentences, using this pair of numbers and their sum: 8, 9.

2. To see more clearly that subtracting is the inverse of adding, complete the following exercise. The square-shaped frame should be replaced in each sentence by the numeral that makes the sentence true.

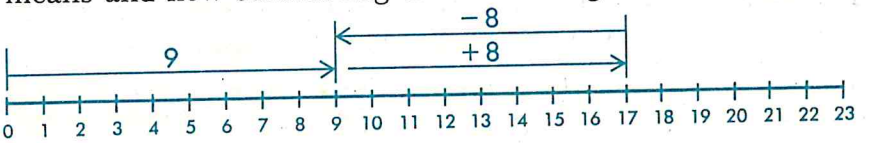
a. $(6 + 9) - 9 = \square$

b. $(65 + 23) - 23 = \square$

c. $(752 + 146) - 146 = \square$

d. $(789 + 210) - 210 = \square$

3. The number line also helps to illustrate what addition means and how subtracting undoes adding.



a. To show the sum of 9 and 8, start at the point labeled 0 and move 9 spaces to the right. To add 8, move 8 more spaces to the right. What is the numeral at the point where the second move ended? Then $9 + 8 = 17$.

b. You have just added 8 to 9; now take 8 from 17 by moving 8 spaces to the left. At what numeral did the last move end? Did you undo your addition move?

1. When you added 8 to 9 on the number line in Exercise **3,** page 15, you saw that the sum of 8 and 9 is 17. But "17" is 1 ten and 7 ones. So you **regrouped** 8 and 9 into 10 and 7.

How would you regroup each of these number pairs?

a. $3 + 9 = 10 + ?$ **b.** $7 + 6 = 10 + ?$ **c.** $5 + 9 = 10 + ?$

2. You can add or subtract by thinking of the numbers in each pair below as so many tens and so many ones. Add in this way in Exercise **a;** in Exercise **b,** undo the addition.

a

25 is ? tens, ? ones
+43 is ? tens, ? ones
? tens, ? ones, or ?

b

68 is ? tens, ? ones
−43 is ? tens, ? ones
? tens, ? ones, or ?

3. Sometimes we have to regroup in adding or subtracting. Copy and complete the addition example and then the subtraction example. The arrows show where to regroup.

58 is 5 tens, 8 ones
+29 is 2 tens, 9 ones
7 tens, 17 ones ⟶ 7 tens + ? ten + ? ones, or ?

87 is 8 tens, 7 ones ⟶ 7 tens, ? ones
−29 is 2 tens, 9 ones 2 tens, ? ones
? tens, ? ones, or ?

4. Regroup as you did in Exercise **3** as you work these addition examples and the related subtraction examples.

a.			**b.**			**c.**		
	96	143		168	594		387	615
	+47	−47		+426	−426		+228	−228

5. Can you do the regrouping mentally? Copy the examples and follow the thinking as you work them.

	Example	*Thinking*		*Example*	*Thinking*
a.	529	$9 + 2 = ?$	**b.**	1,101	$11 - 2 = ?$
	+572	$1 + 2 + 7 = ?$		−572	$9 - 7 = ?$
		$1 + 5 + 5 = ?$			$10 - 5 = ?$

1. You know that 17 + 8 can be regrouped as 10 + 7 + 8, or 10 + 15. What is the sum of 10 and 15?

You also know that 7 + 8 = 15. Then 17 + 8 is like 7 + 8, except that the sum must be in the twenties: 25.

Say the sum for the first example, and use that as the pattern for saying the other sums quickly.

a.

8	18	28	38	48
+6	+6	+6	+6	+6

b.

7	17	27	37	47
+5	+5	+5	+5	+5

c.

4	24	54	74	94
+9	+9	+9	+9	+9

d.

9	89	59	129	119
+2	+2	+2	+2	+2

Each example below is like an addition fact you know. Use this fact to find the sum. Write the answers.

	a	b	c	d	e	f	g	h
2.	56	82	19	49	33	17	48	49
	+4	+8	+6	+9	+8	+7	+5	+1
3.	35	78	26	54	76	87	59	37
	+8	+9	+8	+8	+5	+4	+2	+3

4. Fill in the missing numerals in these sentences. Do the operation inside the parentheses first.

$(7 + 8) + 9 = \square$ $\quad 7 + (8 + 9) = \square$

▶Compare the sums. Are they the same? Does it matter which two neighboring numerals are paired first?

Now change the order of the numerals and add.

$(9 + 8) + 7 = \square$ $\quad 7 + (9 + 8) = \square$

$8 + (7 + 9) = \square$ $\quad (9 + 7) + 8 = \square$

▶Again, compare the sums. Are they the same? Does the order in which the numerals are paired affect the sum?

5. Copy, and fill in the missing numerals. Can you check your work by adding in the opposite direction? Try it.

a. $6 + 8 + 7 = \square$

b. $9 + 5 + 9 = \square$

c. $8 + 8 + 2 + 3 = \square$

d. $6 + 5 + 9 + 4 = \square$

e. $5 + 6 + 6 + 4 = \square$

f. $1 + 9 + 3 + 7 = \square$

Copy, add, and check.

	a	b	c	d	e
1.	57 97 55	30 38 68	79 45 78	273 780 429	976 43 88
2.	3,685 4,997 4,085	6,000 307 995	8,953 939 39	7,626 7,250 5,478	278 5,650 6,208
3.	637 59 704	6,545 6,636 25	8,000 3,768 4,469 903	5,269 874 7,210 7,025	3,620 6,100 2,005 4,073
4.	$3.65 2.80 4.48 5.25	$3.00 .78 1.07 1.20	$3.75 .29 2.50 .05	$1.84 7.40 .75 .07	$86.75 17.33 3.46 .12

Copy, subtract, and check.

5.	636 257	859 365	877 248	808 253	507 125
6.	7,802 988	8,720 6,892	4,752 3,523	3,581 2,611	3,624 2,336
7.	900 548	614 367	7,000 2,484	9,098 689	3,005 2,346
8.	$84.22 40.37	$50.45 28.96	$90.00 64.13	$59.30 50.41	$123.65 89.75
9.	59,146 43,899	72,844 6,895	53,969 39,170	38,275 15,287	68,444 24,468

1. As you know, subtracting undoes adding. So, if you add a number to another number and take it away again, the original number remains. Also, if you subtract a number from a given number and then add it, the original number remains.

Decide what numeral you would use to replace each frame.

a. $(25 + 6) - 6 = \square$ **b.** $(25 - 6) + 6 = \square$

2. In these sentences, what can you put in place of the frames?

a. $\square - 6 = 25$ **b.** $\square + 6 = 25$

In **a**, if you replace the frame with $(25 + 6)$, do you get a true sentence? If you replace the frame with $(25 - 6)$ in **b**, do you get a true sentence?

3. Use the idea suggested in Exercise **2** to substitute for the frame in this sentence.

$$8 + \square = 25$$

4. You know that $(25 + 6)$ and 31 are two names for the same number. What single numeral can you use instead of $(25 + 6)$ to make this a true sentence: $\square - 6 = 25$?

What single numeral can you use instead of $(25 - 6)$ to make this sentence true: $\square + 6 = 25$?

5. In each of the following exercises, use the "doing and undoing" idea to find a pair of numerals that makes the sentence true. Then find the single numeral that can replace the pair of numerals.

a. $\square - 12 = 32$
b. $\square + 35 = 50$
c. $16 + \square = 40$
d. $\square - 45 = 30$
e. $\square - 28 = 46$
f. $31 + \square = 73$
g. $48 + \square = 125$
h. $\square - 67 = 121$
i. $\square - 517 = 68$
j. $231 + \square = 802$
k. $112 + \square = 204$
l. $\square - 38 = 162$
m. $\square - 842 = 524$
n. $220 + \square = 440$
o. $\square + 355 = 640$
p. $\square - 402 = 398$

In solving problems, number sentences are often helpful to show what numbers are given and what we want to find.

1. There were 27 children in the sixth-grade class, but one day only 19 were present. How many were absent?

A number sentence for this problem would be: "Twenty-seven minus nineteen equals what number?" Write the sentence, using digits and N instead of words. Then find the answer to the question in the problem.

Read each problem below. For the first five problems, write number sentences to help you solve them, but solve Problem **7** without writing a number sentence.

2. Students were selected from the three sixth-grade classes to sing in the chorus at the school concert. There were 19 children from Miss Smith's room, 17 from Mrs. Moon's, and 13 from Miss Nelson's. How many students were selected?

3. Miss Smith's class had $20 to spend for parties during the school year. After buying refreshments for their first party, they had $14.05 left. How much did they spend?

4. The principal's office announced that 83 boys and girls were taking instrumental music this year. This is 17 more than last year. How many took instrumental music last year?

5. One day the girls gathered leaves for a science project. Ann found 23 different leaves, Susan found 31, and Marie found 29. How many leaves did the girls gather?

6. During one week, just after the new models appeared, the production of three makes of cars was: Chevrolet, 32,264; Ford, 28,788; Plymouth, 5,832. How many cars of these three makes were produced that week?

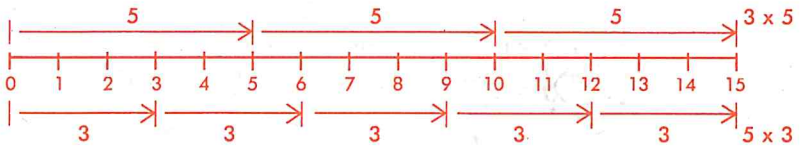
7. Each day Lynn's father gave her all of his nickels and pennies. Monday she got 3 nickels and 3 pennies; Tuesday, 17¢; Wednesday, 6 nickels and 9 pennies; Thursday, nothing. On Friday her father gave her 3 nickels and 7 pennies. How much money did Lynn get in all?

1. What is the product of 4 and 7? As you may know, we say that 4 and 7 are **factors** of the product 28. What is the product of 2 and 14? Then are 2 and 14 factors of 28?

$4 \times 7 = \square$

2. What is the product of 5 and 3? Then are 5 and 3 factors of 15?

3. Multiplication can be thought of as repeated addition. For example, 3×5 means $5 + 5 + 5$. This is shown on the number line below. As you see, the arrangement illustrating 5×3 looks quite different from the arrangement illustrating 3×5. Do "3×5" and "5×3" name the same number?



4. Draw a number line and illustrate 4×6; 6×4.

5. One set of letters at the right has the cardinal number 4; the other set has the cardinal number 3. We can think of multiplication as a pairing of each member of the left-hand set with each member of the right-hand set, as shown below. The product is the number of pairs. As you can see, A from the first set is paired with e, with f, and with g of the second set; B from the first set is also paired with each member of the second set, and so on. How many columns of pairs are there? How many rows? How many pairs are there? Then we have constructed a set to represent 4×3.

A	e
B	f
C	g
D	

Ae	Be	Ce	De
Af	Bf	Cf	Df
Ag	Bg	Cg	Dg

Here is a set constructed by pairing each member of the right-hand set with each member of the left-hand set. How many columns of pairs are there? How many rows? How many pairs are there? Then this set represents 3×4.

eA	fA	gA
eB	fB	gB
eC	fC	gC
eD	fD	gD

1. You know that $4 \times 7 = 28$ and $7 \times 4 = 28$. How many 7's in 28? How many 4's in 28? Does dividing undo what has been done by multiplying? We can write these related sentences.

$4 \times 7 = 28$ $\quad 28 \div 7 = 4$ $\quad 7 \times 4 = 28$ $\quad 28 \div 4 = 7$

2. What is 6×91? What is 91×6? Is the product the same? What is $546 \div 91$? Does $546 \div 91 = 6$ undo $6 \times 91 = 546$?

▶ We say that dividing is the inverse of multiplying.

3. Find the products of the following pairs of numbers. In each case, write the related multiplication sentence and the two division sentences which use the same three numerals.

a. 8×31 **b.** 3×92 **c.** 2×64 **d.** 4×72

4. In each sentence below, find the numeral that makes the sentence true. Do the operation inside the parentheses first.

$(3 \times 5) \times 4 = \square$ $\quad 3 \times (5 \times 4) = \square$

▶ Compare the products. Are they the same? Does it matter which two neighboring numerals are paired first?

Now change the order of the numerals and multiply.

$5 \times (3 \times 4) = \square$ $\quad 4 \times (5 \times 3) = \square$

▶ Again, compare the products. Are they the same? Does the order in which we pair the factors affect the product?

5. What does $4 \times 9 \times 7$ equal? Did you find the product of the three numbers all at once, or did you first find the product of a pair of numbers, and then multiply their product by the third number?

At each step, do we use a pair of numbers in multiplying and get a single number as the product?

6. Below are a multiplication example and a division example with the names of the parts of each.

multiplicand ⟶ 56 ⟶ quotient ⟶ 56
multiplier ⟶ $\times$ 7 ⟶ divisor ⟶ $7\overline{)392}$
product ⟶ 392 ⟶ dividend ⟶ ⤴

Copy these two number sentences and name the parts of each.

a. $8 \times 92 = 736$ **b.** $736 \div 8 = 92$

1. Look at this number sentence. What is another name for $20 + 5$? What is 4×25? Then what numeral will make the sentence true?

$4 \times (20 + 5) = \square$

2. Now study this number sentence. What is 4×20? 4×5? What is the sum of 80 and 20? Then what numeral makes the sentence true?

$(4 \times 20) + (4 \times 5) = \square$

3. Is the following sentence true?

$$4 \times (20 + 5) = (4 \times 20) + (4 \times 5)$$

4. Is "15 + 10" another name for 25? Do you think that $4 \times 25 = (4 \times 15) + (4 \times 10)$? Try it and see.

5. Regroup 25 as the sum of any three numbers. Now multiply each of the numbers by 4 and add the products. Do you get 100? Does it seem that in multiplying we can think of a number as the sum of several numbers, then multiply each of these and add the products?

6. In this example, the product was found by multiplying first the ones and then the tens. What is 7×4? 7×20? What is the sum of 28 and 140? Is this sum the same number as the product in the example? Do you see that in doing this example we thought of 24 as $20 + 4$ and multiplied the ones first?

```
 24
  7
 28 ← 7 × 4
140 ← 7 × 20
168 ← 7 × 24
```

7. Let us see if we can use our discovery to find what 5×346 is. Can we think of 346 as $300 + 40 + 6$? Now study this example and see if you can answer these questions.

How did we get the 30? the 200? the 1500? the 1,730? Then what is 5×346?

```
  346
    5
   30
  200
1 500
1,730
```

8. Use the method of Exercises **6** and **7** to find the products for the following examples.

a.	**b.**	**c.**	**d.**	**e.**	**f.**
69	74	759	468	356	937
5	8	4	7	6	7

1. In this example, the product is being found by the method you used in Exercises **6–8** on page 23. How did we get the 28? the 70? Copy and complete the example. What is the correct numeral to replace the question marks? What is the product for the example?

```
 614
   7
  28
  70
????
```

At the right, the same example has been done by using a short form you may already know. In this form, you do more of the work mentally. You find that 7×4 is 28, and you know that 28 is 2 tens and 8 ones. You write the 8 and remember the 2 tens. Then you multiply again: 7×1 ten $=$? tens. Why do you write a 9 in tens' place? Finally, you multiply again: 7×6 hundreds $=$? hundreds.

```
 614
   7
4,298
```

2. What is 3×0? 11×0? 0×11? 234×0? the product of any number and 0?

3. In Example **a**, the short form has been used again. See if you can explain how the product was obtained. How did we get the 8 in ones' place? Next, did we multiply tens: 4×0 tens $=$? tens? Finally, did we multiply again: 4×9 hundreds $=$? hundreds?

a	b
902	807
4	5
3,608	

Using the short form, find the product for Example **b**. Is it 4,005? If not, what is it?

Find the products, using the short form.

	a	b	c	d	e
4.	219 × 5	802 × 4	742 × 3	503 × 5	418 × 4
5.	251 × 6	400 × 9	609 × 3	618 × 4	967 × 2

	a	b	c
6.	$8 \times 118 = N$	$3 \times 899 = N$	$9 \times 1{,}002 = N$
7.	$7 \times 7{,}240 = N$	$2 \times 8{,}976 = N$	$9 \times 4{,}506 = N$

1. What does 10×3 equal? 10×5? 10×9? 10×13? 10×27? Do you see a pattern you can use when you multiply a number by 10? Then what is 10×31? 10×35? 10×40?

2. You know that 20 is 2 tens. By thinking of 20 as 2 tens, can you find what 20×43 is? Try it: 2 tens $\times$ 43 = ? tens. Write 86 tens as one numeral.

$$\begin{array}{r} 43 \\ \underline{20} \end{array}$$

3. By thinking of 30 as 3 tens, can you find what 30×54 is?

$$\begin{array}{r} 54 \\ \underline{30} \end{array}$$

3 tens $\times$ 54 = ? tens

Is the product for the example 162, 16,200, or 1,620?

Find the products for the following examples.

	a	b	c	d	e
4.	26	74	45	27	99
	20	40	30	60	50
5.	86	95	88	97	90
	70	20	90	80	70

6. What does 100×4 equal? 100×6? 100×9? 100×12? 100×23? Do you see a pattern you can use when you multiply a number by 100? Then tell what 100×56 equals; 100×84; 100×95; 100×107.

7. By thinking of 300 as 3 hundreds, can you find the product for this example? Try it.

$$\begin{array}{r} 132 \\ \underline{300} \end{array}$$

3 hundreds $\times$ 132 = ? hundreds

How do you write 396 hundreds as one numeral?

Using the method of Exercise **7**, find the product for each of these examples.

	a	b	c	d	e
8.	72	312	662	834	1,021
	200	400	500	300	900
9.	738	1,324	2,645	6,523	9,001
	600	500	700	300	800

1.a. What is another name for $10 + 2$?
b. What is 10×25? 2×25? What is the sum of these two products? Study the number sentence below. What numeral can you use to replace the frame so that the sentence will be true?

$$(10 \times 25) + (2 \times 25) = \square \times 25$$

2. At the right is the multiplication example in a form that may be familiar to you. How did we get the 50? the 250? the 300? Do you see that we found the product by thinking of the multiplier as $10 + 2$? Which was used first as a multiplier—the ones or the tens?

$$\begin{array}{r} 25 \\ \underline{12} \\ 50 \\ 250 \\ \hline 300 \end{array}$$

partial products

As you may know, the products 50 and 250 are called **partial products** because each represents a *part* of the final product.

3. Does "25×12" name the same number as "12×25"? Then can you check your answer for Exercise **2** by using 25 as the multiplier? What number is the multiplicand? Copy and complete the check.

$$\begin{array}{r} 12 \\ \underline{25} \end{array}$$

Copy and multiply. Check by the method described in Exercise **3**.

	a	b	c	d	e
4.	$\begin{array}{r} 43 \\ \underline{95} \end{array}$	$\begin{array}{r} 96 \\ \underline{57} \end{array}$	$\begin{array}{r} 58 \\ \underline{29} \end{array}$	$\begin{array}{r} 46 \\ \underline{53} \end{array}$	$\begin{array}{r} 62 \\ \underline{86} \end{array}$
5.	$\begin{array}{r} 73 \\ \underline{67} \end{array}$	$\begin{array}{r} 94 \\ \underline{39} \end{array}$	$\begin{array}{r} 86 \\ \underline{62} \end{array}$	$\begin{array}{r} 57 \\ \underline{84} \end{array}$	$\begin{array}{r} 98 \\ \underline{78} \end{array}$

6.a. 38×48 **b.** 74×58 **c.** 46×68 **d.** 59×65

7. In the box at the right is a "mystery" multiplication example.

Can you discover the correct numerals from the clues that are given?

$$\begin{array}{r} ?? \\ \underline{14} \\ ??? \\ 350 \\ \hline ??? \end{array}$$

1. In Example **a**, the method shown is the method you have been using. In **b**, the form has been shortened. Is the product of 74 and 8 ones shown in the same way in both **a** and **b**? Now compare the way the product of 74 and 2 tens is shown in **a** and **b**. How do they differ? When you multiply by tens, does the product always have a 0 in ones' place? In the short form, we omit the 0. What does the "148" mean?

a	**b**
74	74
28	28
592	592
1 480	1 48
2,072	2,072

2. Copy and multiply. Check by multiplying again.

a.	**b.**	**c.**	**d.**	**e.**	**f.**
25	84	73	64	59	85
78	47	14	37	63	18

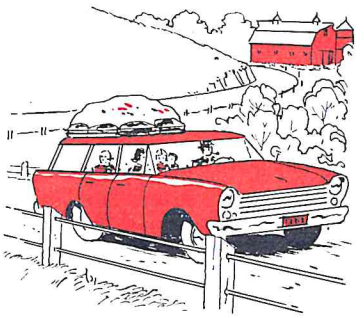
3. A box of typing paper costs \$3.85. How much will 12 boxes of this typing paper cost? To find the answer, can you add? What is another, quicker way to find the answer?

Can you think of \$3.85 as 385 cents? Then, to multiply in an example involving dollars and cents, can you multiply just as you do in any other example? Two steps in this multiplication example have been done for you. Copy and complete the example. To give an answer to the problem, of course, you must put a dollar sign and a point in the product. What is the cost of the 12 boxes of typing paper?

\$3.85
12
770
385

Copy and multiply. Remember to place a dollar sign and a point in your answer whenever necessary.

	a	**b**	**c**	**d**	**e**
4.	\$2.87	\$4.78	\$14.80	\$31.74	\$64.19
	9	8	6	3	7
5.	\$8.36	307	580	809	\$1.49
	4	87	59	47	29



1. The distance between the Lanes' home and their summer place in Canada is 674 miles. One summer they made two round trips. How many miles did they travel in making these trips?

Before you do any work on paper, make sure you understand the problem. What is the distance between the Lanes' home and their summer place? How many times do they drive this distance in a round trip? How many round trips did they make? Which of these number sentences describes the problem?

a. $2 \times 674 = N$ **b.** $2 \times (2 \times 674) = N$

Now must you find the numeral that will make the sentence you have chosen a true sentence? Do this. How many miles did the Lanes travel?

For each of the problems below, make sure you understand the facts that are given and the question that is asked. Write a number sentence if this will help you solve the problem. Then find the answer to the question asked.

2. For hanging a special wallpaper, a paper hanger charges \$5.25 per roll. The number of rolls required for a certain job is 32. What is the total charge for the job?

3. A homeowner paid \$90 for a specially made front door. The carpenter charged \$18 for installing the new door. Hardware cost \$13, and the man who weather-stripped the door charged \$24. What was the total cost of the door?

4. At a hotel, Mr. Lang was charged \$9.50 per night for his room. He stayed three nights. What was his room charge?

5. A young businessman bought two suits at a sale. One suit cost \$39.95, and the other cost \$25.99. How much more did the one suit cost than the other?

1. If a field produces about 60 bushels of corn per acre, about how many bushels would 40 acres produce?

The multiplication example that must be done to answer the question is shown at the right. What is the product of any number and zero? What numeral will you write in ones' place in the product? Will the numeral in tens' place in the product also be zero? What numeral will appear in hundreds' place? thousands' place? Check your answers by copying the example and multiplying.

$$\begin{array}{r} 60 \\ \times\ 40 \\ \hline \end{array}$$

Copy and multiply.

	a	b	c	d	e
2.	58 × 60	75 × 30	70 × 60	40 × 80	48 × 70
3.	\$.69 × 70	\$.49 × 50	\$.97 × 90	\$.40 × 40	\$.06 × 30
4.	700 × 94	807 × 62	149 × 29	906 × 58	623 × 49

5.a. What is 3×117? 50×117? 200×117?

b. What is 2×629? 40×629? 300×629?

6. At the right are two multiplication examples with three-place multipliers. The short form has been used, just as it was used with two-place multipliers.

$$\begin{array}{rr} \textbf{a.}\ 117 & \textbf{b.}\ 629 \\ \times\ 253 & \times\ 342 \\ \hline 351 & 1258 \\ 585 & 2516 \\ 234 & 1887 \\ \hline 29{,}601 & 215{,}118 \end{array}$$

In **a**, what does the "585" mean? What does the "234" mean? You can use your work for **5a** as a guide.

In **b**, what does the "2516" mean? the "1887"?

Do you see why the last figure of the third partial product in each case is in hundreds' place?

Copy and multiply.

	a	b	c	d	e
7.	468 × 316	325 × 248	648 × 725	372 × 165	586 × 957

1. Look at this number sentence. What number is named by "84 ÷ 2"? Then what numeral will make the sentence true? $84 \div 2 = \triangle$

2. Is "80 + 4" another name for 84? Study this number sentence. $(80 \div 2) + (4 \div 2) = \triangle$ What is 80 ÷ 2? 4 ÷ 2? Then what numeral will make the sentence true?

3. Is the following sentence true: $84 \div 2 = (80 \div 2) + (4 \div 2)$?

4. Is "60 + 24" another name for 84? Do you think that $84 \div 2 = (60 \div 2) + (24 \div 2)$? Try it and see.

5. Suppose we regroup 84 as 40 + 30 + 14. Do you think that $84 \div 2 = (40 \div 2) + (30 \div 2) + (14 \div 2)$? Try it as you did in Exercise **4.**

6. Look at this division sentence. If we think of the dividend as 700 + 63, will we have chosen numbers that are easily divided by 7? What is 700 ÷ 7? 63 ÷ 7? What is 100 + 9? Does 763 ÷ 7 equal 109? Check by multiplying. $763 \div 7 = N$

7. In this example, if we think of the dividend as 600 + 30 + 12, will we have chosen numbers that are easily divided by 3? Find the quotient in this way. Check by multiplying. $3\overline{)642}$

Do you see that in division we can think of a dividend as the sum of several numbers and then divide each of these numbers by the divisor? Then, to obtain the quotient for the example, what do we do?

In the following exercises, use the method of Exercises **6** and **7** to find the quotients.

	a	b	c	d
8.	$639 \div 3 = N$	$168 \div 2 = \square$	$264 \div 4 = \triangle$	$175 \div 5 = N$
9.	$990 \div 9 = \square$	$808 \div 8 = N$	$486 \div 6 = \square$	$672 \div 3 = \triangle$
10.	$7\overline{)4{,}907}$	$8\overline{)648}$	$6\overline{)4{,}818}$	$9\overline{)5{,}409}$

1. Usually, in division examples, you do not have to decide how to regroup the entire dividend beforehand. You just make use of place value, and the dividend "regroups itself." This is shown in the work on this page. See if you can discover how this happens as we use a special form for a division example.

a. We begin with hundreds in the dividend. Are there as many as 100 fives in 670? as many as 200 fives? Do we know that the quotient is more than 100 but not as much as 200? Then we write 100 at the side. What is 100×5? We subtract the 500 from 670 to see what there is left to divide.

```
   ???
5)670
  500 | 100   (a)
  170
  150 |  30   (b)
   20
   20 |   4   (c)
      | 134
```

b. Now we think about *tens* rather than hundreds. Are there as many as 20 fives in 170? as many as 30 fives? as many as 40 fives? Then we write 30 at the side. What is 30×5? We subtract 150 from 170 to see what there is left to divide.

c. How many fives are there in 20? We write 4 at the side. What is 4×5? Subtract 20 from 20. Is there anything left to divide?

What is $100 + 30 + 4$? Do you think 134 is the quotient for the example? We write the quotient above the dividend, where the question marks are. Check the answer by multiplying: $134 \times 5 = N$.

2. Look over the work we have done. Do you see we have found that $(100 \times 5) + (30 \times 5) + (4 \times 5) = 670$? In other words, 670 has been regrouped as $500 + ? + ?$.

Using a form similar to that shown in Exercise **1**, find the quotients.

	a	b	c
3.	$5\overline{)890}$	$4\overline{)924}$	$3\overline{)804}$
4.	$6\overline{)984}$	$8\overline{)896}$	$7\overline{)917}$

5. Are there as many as 100 nines in 658? Then here we begin by thinking about tens. Are there as many as 60 nines in 658? as many as 70 nines? as many as 80 nines? What is 70×9? We subtract the 630 from 658 to see what there is left to divide.

```
  ??R?
9)658
 630 | 70
  28
  27 |  3
   ? |  ?
```

How many nines are there in 28? What is 3×9? We subtract the 27 from the 28. What is $28 - 27$? What do we call the 1 we have left?

What is $70 + 3$? In an example such as this, we say we have a quotient of 73 with a remainder of 1. We write our answer as **73R1**.

6. In a division example such as the one in Exercise **5**, can the results be checked just by multiplying: $73 \times 9 = ?$ What must you do next? Do the results check?

7. For the example at the right, suppose we use a $\square$ to represent the quotient and a $\triangle$ to represent the remainder. Can your results be checked, using this sentence: $(\square \times 7) + \triangle = 220$? Do the example, and use the sentence to check.

$7\overline{)220}$

Divide. Check all answers.

	a	b	c	d	e
8.	$6\overline{)328}$	$8\overline{)395}$	$9\overline{)388}$	$5\overline{)228}$	$9\overline{)789}$
9.	$8\overline{)309}$	$6\overline{)2,740}$	$3\overline{)2,384}$	$5\overline{)1,974}$	$7\overline{)6,545}$
10.	$9\overline{)1,856}$	$4\overline{)2,720}$	$3\overline{)2,255}$	$6\overline{)1,206}$	$7\overline{)3,546}$

11. Look back over the examples that have "come out even." If you subtracted in the last step, what number would you get as the remainder? Do you agree that we can think of examples that "come out even" as having a 0 remainder?

12. For the example $275 \div 8$, "33R11" was given as an answer. Do the example. Do you agree with the answer given? If not, why not?

1. Have you noticed that in division examples we are often comparing numbers? Suppose we ask whether there are as many as 7 twenty-ones in 150. Are we really asking whether 7×21 is less than 150? We are comparing 7×21 and 150. What is 7×21? Is it less than 150?

$21\overline{)150}$

Suppose we ask whether there are as many as 8 twenty-ones in 150. Are we really asking whether 8×21 is less than 150? What is 8×21? Is it less than 150?

2. Just as we have the sign "=" to mean "equals," we have a special sign, $<$, to mean "is less than." The point of the sign is aimed at the name for the smaller number. The model sentence at the right is read "7×21 is less than 150." Is the sentence true?

$7 \times 21 < 150$

Read this sentence: $83 < 81$. Is it true?

3. Which sentences are true? Which is not true?

a. $2 \times 13 < 75$ **b.** $4 \times 13 < 75$ **c.** $6 \times 13 < 75$

4. Is 6×13 greater than 75? There is a special sign, $>$, for "is greater than." The sentence at the right is read "6×13 is greater than 75." Is the point in this sign also aimed at the name for the smaller number?

$6 \times 13 > 75$

Read this sentence: $8 \times 21 > 150$. Is it true?

5. Which sentences are true? Which is not true?

a. $7 \times 30 < 242$ **b.** $8 \times 30 < 242$ **c.** $9 \times 30 < 242$

Is Sentence **c** the only one that is not true? In which true sentence is the product closer to 242? Using this information, find the quotient and remainder for this example: $30\overline{)242}$.

In Exercises **6–8**, find which sentences are true and which are not true. Then find the quotient and remainder for each division example.

6. $4 \times 70 < 291$; $5 \times 70 < 291$; $6 \times 70 < 291$ $70\overline{)291}$

7. $2 \times 90 < 326$; $3 \times 90 < 326$; $4 \times 90 < 326$ $90\overline{)326}$

8. $5 \times 61 < 302$; $4 \times 61 < 302$; $3 \times 61 < 302$ $61\overline{)302}$

We have been investigating some ideas that are used in division. But division is not a new topic for you, and you probably remember how to divide by numbers larger than ten.

1. Are there as many as 100 forties in 951? Then we think of tens, not hundreds. Are there as many as 10 forties in 951? as many as 20 forties? 30 forties? What do we write at the side? What is 20×40? Why do we subtract 800 from 951? How many forties are there in 151? What is the quotient? the remainder?

```
40)951
   800 20
   151
   120  3
    31
```

2. There is a shorter method that you have probably used in an earlier grade. Here, more work is done mentally. For example, when we find there are 20 forties —but not 30 forties—in 951, do we know the quotient is some number in the twenties? So we write a 2 above tens' place in the dividend. Then we multiply: $2 \times 40 = 80$. Since the "2" really means 20, what does the "80" mean?

```
   2          23
40)951     40)951
  80         80
  151        151
             120
              31
```

We subtract the 80 and bring down the 1 from the dividend. Are we really subtracting 800 from 951? Next, we find that there are 3 forties in 151 and write the 3 above ones' place in the dividend. How do we get the 120? the remainder 31?

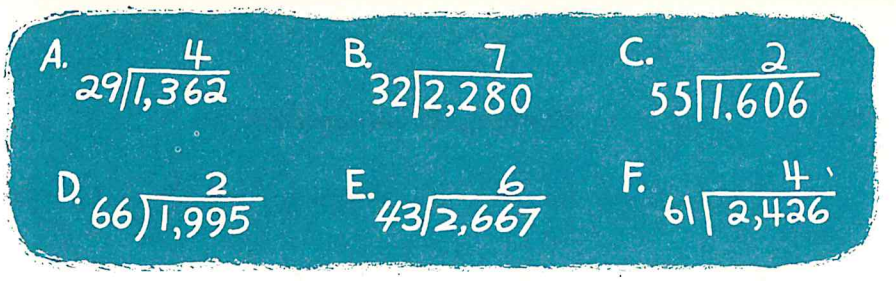
3. Use the longer method to divide 1,762 by 50.

4. This model shows the first step in the shorter form for Exercise **3**. What does the quotient figure "3" mean? the "150"? Copy and complete the example in the short form.

```
    3
50)1,762
  1 50
```

Divide. Try to use the short form only. Check.

	a	b	c	d
5.	$20\overline{)1{,}386}$	$40\overline{)1{,}867}$	$30\overline{)1{,}907}$	$80\overline{)4{,}225}$
6.	$50\overline{)3{,}760}$	$60\overline{)3{,}249}$	$90\overline{)956}$	$60\overline{)572}$



1. This illustration shows chalkboard work done by six pupils. Which is easier to estimate—how many twenty-nines in 1,362 or how many thirties in 1,362? In Example **A**, the pupil rounded the divisor to the nearest ten; he thought of 29 as about 30. What does the quotient figure "4" mean? Do you think it is the correct estimate: $4 \times 29 = \underline{?}$ Copy the example and complete it. What does the "116" in the completed example mean?

2. In **B**, if we round the divisor to the nearest ten, what number do we use to estimate? What does the quotient figure "7" mean? Do you think this is the correct estimate? How can you find out whether it is? Copy and complete the example.

3. Would you round 55 to 50, or to 60? In **C**, do you think the quotient figure 2 is correct? Do the example to find out.

4. Look at **D**. What is 66 rounded to the nearest ten? How was the quotient figure 2 obtained? When you multiply 66 by 2, what do you get? When you subtract "132" from "199," what do you get? Is there something wrong?

Is the following sentence true: $30 \times 66 < 1{,}995$? Then should the quotient figure be 3?

Do you see that when we use a rounded divisor to estimate we do not always get the correct quotient figure? Do the whole example yourself.

5. In one of the remaining examples, another mistake has been made. Do both examples to find the mistake.

Divide. Round the divisors to help in estimating.

	a	b	c	d
6.	$79\overline{)5{,}228}$	$24\overline{)1{,}996}$	$23\overline{)1{,}162}$	$37\overline{)2{,}621}$
7.	$32\overline{)2{,}400}$	$55\overline{)2{,}617}$	$31\overline{)2{,}586}$	$28\overline{)2{,}566}$

Copy and divide. Round the divisors to help in estimating quotients.

	a	b	c	d
1.	$48\overline{)1{,}313}$	$35\overline{)2{,}676}$	$59\overline{)2{,}617}$	$27\overline{)1{,}257}$
2.	$87\overline{)6{,}927}$	$45\overline{)2{,}925}$	$36\overline{)2{,}026}$	$86\overline{)2{,}431}$

In Exercises **1** and **2**, did you round each divisor upward? Was any quotient estimate too large? Were some too small?

Copy and divide. Round the divisors in estimating.

3.	$81\overline{)6{,}561}$	$64\overline{)5{,}135}$	$43\overline{)2{,}432}$	$52\overline{)3{,}886}$
4.	$42\overline{)1{,}624}$	$24\overline{)1{,}392}$	$93\overline{)6{,}666}$	$32\overline{)1{,}839}$

In Exercises **3** and **4**, did you round each divisor downward? Was any quotient estimate too small? Were some too large?

5. You have been rounding divisors to estimate quotients. Have you found a system of your own that makes estimating easier? If so, tell about it, using one of these examples.

a. $48\overline{)1{,}566}$ **b.** $24\overline{)1{,}256}$

For **a**, one system that some people use is to think of 48 as 5 tens, and 1,566 as "150 tens and 66." What is 150 tens ÷ 5 tens? Do the example.

For **b**, some people do not round the divisor. They think: "There are about 5 twenty-fours in 125; then about how many twenty-fours are there in 1,256?" Do the example.

Copy, divide, and check.

	a	b	c	d
6.	$43\overline{)2{,}434}$	$28\overline{)1{,}802}$	$45\overline{)2{,}600}$	$43\overline{)2{,}150}$
7.	$54\overline{)2{,}516}$	$59\overline{)3{,}106}$	$76\overline{)4{,}925}$	$95\overline{)2{,}850}$
8.	$37\overline{)2{,}135}$	$85\overline{)3{,}210}$	$67\overline{)4{,}260}$	$54\overline{)3{,}264}$
9.	$93\overline{)3{,}772}$	$74\overline{)2{,}860}$	$69\overline{)4{,}395}$	$76\overline{)5{,}350}$

1. What is 38 rounded to the nearest ten? Can we think of 12,356 as 1,200 tens + 356? What is 1,200 tens ÷ 4 tens? Then is the quotient for this example a number around 3, 30, 300, or 3,000? Where will you write the first quotient figure? Copy and complete the example.

$38\overline{)12{,}356}$

2. A troop of 31 Girl Scouts collected a total of $475.25 from the sale of cookies. What was the average amount of sales per Girl Scout in this troop?

$31\overline{)\$475.25}$

Can we think of $475.25 as 47,525 cents? Can this number be thought of as 47 thousands + 525? Try to estimate the number of thirty-ones in 47 thousands. Are there as many as 10,000 thirty-ones? as many as 1,000? Where will you write the first quotient figure?

Do the example carefully. Remember, your answer to the problem will need a dollar sign and point.

3. This model sentence is read "What number times 43 is less than 21,603?"

$N \times 43 < 21{,}603$

If we replace N with 1,000, will the sentence be true? Then suppose we think of hundreds of forty-threes. Will the sentence be true if we replace N with 100? with 200? That is, are these sentences true: $100 \times 43 < 21{,}603$? $200 \times 43 < 21{,}603$? Is there more than one replacement that will make the sentence true?

What is the largest number of hundreds that will make the sentence true: 400, 500, or 600? Then what is the first quotient figure for this example: $43\overline{)21{,}603}$? Complete the example.

4. Find the largest number of thousands or of hundreds that will make each sentence true. Then do the example.

a. $N \times 19 < 42{,}603$ $\quad 19\overline{)42{,}603}$

b. $N \times 11 < 57{,}629$ $\quad 11\overline{)\$576.29}$

c. $N \times 39 < 28{,}432$ $\quad 39\overline{)28{,}432}$

d. $N \times 52 < 32{,}968$ $\quad 52\overline{)\$329.68}$

For each of the problems below, write a number sentence (or sentences) to help you solve the problem. Then find the replacement for N (or the frame or the ?) that will make each sentence true. In one problem, you will need information that has been given in another problem.

1. A census of the school children in a city showed that there would be 172 sixth-graders enrolling in a certain school at the beginning of the new school year. The principal wanted each sixth grade to have about 28 pupils. How many sixth-grade teachers were needed for the new school year? Could each class have exactly 28 pupils? Why?

2. On the first day of school, one group of sixth-graders was assigned to rooms on the second floor and another group to rooms on the first floor. The second-floor group had 88 pupils. Were the sixth-grade pupils divided into two equal groups? How many fewer pupils were on the first floor than on the second floor?

3. Altogether there are 36 teachers in the elementary school. If the average number of pupils in a classroom is 26, how many pupils are there in the elementary school?

4. The principal had $46.76 in the budget for supplies for each sixth-grade class. For the 28 pupils in a room, how much is this for each pupil?

5. The elementary school's share of the library budget was $232.50 for the past year. At the end of the year, a count showed that 98 books had been bought. About what was the cost of a book on the average? Does this mean that every book cost the average amount? Could some books have cost more? Could some have cost less?

EXTRA PRACTICE 283–284

If you make any mistakes, study the pages given at the right of the exercises.

1. Round each of the following numbers to the nearest ten. 3

a. 495 **b.** 384 **c.** 9,678 **d.** 74

2. Copy, and replace N with the numeral that will make the sentence true. 7

a. $8 \times 10^2 = N$ **b.** $6 \times 10^3 = N$ **c.** $2 \times 10 = N$

3. Copy, and replace N with 10 or with a power of 10. 7

a. $900 = 9 \times N$ **b.** $7{,}000 = 7 \times N$ **c.** $70 = 7 \times N$

4. Copy, and replace each ? with a power of 10, 10, or 1. 8

$7{,}856{,}496 = (7 \times ?) + (8 \times ?) + (5 \times ?) + (6 \times ?) + (4 \times ?) + (9 \times ?) + (6 \times ?)$

5. Copy, and replace N with a base six numeral. 11

$$(4 \times 6^2) + (5 \times 6) + (4 \times 1) = N$$

6. Copy, and replace N with a base ten numeral. 13

a. $1543_{six} = N$ **b.** $43_{six} = N$

7. Copy, and replace N with the numeral for the largest number of tens that makes the sentence true. 33

a. $N \times 6 < 249$ **b.** $N \times 89 < 7243$

8. Copy and add or subtract as the signs tell you. 15–18

a.
$$\begin{array}{r} \$87.50 \\ 4.35 \\ 9.88 \\ +\ 6.15 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 800 \\ 749 \\ 651 \\ +\ 300 \\ \hline \end{array}$$

c.
$$\begin{array}{r} \$8.75 \\ -\ 3.87 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 400 \\ -\ 197 \\ \hline \end{array}$$

9. Copy, multiply, and check. 21–29

a.
$$\begin{array}{r} \$1.35 \\ 25 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 465 \\ 231 \\ \hline \end{array}$$

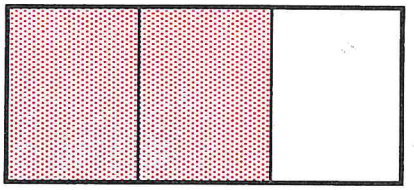
c.
$$\begin{array}{r} 738 \\ 40 \\ \hline \end{array}$$

10. Copy, divide, and check. 30–37

a. $39\overline{)\,31{,}871}$ **b.** $41\overline{)\,31{,}871}$

DISCOVERING MORE ABOUT NUMBERS: PAGES 311–313

2 Naming Fractional Parts

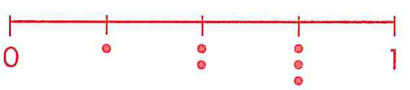


1. The rectangular diagram is divided into three equal parts. What fraction would you use to describe the part of the diagram that is shaded?

Draw a rectangular diagram 3 inches long and 2 inches wide. Divide the diagram into thirds and shade two of these thirds. Can you draw one horizontal line so that the diagram is divided into six equal parts? Do it. Now what fraction can you write to represent the shaded part?

Can you draw two more horizontal lines so that the diagram is divided into twelve equal parts? Do it. What fraction can you write to represent the shaded part?

Do $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{8}{12}$ all name the same fractional part of the diagram?



2. This illustration shows plans for a measuring stick designed by a boy who liked to invent things. He called the distance from the 0 mark to the 1 mark "1 stripe" and divided it into four equal parts. Can you write a fraction that tells what part of a stripe the distance from the 0 mark to the "•" mark is? What part of a stripe is the distance from the 0 mark to the "⁝" mark?

Suppose the boy had divided the stripe into eight equal parts. How many of these eight parts would each $\frac{1}{4}$ stripe contain? Then $\frac{3}{4} = \frac{?}{8}$.

Suppose he had divided the stripe into sixteen equal parts. How many of these sixteen parts would each $\frac{1}{4}$ stripe contain? Then $\frac{3}{4} = \frac{?}{16}$.

3. Look back at the illustration of the stripe. Can you name two fractions that tell what part of a stripe the distance from the 0 mark to the "●" mark is?

By imagining the stripe divided into eight equal parts, and then sixteen equal parts, find the correct numerals for these sentences: $\frac{1}{2} = \frac{?}{8}$; $\frac{1}{2} = \frac{?}{16}$.

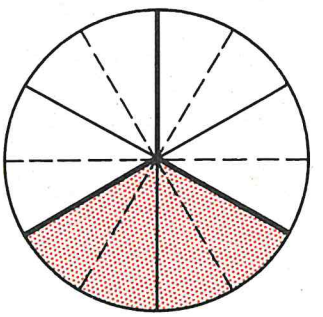
What are four names for $\frac{1}{2}$? three names for $\frac{3}{4}$?

4.a. There are 24 stars in the set at the right. How many rows are there? What is $\frac{1}{8}$ of 24? What is $\frac{3}{8}$ of 24?

b. Suppose we divide the set of 24 stars into four groups, each with the same number of stars. How many stars would be in each group? What is $\frac{1}{4}$ of 24? Do you get the same number when you think of $\frac{1}{4}$ of 24 as when you think of $\frac{2}{8}$ of 24? Is this sentence true: $\frac{1}{4} = \frac{2}{8}$?

c. Now think of the set of 24 as being divided into twelve groups, each with the same number of stars. How many stars would be in each group? What is $\frac{1}{12}$ of 24? What is $\frac{3}{12}$ of 24?

d. Are $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{3}{12}$ names for the same number? Which of the following fractions is another name for the same number: $\frac{4}{24}$, $\frac{5}{24}$, $\frac{6}{24}$?

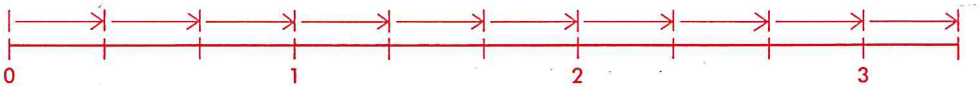


5.a. This diagram has been divided into twelve equal parts. What fraction tells how many twelfths are shaded?

b. Now think of the diagram as being divided into six equal parts. What fraction represents the shaded part?

c. Can you also think of the diagram as being divided into three equal parts? What fraction names the shaded part?

d. Do $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$ all name the same fractional part of the diagram? Do you see a pattern you can use to name other fractions that represent the shaded part? Think of the diagram as being divided into twenty-four equal parts and then forty-eight equal parts: $\frac{1}{3} = \frac{?}{24}$; $\frac{1}{3} = \frac{?}{48}$.



1. On a number line, the space from 0 to 1 is called the **unit space.** This drawing shows moves along a number line. Each move is $\frac{1}{3}$ unit space in length. What numeral would you write at the point where the first move ends? at the point where the second move ends? At the end of the third move, are you at the point labeled 1? Then is this sentence true: $\frac{3}{3} = 1$?

2. How many moves does it take to get from 0 to the point labeled 2? Then is this sentence true: $\frac{6}{3} = 2$? Write a true sentence for each of the following.

a. $\frac{\square}{3} = 3$ **b.** $\frac{\triangle}{3} = 4$ **c.** $\frac{N}{3} = 5$ **d.** $\frac{\square}{3} = 8$

3. Draw a model of a number line and label points with the numerals 0, 1, 2, 3, 4, 5, and 6. Now label a point for $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, and so on. Are there any points labeled twice? In each of these cases, are the numerals different names for the same number?

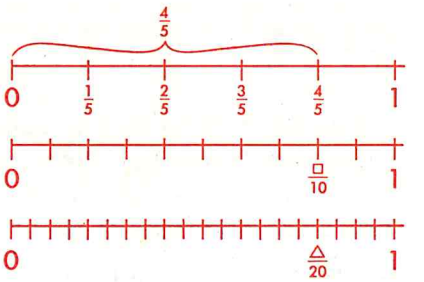
4. You know that 2 is greater than 1. Or, using the sign for "is greater than," we write: $2 > 1$. Then is this sentence true: $\frac{6}{3} > \frac{3}{3}$? Tell whether each of these sentences is true or not true.

a. $\frac{9}{3} > \frac{3}{3}$ **b.** $\frac{12}{3} > \frac{9}{3}$ **c.** $\frac{9}{3} > \frac{6}{3}$ **d.** $\frac{12}{3} > \frac{15}{3}$

5. Study the number line. As we move from left to right (from 0 to 1 to 2 to 3, and so on), do the numerals name larger and larger numbers? Use this idea to tell whether each of these sentences is true or not true.

a. $\frac{9}{3} > \frac{8}{3}$ **b.** $\frac{11}{3} > \frac{10}{3}$ **c.** $\frac{13}{3} > \frac{14}{3}$ **d.** $\frac{17}{3} > \frac{15}{3}$

6. Using the number line you made for Exercise **3,** sketch moves of $\frac{1}{6}$ unit space in length, starting at 0. Label points with the fractions $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, and so on. What name do you find for 1 in this case? What name do you find for 2? for 3?



1. These drawings show the unit space on a number line. In the first drawing, the unit space has been divided into five equal spaces, or fifths. Four of these spaces are what part of the unit space?

2. In the second drawing, the unit space has been divided into tenths. Each fifth of the unit space contains how many tenths of the unit space? What is the correct numeral for this sentence: $\frac{4}{5} = \frac{\square}{10}$?

3. In the third drawing, the unit space has been divided into twentieths. Each fifth of the unit space contains how many twentieths of the unit space? Write the numeral for this sentence: $\frac{4}{5} = \frac{\triangle}{20}$.

4. Suppose the unit space had been divided into fortieths. Then what would be the correct numeral for this sentence: $\frac{4}{5} = \frac{N}{40}$?

5. Suppose the unit space had been divided into two hundred equal spaces. Then each fifth of the unit space would contain how many of these two hundred spaces? What is the correct numeral for each of these sentences: $\frac{1}{5} = \frac{\square}{200}$; $\frac{4}{5} = \frac{\triangle}{200}$?

We have been writing sentences with fractions, using the "=" sign. From your work in an earlier grade, you probably remember:

▶A fraction is a numeral. In a fraction such as $\frac{2}{3}$, the 2 represents the **numerator;** the 3 represents the **denominator.** A fraction bar separates the 2 and the 3. The numerator and the denominator are called the **terms** of the fraction.

$\frac{2}{3}$

6. Do $\frac{4}{5}$, $\frac{8}{10}$, $\frac{16}{20}$, $\frac{32}{40}$, and $\frac{160}{200}$ all name the same number? This number is called a **fractional number.** Fractions that name the same number are called **equivalent fractions.** We use the "=" sign to show that two fractions are equivalent; for example, $\frac{2}{5} = \frac{4}{10}$.

$\frac{1}{2}\ \frac{2}{4}\ \frac{4}{8}\ \frac{8}{16}$ $\frac{3}{4}\ \frac{6}{8}\ \frac{12}{16}$ $\frac{1}{3}\ \frac{2}{6}\ \frac{4}{12}\ \frac{8}{24}$ $\frac{4}{5}\ \frac{8}{10}\ \frac{16}{20}\ \frac{32}{40}\ \frac{160}{200}$

You have been using diagrams and number lines to find equivalent fractions. Shown above are four sets of equivalent fractions. Let us use these sets to discover a way of writing equivalent fractions without a diagram or a number line.

1. Are $\frac{1}{2}$ and $\frac{4}{8}$ equivalent fractions? In which fraction are the numerator and the denominator larger, $\frac{1}{2}$ or $\frac{4}{8}$? How many times as large? By what number could you multiply both terms of $\frac{1}{2}$ to get $\frac{4}{8}$? Is this sentence true: $\frac{4 \times 1}{4 \times 2} = \frac{4}{8}$?

Dividing by 4 is the inverse of multiplying by 4. What can you do to both terms of $\frac{4}{8}$ to get $\frac{1}{2}$?

2. Are $\frac{16}{20}$, $\frac{8}{10}$, and $\frac{4}{5}$ equivalent fractions? What can you do to both terms of $\frac{16}{20}$ to get $\frac{8}{10}$? What can you do to both terms of $\frac{16}{20}$ to get $\frac{4}{5}$?

Copy, and write the missing numerals.

a. $\frac{16 \div \triangle}{20 \div \triangle} = \frac{8}{10}$ **b.** $\frac{\triangle \times 8}{\triangle \times 10} = \frac{16}{20}$

c. $\frac{16 \div \square}{20 \div \square} = \frac{4}{5}$ **d.** $\frac{\square \times 4}{\square \times 5} = \frac{16}{20}$

3. Are $\frac{1}{3}$ and $\frac{8}{24}$ equivalent fractions? What can you do to both terms of $\frac{1}{3}$ to get $\frac{8}{24}$? What can you do to both terms of $\frac{8}{24}$ to get $\frac{1}{3}$?

4. Are $\frac{16}{20}$ and $\frac{160}{200}$ in the same set of equivalent fractions? What can you do to both terms of $\frac{160}{200}$ to get $\frac{16}{20}$? What can you do to both terms of $\frac{16}{20}$ to get $\frac{160}{200}$?

▶If we multiply (or divide) both terms of a fraction by the same number, do we get an equivalent fraction?

5. Suppose you are asked to find the numeral that will make this sentence true: $\frac{1}{5} = \frac{\square}{10}$. What number times 5 equals 10? Then if you multiply 1 by 2, will you have found the missing numerator? Copy the sentence and write the correct numeral. Are $\frac{1}{5}$ and $\frac{2}{10}$ equivalent?

6. Can you find the correct numeral for this sentence: $\frac{12}{20} = \frac{N}{10}$? Try it. What did you do to both terms of $\frac{12}{20}$ to get an equivalent fraction with a denominator of 10?

Check by multiplying: $\frac{2 \times 6}{2 \times 10} = \frac{\triangle}{\square}$.

Copy, and write the correct numerals.

	a	b	c	d
7.	$\frac{12}{16} = \frac{\triangle}{8}$	$\frac{1}{2} = \frac{\square}{16}$	$\frac{3}{5} = \frac{N}{20}$	$\frac{8}{12} = \frac{\square}{3}$
8.	$\frac{2}{\square} = \frac{4}{8}$	$\frac{12}{16} = \frac{N}{4}$	$\frac{4}{3} = \frac{8}{\square}$	$\frac{\triangle}{3} = \frac{12}{6}$
9.	$\frac{8}{20} = \frac{\square}{5}$	$\frac{5}{3} = \frac{\triangle}{6}$	$\frac{80}{200} = \frac{\triangle}{5}$	$\frac{8}{32} = \frac{1}{\square}$

10. For each fraction named in the box, write equivalent fractions with the denominators named.

a. $\boxed{\frac{100}{150}}$ $\frac{?}{15}$ $\frac{?}{3}$ **b.** $\boxed{\frac{16}{64}}$ $\frac{?}{32}$ $\frac{?}{4}$

In Exercises **10a** and **10b,** as you wrote the equivalent fractions, were you writing fractions having smaller numerators and denominators? Such fractions are said to be in **lower terms.**

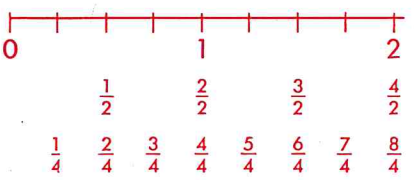
11. Do the fractions at the right name the same number? Can you name another fraction that is equivalent to these but has a smaller numerator and denominator than $\frac{1}{2}$? We say that $\frac{1}{2}$ is the **simplest form** for the number named by this set of fractions. We also say that such a fraction is in **lowest terms.** What is the simplest form for the number named in **10a**? in **10b**? In each case, is there any number other than 1 that will divide both the numerator and the denominator evenly?

$\frac{20}{40}$ $\frac{10}{20}$ $\frac{5}{10}$ $\frac{1}{2}$

12. What is the simplest form for the fractional number named by $\frac{32}{40}$? Is it $\frac{16}{20}$ or $\frac{8}{10}$? If not, what is the simplest form?

Give the simplest form for the number named by each of the following.

	a	b	c	d	e	f
13.	$\frac{2}{10}$	$\frac{8}{24}$	$\frac{10}{12}$	$\frac{2}{9}$	$\frac{20}{32}$	$\frac{3}{7}$
14.	$\frac{4}{16}$	$\frac{8}{6}$	$\frac{3}{5}$	$\frac{16}{32}$	$\frac{18}{21}$	$\frac{14}{12}$



In this drawing of a number line, the spaces from 0 to 1 and from 1 to 2 have been divided into halves and fourths.

1. What two fractions are shown as names for 1? Write two other names for 1, using these sentences: $\frac{2}{2} = \frac{\triangle}{8}$; $\frac{2}{2} = \frac{\square}{16}$.

2. What two fractions are shown as names for 2? Use this sentence to write another name for 2: $\frac{8}{4} = \frac{\triangle}{8}$.

3. Do $\frac{3}{2}$ and $\frac{6}{4}$ name the same number? Use this sentence to write another name for the same number: $\frac{3}{2} = \frac{\square}{20}$.

▶A fraction such as $\frac{2}{2}$ or $\frac{3}{2}$ or $\frac{7}{4}$ or $\frac{8}{4}$—that is, a fraction whose numerator is equal to or greater than its denominator—is called an **improper fraction.** Any fraction whose numerator is less than its denominator is called a **proper fraction.**

4. Give five examples of proper fractions. Does each fraction name a number less than 1, or greater than 1?

5. Look at the point labeled $\frac{5}{4}$. Is this point 1 unit space plus $\frac{1}{4}$ of another unit space to the right of the point labeled 0? We can label this point as $1 + \frac{1}{4}$. Do you think that $\frac{5}{4}$ and $1 + \frac{1}{4}$ are names for the same number? Express $\frac{7}{4}$ in a similar way. Express $\frac{3}{2}$ in a similar way.

We can express $\frac{5}{4}$, or $1 + \frac{1}{4}$, as $1\frac{1}{4}$. The numeral $1\frac{1}{4}$ is called a **mixed fraction.** What mixed fraction is named by $1 + \frac{1}{3}$? by $2 + \frac{5}{6}$?

Express each of these improper fractions as a mixed fraction. If you wish, you may draw a number line to help you.

	a	b	c	d	e	f
6.	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{9}{4}$	$\frac{11}{8}$	$\frac{17}{8}$
7.	$\frac{5}{2}$	$\frac{7}{6}$	$\frac{11}{6}$	$\frac{13}{6}$	$\frac{13}{4}$	$\frac{11}{3}$

1. Another name for 2 is $\frac{8}{4}$. Divide both terms of $\frac{8}{4}$ by 4: $8 \div 4 = \triangle$; $4 \div 4 = \square$. Do you get $\frac{2}{1}$ as a fraction equivalent to $\frac{8}{4}$? The fraction $\frac{2}{1}$ is read "two over one." Since $\frac{2}{1}$ and $\frac{8}{4}$ are names for the same number, $\frac{2}{1}$ is another name for 2.

Copy and complete this chart so that it shows other names for the number 2: $\boxed{\frac{2}{1}}\ \frac{?}{2}\ \frac{?}{3}\ \frac{8}{4}\ \frac{?}{5}\ \frac{?}{6}\ \frac{?}{7}\ \frac{?}{10}\ \frac{?}{15}$.

2. Is $\frac{12}{3}$ another name for 4? Divide both 12 and 3 by 3. Do you obtain $\frac{4}{1}$ as a fraction equivalent to $\frac{12}{3}$? Then is $\frac{4}{1}$ another name for the number 4?

Copy and complete this chart so that it shows other names for the number 4: $\boxed{\frac{4}{1}}\ \frac{?}{2}\ \frac{12}{3}\ \frac{?}{4}\ \frac{?}{5}\ \frac{?}{10}\ \frac{?}{12}\ \frac{?}{25}\ \frac{?}{100}$.

3. What numbers do you think these fractions name: $\frac{1}{1}$, $\frac{3}{1}$, $\frac{5}{1}$, $\frac{8}{1}$? Write five fractions that name the number 3; 5; 8.

4. Since $\frac{14}{7}$ names the number 2, what is the mixed fraction for $\frac{15}{7}$? $\frac{17}{7}$? $\frac{20}{7}$?

5. How many fourths equal 1? Is $\frac{10}{4}$ greater than 1? greater than 2? greater than 3? Write $\frac{10}{4}$ as a mixed fraction. Did you get $2\frac{2}{4}$? What is $\frac{2}{4}$ in simplest form? We say that a number expressed as a mixed fraction is in simplest form when the proper fraction is in simplest form. Thus, $2\frac{1}{2}$ is the simplest form for $2\frac{2}{4}$.

6. Is $\frac{11}{3}$ greater than 2? greater than 3? Write $\frac{11}{3}$ as a mixed fraction. Explain how you got your answer.

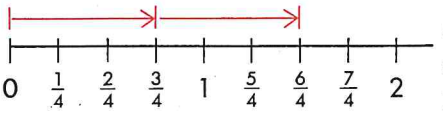
7. Express as a whole number, a proper fraction in simplest form, or a mixed fraction in simplest form.

a. $\frac{6}{15}$ **b.** $\frac{18}{6}$ **c.** $\frac{19}{5}$ **d.** $\frac{8}{64}$ **e.** $\frac{93}{10}$ **f.** $\frac{14}{10}$ **g.** $\frac{60}{12}$

8. Draw a number line and mark points for these fractions: $\frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \frac{5}{5}, \frac{5}{10}, \frac{5}{20}$.

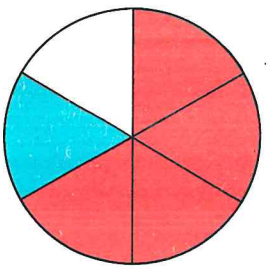
Is $\frac{5}{3}$ less than $\frac{5}{2}$? That is, is it true that $\frac{5}{3} < \frac{5}{2}$? Use the "<" sign to write true sentences involving the following pairs of fractions.

a. $\frac{5}{4}, \frac{5}{3}$ **b.** $\frac{5}{4}, \frac{5}{5}$ **c.** $\frac{5}{5}, \frac{5}{10}$ **d.** $\frac{5}{20}, \frac{5}{10}$



1. Shown on the number line are two moves, each $\frac{3}{4}$ unit space in length. What fraction is at the end of the second move? Is this sentence true: $\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$? What is the denominator of each fraction that represents a move? What is the denominator of the sum? How can you get the numerator of the sum from the numerators of the original fractions? Is this sentence true: $\frac{3}{4} + \frac{3}{4} = \frac{3+3}{4}$, or $\frac{6}{4}$?

2. Now start at 0 again and imagine a move of $\frac{2}{4}$ unit space, then a move of $\frac{1}{4}$ unit space. What fraction is at the end of the second move? Then is this sentence true: $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$? Can we get the answer by writing $\frac{2}{4} + \frac{1}{4} = \frac{2+1}{4}$, or $\frac{3}{4}$?



3. The diagram at the left is divided into six parts of the same size. What fraction represents the blue part? the red part? Can you use an addition sentence to find the fraction that represents the two colored parts: $\frac{1}{6} + \frac{4}{6} = \frac{?}{6}$? Are the denominators of the fractions $\frac{1}{6}$ and $\frac{4}{6}$ the same? Is the denominator of the sum the same? Can we find the numerator of the sum by writing: $\frac{1}{6} + \frac{4}{6} = \frac{1+4}{6}$, or $\frac{5}{6}$?

Fractions in which the denominators are the same are called **like fractions.** Do you see a pattern to use in addition with like fractions? Then find the sums for the following.

	a	b	c	d	e
4.	$\frac{5}{9} + \frac{2}{9}$	$\frac{1}{10} + \frac{2}{10}$	$\frac{1}{5} + \frac{3}{5}$	$\frac{3}{7} + \frac{2}{7}$	$\frac{5}{8} + \frac{2}{8}$
5.	$\frac{6}{5} + \frac{2}{5}$	$\frac{9}{7} + \frac{1}{7}$	$\frac{1}{11} + \frac{4}{11}$	$\frac{5}{16} + \frac{2}{16}$	$\frac{1}{15} + \frac{16}{15}$

Now think of two like fractions, $\frac{\triangle}{5}$ and $\frac{\square}{5}$. Do you agree that $\frac{\triangle}{5} + \frac{\square}{5} = \frac{\triangle+\square}{5}$? Try a few pairs of replacements. How would you represent the sum of two like fractions: $\frac{\triangle}{N}$ and $\frac{\square}{N}$?

1. Look at this number sentence. Can you replace the frame with a fraction that will make the sentence true? Try it. $\frac{5}{12} + \frac{11}{12} = \square$

Did you choose $\frac{16}{12}$? This answer is correct, but it is not in simplest form. Express $\frac{16}{12}$ as an improper fraction in simplest form. Now write a mixed fraction for this number. Is the mixed fraction in simplest form?

2. Find the correct replacement for the frame in this sentence. Can you express your answer as a whole number? Do it. $\frac{4}{7} + \frac{10}{7} = \triangle$

▶When a fraction is another name for a whole number, we use a numeral for the whole number as the simplest form.

Find the sums. Express all sums in simplest form, as proper fractions, mixed fractions, or numerals for whole numbers.

	a	b	c	d	e
3.	$\frac{7}{10} + \frac{9}{10}$	$\frac{11}{16} + \frac{5}{16}$	$\frac{4}{9} + \frac{1}{9}$	$\frac{1}{8} + \frac{3}{8}$	$\frac{11}{12} + \frac{13}{12}$
4.	$\frac{13}{16} + \frac{17}{16}$	$\frac{8}{3} + \frac{4}{3}$	$\frac{5}{6} + \frac{7}{6}$	$\frac{5}{16} + \frac{7}{16}$	$\frac{5}{8} + \frac{3}{8}$

5. Find the missing fractions in these pairs of sentences.

a. $\frac{1}{11} + \frac{5}{11} = \square$ $\frac{5}{11} + \frac{1}{11} = \square$ **b.** $\frac{3}{5} + \frac{9}{5} = \triangle$ $\frac{9}{5} + \frac{3}{5} = \triangle$

c. $\frac{8}{7} + \frac{43}{7} = \mathrm{N}$ $\frac{43}{7} + \frac{8}{7} = \mathrm{N}$ **d.** $\frac{34}{9} + \frac{13}{9} = \square$ $\frac{13}{9} + \frac{34}{9} = \square$

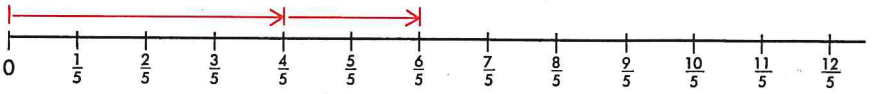
Does the order in which we add a pair of fractional numbers change the sum? Was this true of addition of whole numbers?

6. In a sentence such as **a**, below, what do the parentheses tell you to do? Find the missing fraction for each of these sentences.

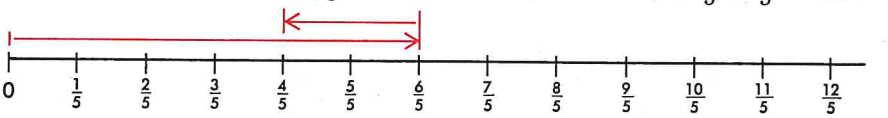
a. $(\frac{2}{9} + \frac{8}{9}) + \frac{1}{9} = \square$ **b.** $\frac{2}{9} + (\frac{8}{9} + \frac{1}{9}) = \square$

c. $\frac{11}{13} + (\frac{3}{13} + \frac{6}{13}) = \triangle$ **d.** $(\frac{11}{13} + \frac{3}{13}) + \frac{6}{13} = \triangle$

Compare your work for Sentences **a** and **b**. Compare your work for Sentences **c** and **d**. Does it seem that in adding three fractional numbers it does not matter which pair of neighboring fractions we group together and use first?



1.a. The drawing shows an addition example on the number line. What is the missing fraction in this sentence: $\frac{4}{5} + \frac{2}{5} = \square$?



b. This drawing shows subtraction undoing the addition: $\frac{6}{5} - \frac{2}{5} = \triangle$. What is the missing fraction in the sentence?

2. What does $\frac{7}{5} + \frac{1}{5}$ equal? Since subtracting undoes what has been done by adding, what does $\frac{8}{5} - \frac{1}{5}$ equal?

3. What does $\frac{2}{5} + \frac{7}{5}$ equal? What do you think is the missing fraction for this sentence: $\frac{9}{5} - \frac{7}{5} = \triangle$?

4. What does $\frac{3}{8} + \frac{4}{8}$ equal? Then what does $\frac{7}{8} - \frac{4}{8}$ equal? (Draw a number line if you wish.) What does $\frac{7}{8} - \frac{3}{8}$ equal?

5.a. Try one without the addition sentence first. What is the missing fraction in this sentence: $\frac{11}{7} - \frac{8}{7} = \mathrm{N}$?
b. Can you check subtraction by addition? Does the fraction you found in **a** make this sentence true: $\mathrm{N} + \frac{8}{7} = \frac{11}{7}$?

6. In each example, have the fractions been like fractions? Do you see a pattern to use when subtracting with like fractions? Does $\frac{13}{9} - \frac{7}{9}$ equal $\frac{13-7}{9}$, or $\frac{6}{9}$? What is $\frac{6}{9}$ in simplest form?

Suppose you have two like fractions, $\frac{\square}{6}$ and $\frac{\triangle}{6}$. Do you think that $\frac{\square}{6} - \frac{\triangle}{6} = \frac{\square - \triangle}{6}$? Suppose you have two like fractions, $\frac{\square}{\mathrm{N}}$ and $\frac{\triangle}{\mathrm{N}}$. How would you represent what $\frac{\square}{\mathrm{N}} - \frac{\triangle}{\mathrm{N}}$ equals?

Copy, and find the missing fractions. Express your answers in simplest form.

	a	b	c	d
7.	$\frac{5}{12} - \frac{1}{12} = \square$	$\frac{7}{20} - \frac{3}{20} = \triangle$	$\frac{6}{7} - \frac{1}{7} = \mathrm{N}$	$\frac{7}{8} - \frac{1}{8} = \square$
8.	$\frac{14}{5} - \frac{2}{5} = \mathrm{N}$	$\triangle - \frac{1}{5} = \frac{2}{5}$	$\frac{7}{8} - \frac{3}{8} = \square$	$\frac{8}{7} - \square = \frac{2}{7}$
9.	$\frac{11}{16} - \frac{5}{16} = \triangle$	$\frac{17}{9} - \frac{2}{9} = \square$	$\frac{26}{3} - \frac{14}{3} = \triangle$	$\square - \frac{16}{5} = \frac{27}{5}$

1. Study Sentence **a**. Now look at Sentence **b**. In **b**, we have rearranged the numerals from **a** and grouped them differently. But are we still thinking about the same numbers? If we find the sum for **b**, will we have found the sum for **a**? What is 2 + 1? What is $\frac{7}{8} + \frac{3}{8}$? Express $\frac{10}{8}$ as a mixed fraction in simplest form. Then do we have this sentence: $3 + 1\frac{1}{4} = \square$? Since $1\frac{1}{4}$ means $1 + \frac{1}{4}$, we can write: $(3 + 1) + \frac{1}{4} = \square$. What is the correct mixed fraction to replace the frame?

a. $(2 + \frac{7}{8}) + (1 + \frac{3}{8}) = \square$

b. $(2 + 1) + (\frac{7}{8} + \frac{3}{8}) = \square$

2. At the right is an addition example in a form you have seen in an earlier grade. Does $2\frac{7}{8}$ mean $2 + \frac{7}{8}$? Express $1\frac{3}{8}$ in this form. Do you see that this example involves the same numbers as Sentence **a** in Exercise **1**?

$$\begin{array}{r} 2\frac{7}{8} \\ 1\frac{3}{8} \\ \hline 3\frac{10}{8} \end{array}$$

When an example is written in vertical form, can you do the addition column by column, beginning with the $\frac{7}{8}$ and $\frac{3}{8}$? Does $3\frac{10}{8}$ mean $3 + \frac{10}{8}$? Express $3 + \frac{10}{8}$ as a mixed fraction in simplest form.

3.a. What does $\frac{11}{3} - \frac{4}{3}$ equal? Express the remainder as a mixed fraction.

b. What is $\frac{11}{3}$ expressed as a mixed fraction? $\frac{4}{3}$? Then should we get the same answer, using this form, that we did in **a**? Can you do this example column by column? What is $\frac{2}{3} - \frac{1}{3}$? $3 - 1$?

$$\begin{array}{r} 3\frac{2}{3} \\ -1\frac{1}{3} \\ \hline \end{array}$$

Do you get the same answer that you did in **a**?

Add or subtract as the signs tell you to do. Express the answers in simplest form.

	a	b	c	d
4.	$7\frac{1}{9} + 5\frac{5}{9}$	$1\frac{3}{4} + 2\frac{3}{4}$	$4\frac{1}{3} + 3\frac{1}{3}$	$7\frac{7}{12} + 8\frac{7}{12}$
5.	$\frac{109}{6} - \frac{85}{6}$	$18\frac{3}{4} - 6\frac{1}{4}$	$20\frac{7}{16} - 4\frac{5}{16}$	$\frac{71}{4} - \frac{59}{4}$

Read each of these problems carefully. For some problems, you may have to use more than one step. For others, you may have to decide which of the facts that are given are needed and which are not needed.

1. Mr. Keats drove his car 987 miles during September, 1,024 miles during October, and 869 miles during November. How many miles per month did he drive his car on the average?

2. During September he used $47\frac{1}{10}$ gallons of gasoline, during October $48\frac{7}{10}$ gallons, and during November $41\frac{3}{10}$ gallons. How many gallons of gasoline did he use in the three months?

3. How many more gallons did he use in October than in September? How many fewer gallons did he use in November than in October?

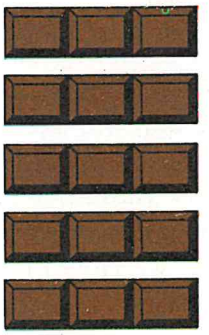
4. Once during the year Mr. Keats was in an automobile accident. Although no one was injured, both cars were damaged. The estimate for repairs to Mr. Keats' car was \$272, and for repairs to the other car \$87.50. The damage to a street sign was estimated to be \$65.75. How much more was the estimate for repairs to Mr. Keats' car than to the other car?

5. It is $\frac{3}{10}$ mile from Mr. Keats' home to the school where his son Paul is a pupil. From the school to Mr. Keats' office it is $2\frac{1}{10}$ miles farther along the same road. What is the length of a round trip between Mr. Keats' home and his office?

6. One afternoon when school was out, Paul bicycled from the school to his home and then to his father's office. His father drove him home that evening with the bicycle in the trunk of the car. How far had Paul bicycled that afternoon?

1. If 1 watermelon is divided equally among 12 people, will each person receive $\frac{1}{12}$ of the watermelon? Do you think this sentence is true: $1 \div 12 = \frac{1}{12}$? Can we think of $1 \div 12 = \frac{1}{12}$ as a division sentence? Then are $1 \div 12$ and $\frac{1}{12}$ names for the same number?

2. Suppose 5 candy bars are to be divided equally among 3 boys. Could we divide each candy bar into thirds and give out the pieces? How many thirds of a bar will each boy receive? Do you agree that this sentence is true: $5 \div 3 = \frac{5}{3}$?



Can we think of $5 \div 3 = \frac{5}{3}$ as a division sentence? Do you see that $5 \div 3$ and $\frac{5}{3}$ are names for the same number?

3. What fractional number is named by $2 \div 12$? Now express your answer in simplest form. Do you get $\frac{1}{6}$ as your answer?

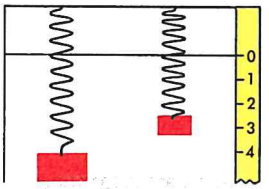
4. Replace the frames with the correct numerals.

a. $1 \div 9 = \frac{\triangle}{\square}$ **b.** $\frac{\diamond}{\square} = 4 \div 7$ **c.** $11 \div 2 = \frac{\triangle}{\square}$

5. What fraction is named by $3 \div 15$? $15 \div 6$? $18 \div 14$? $4 \div 18$? Give the simplest form in each case.

6. What whole number is the quotient for these examples: $0 \div 3$; $0 \div 6$; $0 \div 8$? Suppose you had been asked (as in Exercise **5**) to write the fraction named by these division examples. You might have written: $0 \div 3 = \frac{0}{3}$; $0 \div 6 = \frac{?}{6}$; $0 \div 8 = \frac{?}{8}$. Does it seem that $\frac{0}{3}$, $\frac{0}{6}$, $\frac{0}{8}$, and so on, may be thought of as names for 0?

7. Let us try another approach to the number 0. If we subtract $\frac{2}{3}$ from $\frac{2}{3}$, should the remainder be 0? Suppose we used the pattern for subtracting with like fractions. We might write: $\frac{2}{3} - \frac{2}{3} = \frac{2-2}{3}$, or $\frac{0}{3}$. What might you write for these examples: $\frac{7}{6} - \frac{7}{6}$; $\frac{11}{8} - \frac{11}{8}$? Do you see once again that $\frac{0}{3}$, $\frac{0}{6}$, $\frac{0}{8}$, and so on, are names for 0?



1. A weight stretched a steel spring $4\frac{1}{8}$ inches. Another weight stretched a similar spring $2\frac{5}{8}$ inches. How much farther was the first spring stretched than the second spring?

The subtraction example for this problem is shown at the right. Think carefully. Have we found any way to subtract $\frac{5}{8}$ from $\frac{1}{8}$? Then should we regroup the $4\frac{1}{8}$? Let us see how we can do this.

$$\begin{array}{r} 4\frac{1}{8} \\ -2\frac{5}{8} \\ \hline \end{array}$$

You know that $4\frac{1}{8} = 4 + \frac{1}{8}$. Is it true that $4 + \frac{1}{8} = (3 + 1) + \frac{1}{8}$? Is $\frac{8}{8}$ another name for 1? Then do we know that the following is true?

$$(3 + 1) + \frac{1}{8} = (3 + \frac{8}{8}) + \frac{1}{8} = 3 + (\frac{8}{8} + \frac{1}{8})$$

What is $\frac{8}{8} + \frac{1}{8}$? Then have we found that $4\frac{1}{8} = 3 + \frac{9}{8}$, or $3\frac{9}{8}$?

At the right is the subtraction example with the $4\frac{1}{8}$ regrouped. Subtract. Express the remainder in simplest form. Give your answer to the question asked in the problem.

$$\begin{array}{r} 4\frac{1}{8} = 3\frac{9}{8} \\ -2\frac{5}{8} = 2\frac{5}{8} \\ \hline \end{array}$$

2. Suppose you are asked to find the remainder for this example: $1 - \frac{5}{12}$. What name can you use for 1 in this case so that the subtraction can be done easily? Work the example.

3. Can you find the remainder for this example by thinking of 7 as $6 + 1$, and then expressing the 1 as thirds? Work the example.

$$\begin{array}{r} 7 \\ -1\frac{2}{3} \\ \hline \end{array}$$

See whether you understand regrouping with fractions. Copy the exercises below and write the missing numerals.

4.

a	b	c
$5\frac{3}{8} = 5 + \frac{3}{8}$	$6 = 5 + 1$	$4\frac{1}{4} = 4 + \frac{1}{4}$
$= (4 + 1) + \frac{3}{8}$	$= 5 + \frac{?}{9}$	$= (3 + 1) + \frac{1}{4}$
$= 4 + (\frac{?}{8} + \frac{3}{8})$	$= 5\frac{?}{9}$	$= 3 + (\frac{?}{4} + \frac{1}{4})$
$= 4\frac{\triangle}{8}$		$= 3\frac{\triangle}{4}$

1.a. What is $\frac{19}{8} - \frac{11}{8}$? Give the answer in simplest form.
b. Express $\frac{19}{8}$ and $\frac{11}{8}$ as mixed fractions. Subtract, using the form at the right. What is $\frac{3}{8} - \frac{3}{8}$? $2 - 1$? Is the answer the same as in **a**? Why do you think we do not write the 0 under the $\frac{3}{8}$'s?

$$\begin{array}{r} 2\frac{3}{8} \\ -1\frac{3}{8} \\ \hline 1 \end{array}$$

2. Copy, and write the missing numerals. Try to do as much of the work mentally as possible.

a. $3\frac{1}{4} = 2\frac{\square}{4}$ **b.** $7 = 6\frac{\triangle}{5}$ **c.** $3\frac{3}{8} = 2\frac{\triangle}{8}$ **d.** $8\frac{2}{5} = 7\frac{\triangle}{5}$

e. $5 = 4\frac{\square}{8}$ **f.** $23 = 22\frac{\square}{12}$ **g.** $12\frac{1}{9} = 11\frac{\triangle}{9}$ **h.** $1\frac{13}{16} = \frac{\triangle}{16}$

Copy and subtract. Express the remainders in simplest form.

	a	b	c	d	e	f
3.	$8\frac{1}{5}$ $5\frac{2}{5}$	$6\frac{3}{4}$ $4\frac{3}{4}$	$4\frac{3}{8}$ $2\frac{7}{8}$	$4\frac{2}{5}$ $1\frac{4}{5}$	$3\frac{7}{12}$ $1\frac{11}{12}$	$8\frac{5}{6}$ $3\frac{5}{6}$
4.	$8\frac{7}{10}$ $5\frac{9}{10}$	7 $5\frac{3}{8}$	$3\frac{3}{10}$ $1\frac{7}{10}$	$5\frac{2}{5}$ $2\frac{3}{5}$	$9\frac{5}{12}$ $2\frac{7}{12}$	$6\frac{1}{3}$ $3\frac{2}{3}$
5.	1 $\frac{7}{9}$	8 $\frac{15}{16}$	$9\frac{1}{7}$ $\frac{4}{7}$	$6\frac{1}{2}$ 3	1 $\frac{1}{11}$	$2\frac{2}{3}$ 1
6.	2 $1\frac{3}{16}$	$6\frac{1}{7}$ $\frac{5}{7}$	$16\frac{7}{10}$ 4	6 $\frac{1}{5}$	$11\frac{3}{16}$ $\frac{7}{16}$	3 $\frac{1}{4}$
7.	5 $4\frac{3}{4}$	$17\frac{1}{8}$ $8\frac{3}{8}$	7 $\frac{1}{10}$	$6\frac{1}{8}$ 5	12 $\frac{9}{16}$	11 $4\frac{2}{9}$

Find the missing numerals.

	a	b	c
8.	$2\frac{2}{7} + 1\frac{3}{7} + 1\frac{5}{7} = \mathrm{N}$	$\frac{3}{8} + \square = \frac{13}{8}$	$4\frac{1}{3} - 2\frac{1}{3} = \triangle$
9.	$\frac{8}{3} + \frac{14}{3} + \square = 8$	$\square - 3\frac{3}{5} = 1\frac{1}{5}$	$\frac{15}{4} = 3 + \triangle$
10.	$(\frac{7}{9} + 1) - \frac{8}{9} = \square$	$(1 + \frac{2}{5}) + 3 = \triangle$	$2\frac{2}{5} - \triangle = \frac{4}{5}$
11.	$(3\frac{3}{8} + 2\frac{1}{8}) + 11\frac{7}{8} = \square$	$2\frac{4}{7} + \triangle = 3\frac{1}{7}$	$\square + \frac{3}{2} = 6$

Tell which of the following are true and which are false.

12. $\frac{10}{5} > 2\frac{2}{5}$ $3\frac{1}{3} < \frac{11}{3}$ $1\frac{5}{9} < 2 - \frac{2}{9}$

13. $\frac{5}{6} + \frac{6}{6} > 3 - 1\frac{5}{6}$ $\frac{5}{2} + \frac{7}{2} + \frac{1}{2} < 6$ $1\frac{7}{8} + 2\frac{5}{8} < 5\frac{1}{8} - \frac{3}{8}$

1. The gasoline gauge of Mr. Johnson's automobile registered "empty" when he stopped at the filling station. The attendant pumped $13\frac{7}{10}$ gallons into the tank. Mr. Johnson knew that the tank held only 14 gallons. How much gasoline did he have left when he stopped?

2. One route to Mr. Johnson's office is $20\frac{7}{10}$ miles. An alternate route is $18\frac{9}{10}$ miles. How much longer is the first route?

3. What is the total distance that Mr. Johnson drives in a day if he takes one route in the morning to his office and the other route in the evening when he returns home?

4. How much greater is this mileage than the shorter route both ways?

5. The school Betty Johnson attends is on the other side of a lake, $5\frac{3}{4}$ miles from the Johnson home. Betty rides on a school bus but has to walk $\frac{1}{4}$ mile to the bus stop. How far does she ride on the bus each day?

6. In walking to and from the bus stop during the school week, how many miles does Betty walk?

7. Mrs. Johnson wanted to know the length of a rod she needed to hang some new draperies in the dining room. Mr. Johnson helped measure. He found that the distance across the window was $7\frac{2}{3}$ feet. Mrs. Johnson told him she wanted the rod to extend $\frac{2}{3}$ foot on each side of the window. How long a rod did he tell her to buy?

1. You know that if you multiply (or divide) both terms of a fraction by the same number, you get an equivalent fraction.

By what number can you multiply both terms of $\frac{5}{7}$ to get $\frac{15}{21}$? Then are $\frac{5}{7}$ and $\frac{15}{21}$ equivalent fractions? Check by dividing: $\frac{15 \div 3}{21 \div 3} = \frac{\triangle}{\square}$.

2. To find the simplest form for $\frac{12}{30}$, do you look for numbers that can divide both 12 and 30 evenly? Can both 12 and 30 be divided evenly by 2? by 3? by 6? Which of these numbers can you use to find the simplest form in the fewest steps?

On paper, show the steps needed when you divide both terms of $\frac{12}{30}$ by 2 first; when you divide by 3 first; when you divide by 6.

3. To express $\frac{78}{15}$ as a mixed fraction in simplest form, can you begin by finding an equivalent improper fraction in simplest form? Can you also begin by expressing $\frac{78}{15}$ as a mixed fraction?

How many 15's in 78? Then is $\frac{78}{15}$ greater than 5? Does $\frac{78}{15}$ equal $5 + \frac{3}{15}$? Express $\frac{78}{15}$ as a mixed fraction in simplest form.

4. Express in simplest form, as a proper fraction, a mixed fraction, or a numeral for a whole number.

a. $\frac{24}{30}$ **b.** $\frac{68}{16}$ **c.** $\frac{48}{64}$ **d.** $\frac{36}{84}$

e. $\frac{98}{14}$ **f.** $\frac{44}{20}$ **g.** $\frac{20}{44}$ **h.** $\frac{45}{36}$

Copy, and replace the frames with the correct numerals.

	a	b	c	d	e
5.	$\frac{3}{8} = \frac{\square}{40}$	$\frac{1}{4} = \frac{\triangle}{20}$	$\frac{7}{8} = \frac{\triangle}{56}$	$\frac{\triangle}{39} = \frac{11}{13}$	$\frac{4}{5} = \frac{\triangle}{25}$
6.	$\frac{\square}{45} = \frac{7}{15}$	$\frac{11}{16} = \frac{\square}{64}$	$\frac{5}{8} = \frac{\triangle}{32}$	$\frac{1}{5} = \frac{\square}{100}$	$\frac{5}{6} = \frac{\square}{48}$

7. In Exercise **5a**, when you wanted to express $\frac{3}{8}$ as an equivalent fraction with a denominator of 40, did you think: "What number times 8 equals 40?" Then what did you do to find the missing numerator? Do you see that the procedure is easy because 8 is a *factor* of 40? In each case in Exercise **5**, was the smaller denominator a factor of the larger denominator? Is the number that we use as the multiplier the other factor necessary to obtain the larger denominator?

1.a. What is $\frac{8}{8} + \frac{2}{8}$? Express $\frac{10}{8}$ as a mixed fraction in simplest form. $\frac{8}{8} + \frac{2}{8}$

b. Do you think that if we used other names for $\frac{8}{8}$ and $\frac{2}{8}$ we would still have an expression of equal value; that is, an expression that names the same number? Let us see. Is $\frac{8}{8}$ equivalent to $\frac{4}{4}$? Is $\frac{2}{8}$ equivalent to $\frac{1}{4}$? What is $\frac{4}{4} + \frac{1}{4}$ expressed as a mixed fraction? Do we get the same value for the expression?

2.a. Now let us work with another expression. Does $\frac{8}{4}$ name the number 2? What is the mixed fraction for $2 + \frac{1}{2}$? Then is $2\frac{1}{2}$ the value of the expression $\frac{8}{4} + \frac{1}{2}$? $\frac{8}{4} + \frac{1}{2}$

b. Now begin with $\frac{8}{4} + \frac{1}{2}$ again. Is $\frac{2}{4}$ another name for $\frac{1}{2}$? What is $\frac{8}{4} + \frac{2}{4}$? Express your answer as a mixed fraction in simplest form. Do you get the same value as in **a**?

▶ Different names for the same number can be substituted in an expression without changing the value of the expression.

3. For each fraction in the box, write equivalent fractions with the denominators named.

$\boxed{\frac{1}{2}}$ $\frac{?}{4}$ $\frac{?}{6}$ $\frac{?}{8}$ $\frac{?}{10}$ $\frac{?}{12}$ $\frac{?}{14}$ $\frac{?}{16}$

$\boxed{\frac{5}{6}}$ $\frac{?}{12}$ $\frac{?}{18}$ $\frac{?}{24}$ $\frac{?}{30}$ $\frac{?}{36}$

$\boxed{\frac{2}{3}}$ $\frac{?}{6}$ $\frac{?}{9}$ $\frac{?}{12}$ $\frac{?}{15}$ $\frac{?}{18}$ $\frac{?}{21}$ $\frac{?}{24}$

$\boxed{\frac{1}{4}}$ $\frac{?}{8}$ $\frac{?}{12}$ $\frac{?}{16}$ $\frac{?}{20}$ $\frac{?}{24}$

4. You have been working with addition involving like fractions. Now test your imagination and your understanding of the work in Exercises **1–2.** Using the sets of equivalent fractions you wrote for Exercise **3**, find the sums for these examples.

a. $\frac{1}{2} + \frac{1}{4} = \triangle$ **b.** $\frac{2}{3} + \frac{5}{6} = \square$ **c.** $\frac{1}{2} + \frac{5}{6} = \square$

d. $\frac{1}{2} + \frac{2}{3} = \triangle$ **e.** $\frac{1}{4} + \frac{2}{3} = \square$ **f.** $\frac{1}{4} + \frac{5}{6} = \triangle$

5. Copy each of the sentences and replace the frame with a numeral that will make the sentence true.

a. $\frac{1}{2} + \frac{3}{8} = \frac{\square}{8} + \frac{3}{8}$ **b.** $\frac{2}{3} + \frac{1}{9} = \frac{\triangle}{9} + \frac{1}{9}$

c. $\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{\square}{6}$ **d.** $\frac{5}{8} + \frac{1}{16} = \frac{\square}{16} + \frac{1}{16}$

1. Are the fractions $\frac{1}{3}$ and $\frac{1}{6}$ like fractions? Why not? Fractions that have different denominators are called **unlike fractions.**

$\frac{1}{3} + \frac{1}{6}$

Suppose you are asked to find the sum of $\frac{1}{3}$ and $\frac{1}{6}$. What do you think must be done before you can use the method you know for adding with like fractions? Since 3 is a factor of 6, can you express $\frac{1}{3}$ as sixths? The form below can be used in working the example. Copy the sentence, using the correct numerals to replace the frames.

$$\frac{1}{3} + \frac{1}{6} = \frac{\square}{6} + \frac{1}{6} = \frac{\square + 1}{6}, \text{ or } \frac{\triangle}{6}$$

What is $\frac{3}{6}$ expressed in simplest form?

2. Look at this example. Is 2 a factor of 18? Is 3 a factor of 18? Then can we rewrite the example with the three fractions expressed with a denominator of 18? We say that 18 is a **common denominator** for the fractions. Work the example, using this sentence as a guide.

$\frac{1}{2} + \frac{2}{3} + \frac{5}{18} = ?$

$$\frac{1}{2} + \frac{2}{3} + \frac{5}{18} = \frac{\square}{18} + \frac{\triangle}{18} + \frac{5}{18} = \frac{\square + \triangle + 5}{18}$$

Express the sum as a mixed fraction in simplest form.

Rewrite each example so that the fractions have the same denominator; that is, a common denominator. Then find the sum and express it in simplest form.

3. **a.** $\frac{2}{3} + \frac{1}{12}$ **b.** $\frac{1}{16} + \frac{1}{2}$ **c.** $\frac{7}{8} + \frac{1}{4}$ **d.** $\frac{1}{8} + \frac{3}{16}$

4. **a.** $\frac{7}{5} + \frac{3}{10}$ **b.** $\frac{2}{15} + \frac{1}{5}$ **c.** $\frac{1}{6} + \frac{3}{2}$ **d.** $\frac{1}{4} + \frac{3}{8}$

5. **a.** $\frac{3}{4} + \frac{1}{2} + \frac{1}{12}$ **b.** $\frac{3}{8} + \frac{5}{4} + \frac{1}{16}$

6. Can you work this subtraction example by a method similar to the one used in addition with unlike fractions? Try it. What is the remainder? Check by adding: $\frac{1}{8} + \frac{1}{4} = \triangle$.

$\frac{3}{8} - \frac{1}{4} = \square$

Find the remainders and express in simplest form.

7. **a.** $\frac{1}{3} - \frac{1}{6}$ **b.** $\frac{7}{8} - \frac{1}{2}$ **c.** $\frac{14}{15} - \frac{2}{5}$ **d.** $\frac{4}{5} - \frac{11}{20}$

8. **a.** $\frac{5}{3} - \frac{5}{6}$ **b.** $\frac{3}{5} - \frac{1}{10}$ **c.** $\frac{2}{3} - \frac{4}{15}$ **d.** $\frac{3}{2} - \frac{1}{4}$

1. What fraction with the denominator 8 names the same number as 9, or $\frac{9}{1}$? What fraction with the denominator 8 is equivalent to $\frac{1}{4}$? Then can you find the sum for this model example by writing it as $\frac{72}{8} + \frac{7}{8} + \frac{2}{8} = \square$? What is the sum, expressed as a mixed fraction in simplest form?

$9 + \frac{7}{8} + \frac{1}{4} = \square$

Could you also find this sum by working with the fractions $\frac{7}{8}$ and $\frac{1}{4}$ first? What is $\frac{7}{8} + \frac{1}{4}$ expressed as a mixed fraction in simplest form? Now add this number to 9. Do you need to express 9 as an improper fraction to perform this addition? Which of the methods for this example seems easier to you?

2. Use any method you wish to find the sums.

a. $2 + \frac{1}{2} + \frac{1}{6}$ **b.** $5 + \frac{2}{3} + \frac{5}{12}$ **c.** $\frac{1}{2} + \frac{3}{10} + 8$

d. $\frac{1}{12} + \frac{1}{6} + 4$ **e.** $\frac{5}{8} + \frac{11}{16} + 3$ **f.** $4 + \frac{4}{5} + \frac{7}{15}$

3. Does $1\frac{7}{8} + 2\frac{7}{24}$ equal $(1 + \frac{7}{8}) + (2 + \frac{7}{24})$? Can we rewrite the example as $(1 + 2) + (\frac{7}{8} + \frac{7}{24})$? Why? Find the sum and express it in simplest form.

4. At the right is an addition example in vertical form. Copy, and complete the work for this example.

$$\begin{array}{r} 7\frac{1}{4} = 7\frac{\triangle}{32} \\ \underline{6\frac{27}{32} = 6\frac{27}{32}} \end{array}$$

5. The vertical form is often used in subtraction examples. Look at Example **a**, below. Is the work correct as far as it goes? Copy the example and find the remainder.

a.
$$\begin{array}{r} 8\frac{5}{6} = 8\frac{5}{6} \\ \underline{-5\frac{1}{3} = 5\frac{2}{6}} \end{array}$$

b.
$$\begin{array}{r} 11\frac{1}{4} = 11\frac{3}{12} \\ \underline{-8\frac{5}{12} = 8\frac{5}{12}} \end{array}$$

Now study Example **b**. Is the work correct as far as it goes? What must you do before you can work this example, column by column? What is the missing numeral in this sentence: $11\frac{3}{12} = 10\frac{\triangle}{12}$? Copy the example and find the remainder.

6. What does $\frac{10}{3} - \frac{11}{6}$ equal? Express your answer as a mixed fraction in simplest form.

Will you get the same answer if you express $\frac{10}{3}$ and $\frac{11}{6}$ as mixed fractions and subtract, using the vertical form? Try it.

Subtract, using any method you wish. Express remainders in simplest form. Check, using addition.

	a	b	c	d
7.	$7\frac{5}{16} - 4\frac{1}{8}$	$4\frac{1}{3} - 1\frac{1}{12}$	$\frac{49}{5} - \frac{63}{10}$	$\frac{23}{8} - 1$
8.	$12\frac{1}{6} - 9\frac{2}{3}$	$16\frac{3}{8} - 9\frac{9}{16}$	$7 - 1\frac{5}{9}$	$14\frac{1}{3} - 5\frac{7}{12}$
9.	$27\frac{9}{16} - 8\frac{1}{2}$	$8\frac{5}{8} - 7\frac{3}{4}$	$\frac{85}{8} - \frac{25}{4}$	$9\frac{7}{10} - 9\frac{1}{2}$

10. Is this model sentence true? One way to find out is to find the sum of $\frac{9}{4}$ and $\frac{17}{2}$, and the sum of $1\frac{5}{12}$ and $2\frac{1}{4}$, and then compare the sums. Can you find another, easier way? If so, tell about it. If not, find and compare the two sums.

$$\frac{9}{4} + \frac{17}{12} = 1\frac{5}{12} + 2\frac{1}{4}$$

Copy and add or subtract as the signs tell you. Express answers in simplest form.

	a	b	c	d	e	f
11.	$8\frac{5}{6}$ $+ 8\frac{2}{3}$	$6\frac{9}{16}$ $- 4\frac{5}{8}$	$8\frac{1}{2}$ $+ 4\frac{1}{6}$	$5\frac{7}{8}$ $+ 7\frac{1}{4}$	$8\frac{5}{12}$ $- 7\frac{3}{4}$	$3\frac{7}{10}$ $+ 5\frac{1}{2}$
12.	$3\frac{1}{16}$ $8\frac{3}{8}$ $+ 5\frac{3}{4}$	$4\frac{1}{2}$ $9\frac{2}{5}$ $+ 5\frac{3}{10}$	$5\frac{3}{8}$ $5\frac{3}{8}$ $+ 6\frac{1}{4}$	$12\frac{1}{4}$ $+ 5\frac{1}{20}$	$8\frac{1}{2}$ $+ \frac{5}{6}$	$9\frac{1}{3}$ $- 2\frac{11}{12}$
13.	$8\frac{3}{4}$ $5\frac{5}{6}$ $+ 6\frac{5}{12}$	$11\frac{1}{2}$ $- 2\frac{3}{4}$	$12\frac{5}{24}$ $+ 8\frac{5}{6}$	$4\frac{3}{5}$ $- 4\frac{1}{10}$	$4\frac{1}{6}$ $6\frac{2}{3}$ $+ 6\frac{1}{3}$	$1\frac{5}{8}$ $4\frac{5}{16}$ $+ 3\frac{1}{8}$

Find the numeral that will make each sentence true.

	a	b	c
14.	$7\frac{1}{5} + \square = 9\frac{1}{10}$	$3\frac{1}{6} + 2\frac{5}{12} = \triangle$	$\square - 2\frac{3}{4} = 1\frac{1}{8}$
15.	$\frac{3}{7} + \frac{13}{14} = \triangle$	$\square - 2\frac{1}{2} = 5\frac{1}{4}$	$N + 1\frac{5}{16} = 3\frac{1}{4}$
16.	$\frac{8}{3} + \frac{11}{6} = \square + \frac{1}{2}$	$4\frac{3}{5} + \triangle = 8\frac{1}{15}$	$1\frac{2}{3} + 3\frac{2}{9} + \frac{1}{18} = \square$

17.a. $(6\frac{1}{3} + 1\frac{1}{6}) - 1\frac{1}{6} = \triangle$ **b.** $(\square + 3\frac{1}{4}) - 3\frac{1}{4} = 8$

18. Without adding, try to tell whether these sentences are true or false. Explain your reasoning.

a. $\frac{11}{12} + \frac{5}{6} > 2$ **b.** $\frac{99}{100} + \frac{49}{50} < 2$ **c.** $(\frac{19}{16} + \frac{5}{4}) + \frac{3}{2} < 3$

1. The Thomas family went on an overnight fishing trip in Georgian Bay. They traveled for $3\frac{3}{4}$ hours to a small island, fished for $1\frac{1}{2}$ hours, and then explored the island for 2 hours. How much time did they spend in these activities?

2. Dave caught a bass that weighed $3\frac{7}{8}$ pounds. Then his brother Glenn caught a fish that weighed $4\frac{1}{4}$ pounds. Their younger sister Mary caught one that weighed $4\frac{11}{16}$ pounds. What is the difference in weight between the fish caught by Mary and the fish caught by Dave?

3. What is the difference in weight between Mary's fish and Glenn's fish? What is the total weight of the three fish?

4. Below are two number sentences that use facts given in Problem **2**. Make up a problem that each sentence might be used to solve. Then answer the question asked.

a. $4\frac{1}{4} - 3\frac{7}{8} = \square$ **b.** $(3\frac{7}{8} + 4\frac{1}{4}) - 4\frac{11}{16} = \triangle$

5. The large gasoline tank in the boat held 20 gallons. There were also two small reserve tanks with 5 gallons in each. Mr. Thomas estimated that $18\frac{3}{4}$ gallons had been used from the large tank. How many gallons of gasoline were left in it? How many gallons were left altogether?

6. When the Thomas family stopped to refuel at a neighboring island the next day, $17\frac{9}{10}$ gallons were put into the big tank and $3\frac{1}{5}$ gallons into the reserve tank they had been using. How many gallons of gasoline did they buy?

7. The two boys had the job of taking down the tents and packing them. In the race to see who would finish first, Dave took $11\frac{1}{6}$ minutes and Glenn took $11\frac{1}{3}$ minutes. Who finished first? How much longer did his brother take?

1. The table shows the number of cars produced by four manufacturers during the spring months of a recent year.

American Motors	92,331
Chrysler	106,212
Ford	390,514
General Motors	755,702

Round each number to the nearest thousand; to the nearest ten thousand; to the nearest fifty thousand.

2. You know that 10^2 means 10×10, or 100. In a similar way, express what each of the following means.

a. 10^3 **b.** 10^4 **c.** 10^5 **d.** 10^6 **e.** 10^7

Tell what the relationship is between the exponent and the number of zeros in the numeral you write for the product.

3. Express each of the following numbers as a power of 10.

a. 10,000,000 **b.** 100,000 **c.** 100,000,000

4. As you know, $3{,}000 = 3 \times 1{,}000$, or 3×10^3. Express each of the following as a single digit times 10 or a power of 10.

a. 70 **b.** 4,000 **c.** 500 **d.** 60,000 **e.** 800

5. Is the following sentence true: $482 = 400 + 80 + 2$? Is it also true that $400 + 80 + 2 = (4 \times 10^2) + (8 \times 10) + 2$?

Express each of these numbers in the same two ways.

a. 375 **b.** 8,736 **c.** 13,479 **d.** 804 **e.** 54

Copy each sentence and replace N with a single numeral that will make the sentence true.

	a	b
6.	$3 \times 10^2 = N$	$(5 \times 10^3) + (2 \times 10^2) + 1 = N$
7.	$N = (4 \times 10^4) + (2 \times 10^2)$	$(3 \times 10^6) + (9 \times 10^3) + 6 = N$

8. Find the number named by each of the following.

a. 6^2 **b.** 3^4 **c.** 2^3 **d.** 2^4 **e.** 6^3 **f.** 5^3

9. What number is named by 2^5? by 5^2? Is this sentence true: $5^2 < 2^5$? Is this sentence also true: $2^5 > 5^2$? Tell which of the following are true and which are not.

a. $3^5 < 15^2$ **b.** $6^3 < 4^4$ **c.** $2^4 > 4^2$ **d.** $16^2 > 8^3$

Copy and add. Check by adding in the other direction.

	a	b	c	d	e
1.	\$8.76	\$6.00	205	89	300
	2.49	.47	490	447	682
	.98	2.05	576	50	80
	.09	5.50	607	852	239
2.	\$50.70	3,504	\$54.08	\$ 3.49	1,980
	8.73	7,876	8.07	55.59	608
	6.57	653	2.03	48.00	7,090
3.	8,256	82,256	809	\$48.47	426
	9,409	49,683	358	4.96	75
	3,505	57,658	482	65.25	50
	8,629	46,573	507	7.08	728

Copy, subtract, and check.

4.	3,126	1,525	7,305	6,300	4,000
	1,748	976	2,548	2,854	2,976
5.	\$124.40	55,325	68,478	\$700.00	80,500
	66.08	28,769	48,509	328.36	32,009
6.	55,487	943,045	63,245	348,126	520,430
	19,498	646,359	9,486	139,348	5,786

7. Find the correct numerals for these sentences.

a. $95 + \square = 172$
b. $146 + 327 = 327 + \triangle$
c. $\square - 386 = 496$
d. $(\triangle + 83) + 47 = 155$
e. $(263 + 75) - 75 = N$
f. $(824 - \square) + 369 = 824$
g. $N - 2{,}483 = 879$
h. $46{,}365 + \square = 46{,}365$

8. During two weeks the number of cars produced in the United States was 134,084 the first week and 137,944 the second week. How many more cars were produced the second week than the first? How many cars were produced during the two weeks?

Find a numeral that will make each sentence true.

	a	b
1.	$7 \times 67 = \square \times 7$	$(13 \times 25) \times 4 = \triangle \times 100$
2.	$372 \times \triangle = 0$	$3 \times (8 + \square) = (3 \times 8) + (3 \times 5)$
3.	$\square \times 36{,}372 = 36{,}372$	$18 \times 42 = (18 \times \square) + (18 \times 2)$

Copy, multiply, and check.

	a	b	c	d	e
4.	406 38	327 126	8,025 53	580 29	647 305
5.	439 306	729 478	9,305 64	867 870	6,085 52

6. In your work for Exercise **4b**, does the first partial product mean 1,962? What does the second partial product, "654," mean? What does the third partial product, "327," mean?

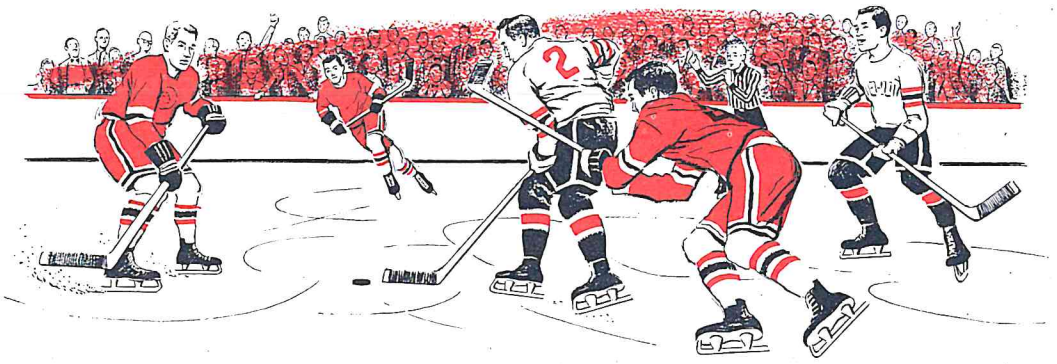
7. Is this sentence true: $52 \times 39 = 2{,}028$? Then what is the quotient for this example: $39\overline{)2{,}028}$? What is the quotient and remainder for this example: $39\overline{)2{,}058}$? How can you check your answer to the second example?

8. What is the largest number of hundreds that will make this sentence true: $N \times 27 < 21{,}720$? Then what is the first quotient figure for this example: $27\overline{)21{,}720}$? Copy the example and find the quotient. Check your answer.

Copy, divide, and check.

	a	b	c	d	e
9.	$44\overline{)1{,}305}$	$62\overline{)4{,}940}$	$35\overline{)32{,}011}$	$87\overline{)3{,}505}$	$59\overline{)19{,}331}$
10.	$56\overline{)35{,}300}$	$53\overline{)5{,}079}$	$58\overline{)2{,}917}$	$80\overline{)16{,}012}$	$29\overline{)26{,}850}$
11.	$74\overline{)\$50.64}$	$24\overline{)81{,}920}$	$66\overline{)\$463.23}$	$97\overline{)1{,}859}$	$29\overline{)10{,}498}$

12. The crew of a lighthouse is on duty 20 days and off 6 days. About how many 26-day periods are there in a year?



1. At an international amateur ice hockey game, there was a sellout crowd of 4,065. At another game in the same tournament, the crowd was only 2,867. How many fewer persons were at the second game?

2. The total income from sales of tickets was $62,482 to the nearest dollar, and expenses were $70,589. How much more were the expenses than the income from sales of tickets?

3. The average price of a ticket was $3.00. About how many tickets would it have been necessary to sell in order to meet the total expenses of $70,589?

4. The sponsors of the American team estimated that the cost per man for the tournament would be $780. The team had 16 men. What was the estimated cost for the team?

5. Only a year earlier, one of the players had bought an outfit on his own. The prices were as follows: shoes and skates, $29.50; pants, $6.88; stockings, $1.59; shin guards, $6.89; gloves, $17.88; sweater, $3.98. What was the total cost of the outfit?

6. The estimated cost for bringing the team representing Sweden to the tournament was $17,728. The team had 16 men. What was the estimated cost per man?

7. For each game there were two officials. One received a fee of $50 plus expenses, and the other $45 plus expenses. For the 28 games played, what was the amount of the officials' fees?

1. To find the sum of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{1}{12}$, do we rewrite the example so that it has like fractions? Why do we do this?

$\frac{2}{3} + \frac{5}{6} + \frac{1}{12} = \square$

Study the denominators. Can 12 be divided evenly by 3? Can 12 be divided evenly by 6? In other words, is each of the smaller denominators a factor of the larger denominator? Then what do we use as a common denominator? Work the example.

2. Now look at this example. Can 5 be divided evenly by 4? Then can we use 5 as a common denominator? Do you see that we need a different method if we are to write the example with like fractions?

$\frac{2}{5} + \frac{3}{4} = N$

One way to find like fractions that are names for $\frac{2}{5}$ and $\frac{3}{4}$ is to make a chart of fractions that are equivalent to each of these fractions. Do this, using the form shown at the right.

$\boxed{\frac{2}{5}}$	$\frac{?}{10}$	$\frac{?}{15}$	$\frac{?}{20}$	$\frac{?}{25}$	$\frac{?}{30}$
$\boxed{\frac{3}{4}}$	$\frac{?}{8}$	$\frac{?}{12}$	$\frac{?}{16}$	$\frac{?}{20}$	$\frac{?}{24}$

Which two fractions in your chart are *like fractions*? Does $\frac{2}{5} + \frac{3}{4}$ equal $\frac{8}{20} + \frac{15}{20}$? Find the sum.

3. The common denominator to use with $\frac{2}{5}$ and $\frac{3}{4}$ may also be found mentally. Must the common denominator be greater than 5? Do we want to be able to divide this number evenly by both 4 and 5? In other words, should it have both 4 and 5 as factors? You have already found that 20 is a common denominator. Do you see how 20 is related to 4 and 5?

4. For a common denominator to use with these fractions, must we choose a number that has both 3 and 7 as factors? Can we use 21 as a common denominator: $\frac{1}{3} = \frac{\square}{21}$; $\frac{5}{7} = \frac{\triangle}{21}$? Work the example.

$\frac{1}{3} + \frac{5}{7} = N$

5. Do you see a pattern? Then tell what common denominator you would use if the fractions were $\frac{8}{9}$ and $\frac{1}{2}$; $\frac{1}{5}$ and $\frac{1}{3}$; $\frac{1}{4}$ and $\frac{5}{7}$. Now find the sums or remainders for these examples.

a. $\frac{8}{9} + \frac{1}{2}$ **b.** $\frac{1}{3} - \frac{1}{5}$ **c.** $\frac{1}{4} + \frac{5}{7}$ **d.** $\frac{7}{8} - \frac{2}{3}$

1. Complete this chart so that the fractions in each set are equivalent to the fraction in the box.

$\boxed{\frac{3}{4}}$ $\frac{?}{8}$ $\frac{?}{12}$ $\frac{?}{16}$ $\frac{?}{20}$ $\frac{?}{24}$ $\frac{?}{28}$ $\frac{?}{32}$ $\frac{?}{36}$ $\frac{?}{40}$ $\frac{?}{44}$ $\frac{?}{48}$ $\frac{?}{52}$ $\frac{?}{56}$ $\frac{?}{60}$

$\boxed{\frac{5}{6}}$ $\frac{?}{12}$ $\frac{?}{18}$ $\frac{?}{24}$ $\frac{?}{30}$ $\frac{?}{36}$ $\frac{?}{42}$ $\frac{?}{48}$ $\frac{?}{54}$ $\frac{?}{60}$

2. Using the method you discovered on page 67 in Exercises **3–5**, what common denominator would you use for this example? $\frac{3}{4} + \frac{5}{6} = N$

Now look at your completed chart. Does it show other numbers that could be used as a common denominator? Could 36 be used? 48? 60?

Do you think there are many numbers that could be used as a common denominator? Is there a number less than 24 that can be used as a common denominator? Name it. Is there a number less than 12 that can be used as a common denominator? We call 12 the **least common denominator.** Work the example.

3.a. What is a common denominator to use for the example $\frac{5}{6} + \frac{1}{8} = \square$? What is the least common denominator?

b. Must the least common denominator be larger than 8? Must it have 8 as a factor? Do you think we can find the least common denominator by doubling 8, tripling 8, and so on, until we find a number that also has 6 as a factor? What is 2×8? Does 16 have 6 as a factor? What is 3×8? Does 24 have 6 as a factor? Then 24 is the least common denominator. Work the example.

4. For an example such as $\frac{5}{6} + \frac{1}{3} + \frac{4}{9} = \square$, could we follow the same procedure as in Exercise **3b**, but use 9 (the largest denominator)? What is 2×9? Is 3 a factor of 18? Is 6 a factor of 18? Work the example.

Find the least common denominators; then add or subtract as the signs tell you.

	a	b	c	d
5.	$\frac{5}{8} + \frac{3}{12}$	$\frac{1}{8} + \frac{3}{10}$	$\frac{5}{6} - \frac{5}{9}$	$\frac{5}{6} + \frac{5}{8}$
6.	$35\frac{5}{6} - 43\frac{3}{8}$	$\frac{19}{6} + \frac{1}{3} + \frac{3}{5}$	$\frac{7}{4} + \frac{5}{6} + \frac{1}{3}$	$\frac{13}{3} - \frac{14}{5}$

Copy, find the least common denominator, and add or subtract as the signs tell you.

	a	b	c	d	e
1.	$16\frac{5}{8}$ $-\ 7\frac{1}{3}$	$10\frac{3}{4}$ $-\ 7\frac{2}{5}$	$32\frac{5}{6}$ $-\ 7\frac{5}{8}$	$52\frac{3}{5}$ $-\ 5\frac{1}{3}$	$9\frac{2}{3}$ $-\ 7\frac{1}{4}$
2.	$15\frac{1}{3}$ $6\frac{3}{5}$ $+\ 3\frac{1}{3}$	$2\frac{1}{6}$ $9\frac{1}{5}$ $+\ 4\frac{1}{6}$	$6\frac{1}{4}$ $4\frac{1}{6}$ $+\ 5\frac{1}{2}$	$5\frac{3}{10}$ $7\frac{1}{2}$ $+\ 8\frac{1}{3}$	$5\frac{1}{3}$ $7\frac{1}{2}$ $+\ 3\frac{5}{8}$

Using what you know about addition and subtraction, find the missing fractions.

	a	b
3.	$16\frac{3}{5} - 8\frac{1}{2} = \triangle$	$\square - 4\frac{2}{3} = 1\frac{1}{4}$
4.	$17\frac{1}{4} - \triangle = 6\frac{5}{8}$	$10\frac{2}{7} + \square = 11\frac{1}{3}$
5.	$\square - 3\frac{1}{6} = 4\frac{7}{8}$	$\frac{5}{6} - \frac{2}{9} = \triangle$
6.	$N - 8\frac{1}{8} = 5\frac{3}{10}$	$3\frac{1}{9} - 2\frac{1}{15} = \square$
7.	$\frac{7}{5} - \frac{11}{9} = \square$	$(3\frac{1}{5} + \frac{5}{7}) - \triangle = 3\frac{1}{5}$
8.	$(2\frac{1}{6} + \square) + \frac{4}{5} = 2\frac{1}{6}$	$\frac{3}{2} + \frac{5}{6} + \frac{6}{5} = \triangle$
9.	$8\frac{1}{5} + N = 12\frac{1}{3}$	$\frac{5}{9} + 3\frac{1}{2} = \triangle + \frac{1}{18}$
10.	$7\frac{3}{4} - \triangle = 3\frac{3}{5}$	$\square - 2\frac{3}{8} = 3\frac{3}{7}$
11.	$\frac{2}{3} + \frac{1}{7} + 1\frac{5}{14} = \square$	$2\frac{1}{2} + \square = 2\frac{3}{5}$

Copy and add or subtract as the signs tell you.

	a	b	c	d	e
12.	$14\frac{3}{8}$ $+\ 2\frac{11}{12}$	$3\frac{1}{10}$ $+\ 1\frac{3}{8}$	$15\frac{3}{4}$ $-\ 6\frac{5}{6}$	$8\frac{1}{6}$ $-\ 3\frac{3}{10}$	$\frac{5}{6}$ $+\ \frac{7}{9}$
13.	$5\frac{3}{10}$ $+\ 4\frac{5}{6}$	$13\frac{1}{6}$ $-\ 3\frac{4}{9}$	$15\frac{1}{8}$ $-\ 7\frac{1}{6}$	$7\frac{3}{4}$ $+\ 9\frac{5}{9}$	$12\frac{1}{4}$ $-\ 4\frac{7}{10}$
14.	$13\frac{1}{2}$ $-\ 5\frac{5}{7}$	$10\frac{7}{8}$ $+\ 7\frac{5}{6}$	$12\frac{3}{8}$ $-\ 3\frac{1}{3}$	$3\frac{5}{6}$ $+\ 4\frac{4}{9}$	$12\frac{3}{10}$ $-\ 2\frac{5}{6}$

15. A tank will hold $6\frac{1}{4}$ gallons. How many more gallons are needed to fill the tank after $2\frac{7}{10}$ gallons are put into it?

1. For his science show project, Jim needed three pieces of board. One had to be $2\frac{1}{2}$ feet long, another $3\frac{1}{4}$ feet long, and the other $3\frac{2}{3}$ feet long. Jim bought a board 12 feet long. What length of board will be left over after the pieces are cut?

In solving this problem, there are two questions to be answered. First, what is the combined length of board needed for the three pieces? Second, what length of board will be left over?

Suppose we let $\square$ represent the number of feet in the combined length of the three pieces of board, and $\triangle$ represent the number of feet of board that will be left over. Do the following two number sentences describe the situation?

a. $2\frac{1}{2} + 3\frac{1}{4} + 3\frac{2}{3} = \square$ **b.** $12 - \square = \triangle$

Find the correct numeral for Sentence **a**; then use this result for your work in Sentence **b**. What length of board will be left over?

Do you see that number sentences can help you analyze a problem before you do any calculating?

2. Anne and Jessie made decorations for the show. Anne worked $\frac{3}{4}$ hour one afternoon and $1\frac{1}{3}$ hours another afternoon. Jessie worked just one afternoon, but it was a Saturday and she spent $2\frac{4}{5}$ hours making decorations. How much more time did Jessie spend than Anne?

3. In describing their work, Jessie said: "In all, we worked a little less than 5 hours." How much less than 5 hours did they work?

If you make any mistakes, study the pages given at the right of the exercises.

EXTRA PRACTIC 285–286

1. Copy, and write the correct numeral. 45

a. $\frac{7}{8} = \frac{\square}{24}$ **b.** $\frac{\square}{30} = \frac{4}{5}$ **c.** $\frac{18}{45} = \frac{2}{\triangle}$

2. Express as a whole number, a proper fraction in simplest form, or a mixed fraction in simplest form. 47

a. $\frac{4}{64}$ **b.** $\frac{13}{5}$ **c.** $\frac{81}{6}$ **d.** $\frac{36}{9}$ **e.** $\frac{21}{14}$ **f.** $\frac{18}{36}$

3. Copy, add, and express the sums in simplest form. 48–49

a. $\frac{5}{16} + \frac{7}{16}$ **b.** $\frac{4}{7} + \frac{3}{7} + \frac{8}{7}$ **c.** $\frac{8}{3} + \frac{2}{3}$

4. Subtract. Express remainders in simplest form. 50

a. $\frac{5}{16} - \frac{3}{16}$ **b.** $\frac{17}{19} - \frac{5}{19}$ **c.** $\frac{7}{8} - \frac{5}{8}$

5. Add or subtract as the signs tell you. Express your answers in simplest form. 51

a. $7\frac{1}{9} + 5\frac{5}{9}$ **b.** $18\frac{3}{4} - 6\frac{1}{4}$ **c.** $8\frac{5}{8} - 7\frac{3}{8}$

6. Copy, and write the correct numeral. 55

a. $4\frac{1}{3} - 2\frac{1}{3} = \triangle$ **b.** $\frac{8}{5} + \frac{14}{5} + \triangle = 8$ **c.** $2\frac{3}{7} + \square = 3\frac{1}{7}$

For each of the following, add or subtract as the signs tell you. Write answers in simplest form.

	a	b	c	d	e	
7.	$17\frac{9}{16}$ $+ 8\frac{9}{16}$	$12\frac{4}{15}$ $- 7\frac{1}{15}$	$4\frac{1}{5}$ $+ 8\frac{1}{5}$	$9\frac{1}{8}$ $- 3\frac{5}{8}$	$23\frac{3}{8}$ $- 15\frac{1}{8}$	59–60
8.	$1\frac{2}{3}$ $+ 15\frac{1}{12}$	$7\frac{1}{2}$ $- 2\frac{3}{5}$	$15\frac{5}{16}$ $- 5\frac{1}{2}$	$16\frac{2}{5}$ $- 8\frac{1}{2}$	$3\frac{1}{2}$ $+ 7\frac{1}{5}$	67–69
9.	$11\frac{1}{6}$ $- 3\frac{7}{8}$	$2\frac{1}{3}$ $3\frac{1}{5}$ $+ 5\frac{1}{10}$	$6\frac{1}{3}$ $8\frac{1}{2}$ $+ 3\frac{5}{6}$	$16\frac{1}{4}$ $- 9\frac{5}{6}$	$13\frac{1}{10}$ $- 8\frac{5}{6}$	67–69

10. The weights of three boys are $87\frac{7}{8}$ pounds, $90\frac{1}{4}$ pounds, and $89\frac{1}{2}$ pounds. What is the weight added to an elevator's load after they enter? 70

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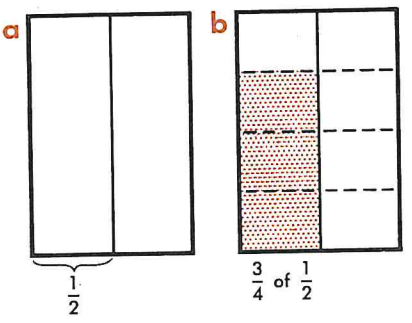
3 A New Kind of Sentence

Do the sentences at the right seem strange to you? You have not worked with such sentences before. But when you worked with a sentence such as $\frac{2}{5} + \frac{1}{10} = \square$, did you look for a single fraction to replace the frame? Let us do the same thing with a sentence such as $\frac{3}{4}$ of $\frac{1}{2} = \square$; that is, let us look for a single fraction that means the same as "$\frac{3}{4}$ of $\frac{1}{2}$."

$\frac{3}{4}$ of $\frac{1}{2} = \square$

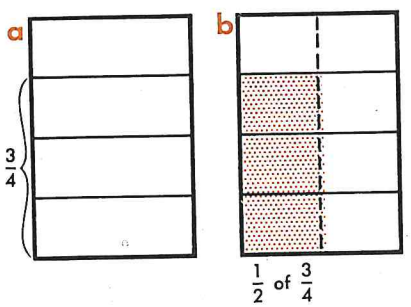
$\frac{2}{3}$ of $\frac{4}{5} = \triangle$



1. We begin with the expression $\frac{3}{4}$ of $\frac{1}{2}$. Illustration **a** shows a sheet of paper that has been marked off into halves. In **b**, horizontal lines have been drawn so that each half is marked off into four equal parts. Has $\frac{3}{4}$ of one of the halves been shaded? We can describe this as "$\frac{3}{4}$ of $\frac{1}{2}$."

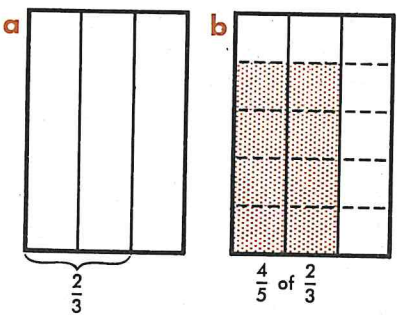
Study illustration **b**. How many equal parts does the sheet of paper have? How many of these parts are shaded? Then what fractional part of the entire sheet of paper is shaded? Do both "$\frac{3}{8}$" and "$\frac{3}{4}$ of $\frac{1}{2}$" tell what fractional part of the sheet is shaded? Then do you agree that $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$?

2. You have just found what $\frac{3}{4}$ of $\frac{1}{2}$ is. Now let us "reverse" the situation and find what $\frac{1}{2}$ of $\frac{3}{4}$ is. This time, illustration **a** shows a sheet of paper marked off into fourths. In **b**, a vertical line has been drawn so that each fourth is marked off into two equal parts. Has $\frac{1}{2}$ of $\frac{3}{4}$ been shaded?



Study illustration **b**. How many equal parts does the paper have? How many are shaded? Then what part of the paper is shaded? Is this sentence true: $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$?

3. As another example, let us find what $\frac{4}{5}$ of $\frac{2}{3}$ is. In **a**, a sheet of paper has been marked off into thirds. In **b**, horizontal lines have been drawn so that each third is marked off into five equal parts. Has $\frac{4}{5}$ of $\frac{2}{3}$ been shaded?

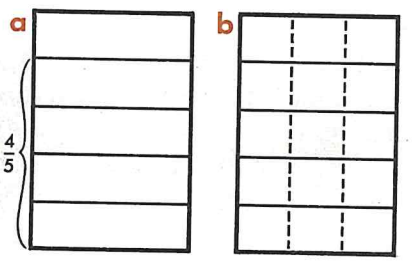


Study illustration **b**. How many equal parts does the paper have? How many are shaded? Then what fractional part of the paper is shaded? Is this sentence true: $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15}$?

4. Now try this sentence.

$$\frac{2}{3} \text{ of } \frac{4}{5} = \triangle$$

In **a**, the sheet of paper has been marked off into fifths. In **b**, vertical lines have been drawn so that each fifth is marked off into three equal parts. Copy illustration **b** and shade $\frac{2}{3}$ of $\frac{4}{5}$.



How many equal parts does the sheet of paper have? How many are shaded? Then what is the correct replacement for the frame in this sentence: $\frac{2}{3}$ of $\frac{4}{5} = \triangle$?

5. Look back over the work you have been doing. In each case, did you find a single fraction that meant the same as "$\frac{3}{4}$ of $\frac{1}{2}$," "$\frac{2}{3}$ of $\frac{4}{5}$," and so on? When you did this, you were naming the *product* of two fractional numbers. In other words, you were multiplying with fractions.

Instead of writing $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$, we can use a "×" sign and write: $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$. Instead of $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$, we can write: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. Copy your other two sentences, using the "×" sign instead of the word "of."

1. Suppose you are asked to find the correct replacement for the frame in this sentence: $\frac{5}{6} \times \frac{1}{2} = \square$.

One way to find the answer is to think of "$\frac{5}{6}$ of $\frac{1}{2}$" and use diagrams. That is, take a sheet of paper and mark it off into halves; then draw lines so that each half is marked off into six equal parts. Shade $\frac{5}{6}$ of $\frac{1}{2}$. What fractional part of the paper is shaded? Then what does $\frac{5}{6} \times \frac{1}{2}$ equal?

2. Suppose you wanted to find the correct replacement in this sentence: $\frac{1}{2} \times \frac{5}{6} = \square$. Can you think of "$\frac{1}{2}$ of $\frac{5}{6}$" and use diagrams to find the answer? First, mark off your sheet of paper into sixths, then draw lines so that each sixth is marked off into two equal parts.

Now shade $\frac{1}{2}$ of $\frac{5}{6}$. What is the correct replacement for the frame in this sentence: $\frac{1}{2} \times \frac{5}{6} = \square$?

3. You have been using diagrams to find the product of two fractional numbers. Is there a pattern we can use to find the answers without drawing diagrams? Let us see. Shown below are three of the multiplication sentences with which you have been working.

a. $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ **b.** $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ **c.** $\frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$

Look at Sentence **a**. Do you see how the numerator of the answer can be found by using the numerators of the original fractions? Do you see how the denominator of the answer can be found by using the denominators of the original fractions? Is this sentence true: $\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2}$, or $\frac{3}{8}$?

Look at Sentence **b**. How can the numerator of the answer be obtained from the numerators of the original fractions? How can the denominator of the answer be obtained from the denominators of the original fractions? Write a sentence similar to the one at the end of the preceding paragraph.

Can you write a similar sentence, using the original fractions in Sentence **c**? Do it.

Do Sentences **a**, **b**, and **c** give the same results as the diagrams you drew?

Do you see a pattern for finding the product of fractional numbers? Then write the fraction that you think is the correct replacement in each of these sentences.

	a	b	c	d
4.	$\frac{1}{4} \times \frac{1}{3} = \triangle$	$\frac{7}{8} \times \frac{1}{2} = \square$	$\frac{3}{4} \times \frac{1}{5} = N$	$\frac{1}{3} \times \frac{2}{9} = \square$

5. Check your answers to Exercises **4a–c** by drawing diagrams.

Use the pattern you have discovered to find the missing fraction in each of these sentences.

	a	b	c	d
6.	$\frac{1}{4} \times \frac{5}{6} = \triangle$	$\frac{2}{5} \times \frac{3}{5} = \square$	$\frac{1}{2} \times \frac{1}{2} = \square$	$\frac{1}{4} \times \frac{1}{4} = \triangle$
7.	$\frac{1}{7} \times \frac{2}{3} = N$	$\frac{4}{5} \times \frac{1}{3} = \triangle$	$\frac{3}{8} \times \frac{1}{2} = \square$	$\frac{2}{7} \times \frac{3}{7} = N$
8.	$\frac{1}{5} \times \frac{1}{2} = \square$	$\frac{1}{2} \times \frac{1}{5} = \square$	$\frac{3}{4} \times \frac{1}{4} = \triangle$	$\frac{1}{4} \times \frac{3}{4} = \triangle$

9. You know that the order in which you multiply with two whole numbers does not change the product. Is the same thing true in multiplication of fractional numbers? You can check your answer by comparing the products you got for Exercises **8a** and **b**, and for **8c** and **d**. Or try a few other pairs of examples if you are not sure.

10. Sometimes, as you use the pattern, the fraction you get may not be the simplest form for the answer. What is $\frac{2}{3} \times \frac{1}{4}$? What is the simplest form for $\frac{2}{12}$? Then we can write: $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$.

Find the products and express them in simplest form.

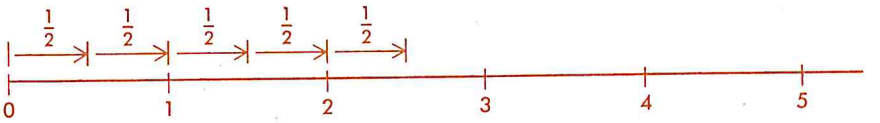
	a	b	c	d
11.	$\frac{7}{10} \times \frac{2}{3} = \square$	$\frac{8}{9} \times \frac{3}{4} = \triangle$	$\frac{3}{8} \times \frac{2}{9} = N$	$\frac{2}{5} \times \frac{5}{6} = \triangle$
12.	$\frac{3}{5} \times \frac{1}{6} = \triangle$	$\frac{5}{9} \times \frac{4}{5} = N$	$\frac{7}{8} \times \frac{2}{7} = \square$	$\frac{1}{3} \times \frac{3}{10} = N$

13. Mrs. Brown left $\frac{2}{3}$ of an apple pie for Jim and his three friends. The boys cut the $\frac{2}{3}$ of the pie carefully, so that each of them received $\frac{1}{4}$ of it. What fractional part of the *entire* pie was each piece? Express your answer in simplest form.

1. Suppose you are asked to find the product for this sentence. You know that 5 can be written in fraction form as $\frac{5}{1}$. Then, using the pattern, can you write this sentence: $\frac{5}{1} \times \frac{1}{2} = \frac{5}{2}$? What is $\frac{5}{2}$ expressed as a mixed fraction?

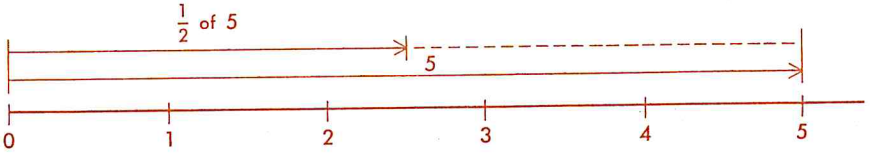
$5 \times \frac{1}{2} = \square$

2. You have seen that with whole numbers a multiplication example can be shown as repeated addition on the number line. That is, 5×3 can be shown as $3 + 3 + 3 + 3 + 3$.



Can $5 \times \frac{1}{2}$ be shown as $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$? Study the drawing. Do you again get $\frac{5}{2}$, or $2\frac{1}{2}$, as the answer?

3. Now let us look at $\frac{1}{2} \times 5$. It is not possible to show $\frac{1}{2} \times 5$ as repeated addition. But if we think of $\frac{1}{2} \times 5$ as "$\frac{1}{2}$ of 5," then we can use the number line as shown here.



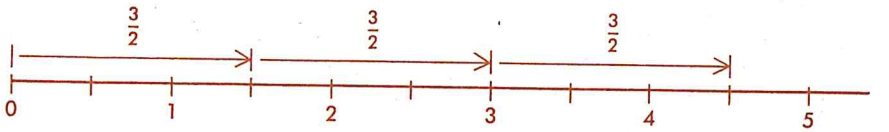
Do we again get $2\frac{1}{2}$ as the answer? Would we also get this answer by using the pattern and multiplying? Use this sentence to see: $\frac{1}{2} \times 5 = \frac{1}{2} \times \frac{5}{1} = \frac{?}{2}$, or $\triangle$.

4. Using the pattern, can you find what $4 \times \frac{3}{5}$ equals? What fraction will you use as a name for 4? Work the example.

Find the products. As you work, try to discover a short cut you can use.

	a	b	c	d
5.	$3 \times \frac{1}{4}$	$\frac{1}{2} \times 7$	$4 \times \frac{6}{7}$	$\frac{2}{5} \times 6$
6.	$9 \times \frac{3}{4}$	$\frac{7}{9} \times 4$	$8 \times \frac{4}{5}$	$\frac{3}{8} \times 11$

7. Look back over your work for Exercises **5** and **6**. Was it really necessary to express 3 as $\frac{3}{1}$, 7 as $\frac{7}{1}$, and so on? How could you have obtained the answer without doing this?



8. Using this drawing of a number line, can you find what $3 \times \frac{3}{2}$ equals? Do it.

9. You know that you can substitute different names for a number in an expression without changing its value. Is $\frac{12}{4}$ another name for 3? Then can you find the product for Exercise **8**, using this sentence: $\frac{12}{4} \times \frac{3}{2} = \triangle$? Do the multiplication. Do you again get $4\frac{1}{2}$ as the answer?

10. What is 5×2? Suppose we rename 5 and 2 as $\frac{15}{3}$ and $\frac{4}{2}$. Can we find the product of 5 and 2 by using this sentence: $\frac{15}{3} \times \frac{4}{2} = \square$? Do the multiplication and express your answer in simplest form. Is 10 again the answer?

▶You first discovered the pattern for multiplication with fractions, using proper fractions. Now look at the sentences you have been working with. In each sentence, are the multiplier and the multiplicand improper fractions? Does using the pattern still result in the correct answer?

$$\frac{12}{4} \times \frac{3}{2} = \triangle$$

$$\frac{15}{3} \times \frac{4}{2} = \square$$

Find the products and express them in simplest form.

11. **a.** $\frac{25}{4} \times \frac{1}{5} = \square$ **b.** $\frac{4}{3} \times \frac{9}{8} = N$ **c.** $\frac{9}{4} \times \frac{1}{2} = \triangle$

12. **a.** $\frac{2}{3} \times \frac{11}{6} = N$ **b.** $\frac{3}{4} \times \frac{4}{3} = \square$ **c.** $\frac{6}{5} \times \frac{5}{3} = \triangle$

13. In Sentence **a**, below, what do the parentheses tell you to do first? What is the product of $\frac{1}{3}$ and $\frac{5}{2}$? of $\frac{5}{6}$ and $\frac{1}{4}$?

a. $(\frac{1}{3} \times \frac{5}{2}) \times \frac{1}{4} = \square$ **b.** $\frac{1}{3} \times (\frac{5}{2} \times \frac{1}{4}) = \square$

Do Sentences **a** and **b** have the same fractions? Are they paired in the same way? Is the product for **b** the same as that for **a**?

14. Now find the product for each sentence below.

a. $\frac{3}{4} \times (\frac{1}{2} \times \frac{7}{5}) = \square$ **b.** $(\frac{3}{4} \times \frac{1}{2}) \times \frac{7}{5} = \square$

▶In finding the product of three fractional numbers, does the way in which we pair them change the result?

1. Barbara and Mary live about $\frac{3}{4}$ of a mile apart. The library where they often arrange to meet is about $\frac{3}{4}$ of the distance from Barbara's house to Mary's house. If Barbara walks this distance, what fractional part of a mile does she walk?

2. When Mary walks from her house to the library, what fractional part of a mile does she walk?

3.a. Harold had a board that was $\frac{5}{6}$ of a yard long. He cut a piece that was $\frac{1}{3}$ of the total length of the board. What fractional part of a yard is the length of the piece he cut?
b. What fractional part of a yard is the length of the board that was left? Express your answer in simplest form.

4. Carl bought $\frac{1}{2}$ pound of candy. He gave $\frac{1}{2}$ of his candy to his younger brother, Larry. Larry ate $\frac{1}{2}$ of the candy he had been given. What fractional part of a pound did Larry eat?

5. Mrs. Martin receives Yellow Stamps with her purchases at the supermarket. In a month she can fill about $\frac{2}{5}$ of a book of stamps. About how many books can she fill in a year?

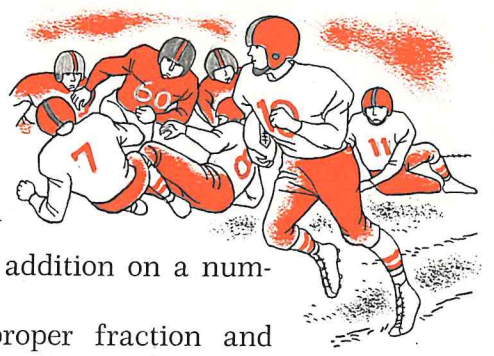
6. You have seen that, in finding the product of three fractional numbers, you can pair the numbers in different ways and still get the same result. For example, what does $(\frac{7}{3} \times \frac{1}{2}) \times \frac{5}{2}$ equal? What does $\frac{7}{3} \times (\frac{1}{2} \times \frac{5}{2})$ equal?

Do you think you can omit parentheses and find the product of the numerators, the product of the denominators, and then write the results as a fraction? That is, is this sentence true?

$$\frac{7}{3} \times \frac{1}{2} \times \frac{5}{2} = \frac{7 \times 1 \times 5}{3 \times 2 \times 2}, \text{ or } \frac{35}{12}$$

Was $\frac{35}{12}$ (or $2\frac{11}{12}$) the answer you got earlier? Now find the products for the following.

a. $\frac{7}{3} \times \frac{1}{5} \times \frac{7}{2}$ **b.** $\frac{2}{7} \times \frac{5}{3} \times \frac{4}{3}$ **c.** $\frac{1}{3} \times \frac{4}{5} \times \frac{3}{2}$



1. During the three months of the football season, the Harrison High quarterback lost an average of $2\frac{1}{2}$ pounds a month. If we want to find how many pounds he lost altogether, can we write this number sentence for the problem: $3 \times 2\frac{1}{2} = \square$?

a. Can the example be shown as repeated addition on a number line? Try it. What answer do you get?

b. Can you also express $2\frac{1}{2}$ as an improper fraction and multiply just as you have been doing? Try this method, too. Do you get the same answer that you did in **a**?

2. In multiplication with whole numbers, is it true that 3×9, or $3 \times (7 + 2)$, equals $(3 \times 7) + (3 \times 2)$? Let us investigate $3 \times 2\frac{1}{2}$ again and see whether we can use this idea with fractional numbers. You know that another name for $2\frac{1}{2}$ is $2 + \frac{1}{2}$. What is 3×2? What is $3 \times \frac{1}{2}$? What is the sum of these two products? Do you get the same answer as before? Then is this sentence true?

$$3 \times 2\frac{1}{2} = (3 \times 2) + (3 \times \frac{1}{2})$$

3.a. Now see whether you can use this idea to find what $2 \times 1\frac{2}{3}$ equals. Can we think of $1\frac{2}{3}$ as $1 + \frac{2}{3}$? What is 2×1? What is $2 \times \frac{2}{3}$? What is $2 + \frac{4}{3}$ expressed as a mixed fraction?

Do you think this sentence is true?

$$2 \times 1\frac{2}{3} = (2 \times 1) + (2 \times \frac{2}{3})$$

b. To check, express $1\frac{2}{3}$ as an improper fraction and multiply again. What improper fraction is equivalent to $1\frac{2}{3}$? What is $2 \times \frac{5}{3}$? Express your answer as a mixed fraction. Does the result agree with your result in **a**?

4. Find the product by renaming each mixed fraction as you did in Exercise **3a**. Use the form of the model sentence.

a. $3 \times 4\frac{1}{3} = \square$ **b.** $6 \times 1\frac{2}{5} = \triangle$ **c.** $5 \times 1\frac{1}{4} = N$

5. Copy the sentences and replace the frames.

a. $3 \times 2\frac{2}{7} = (3 \times \square) + (3 \times \frac{2}{7})$
b. $7 \times 8\frac{1}{5} = (7 \times 8) + (7 \times \triangle)$
c. $4 \times \square = (4 \times 3) + (4 \times \frac{7}{9})$
d. $2\frac{1}{4} \times 5 = (\square \times 5) + (\frac{1}{4} \times 5)$
e. $4\frac{1}{2} \times \frac{3}{4} = (4 \times \triangle) + (\frac{1}{2} \times \triangle)$

6. Look at Exercises **5a–e**. In each case, can you find a single numeral to represent the sum named at the right of the equals sign? Do it. Does this numeral represent the product of the two numbers named at the left of the equals sign?

7. Sometimes we use a vertical form for multiplication with mixed fractions. Study this example. Do you see that $4\frac{1}{4}$ was thought of as $4 + \frac{1}{4}$? Which was multiplied first, the $\frac{1}{4}$ or the 4? What is $7 \times \frac{1}{4}$ expressed as a mixed fraction? What is 7×4? What is the sum of $1\frac{3}{4}$ and 28? Is this the product for the example?

$$\begin{array}{r} 4\frac{1}{4} \\ \times\ 7 \\ \hline 1\frac{3}{4} \\ 28 \\ \hline \end{array}$$

Did we have to express $\frac{7}{4}$ as $1\frac{3}{4}$ immediately? Would we get the same answer by adding 28 and $\frac{7}{4}$? One advantage to the method used is that we have the partial products in column form.

8. In this example, can we think of $3\frac{2}{3}$ as $3 + \frac{2}{3}$? Does it matter whether we use the $\frac{2}{3}$ or the 3 as a multiplier first? Copy the example. What is $\frac{2}{3} \times 12$? What is 3×12? Now complete the example.

$$\begin{array}{r} 12 \\ \times\ 3\frac{2}{3} \\ \hline \end{array}$$

Find the products, using any method you wish.

	a	b	c
9.	$8 \times 3\frac{1}{5} = \square$	$8\frac{5}{6} \times 10 = \triangle$	$12 \times 9\frac{7}{12} = \square$
10.	$13\frac{5}{6} \times 6 = \mathrm{N}$	$9 \times 4\frac{4}{5} = \triangle$	$15\frac{1}{3} \times \frac{1}{4} = \square$

11. Mr. Walker has $5\frac{1}{2}$ acres available for planting pine tree seedlings. If he plants $\frac{1}{6}$ of the $5\frac{1}{2}$ acres each spring, what part of an acre would he plant each spring?

1. Suppose you are asked to find what $1\frac{1}{4} \times 2\frac{1}{3}$ equals. Do you think the following sentences are true?

$$1\tfrac{1}{4} \times 2\tfrac{1}{3} = 1\tfrac{1}{4} \times (2 + \tfrac{1}{3})$$

$$= (1\tfrac{1}{4} \times 2) + (1\tfrac{1}{4} \times \tfrac{1}{3})$$

a. Find what $1\frac{1}{4} \times 2$ equals by any method you wish.
b. Find what $1\frac{1}{4} \times \frac{1}{3}$ equals.
c. Now find what $2\frac{1}{2} + \frac{5}{12}$ equals. Is this number, $2\frac{11}{12}$, the product of $1\frac{1}{4}$ and $2\frac{1}{3}$?

2. Let us do the same example another way. Is the improper fraction $\frac{5}{4}$ another name for $1\frac{1}{4}$? Is $\frac{7}{3}$ another name for $2\frac{1}{3}$? Now find the product: $\frac{5}{4} \times \frac{7}{3} = \square$. Express the answer as a mixed fraction. Do you again get $2\frac{11}{12}$ as the answer? Was it easier to multiply with the improper fractions than with the mixed fractions?

3. It is often more convenient to multiply with improper fractions. So let us explore some ideas that you may find useful in expressing mixed fractions as improper fractions.

a. How many fourths equal 1? Then does 2 equal $2 \times \frac{4}{4}$, or $\frac{8}{4}$? Does $2\frac{3}{4}$ equal $\frac{8}{4} + \frac{3}{4}$, or $\frac{11}{4}$?
b. How many sixths equal 1? Is this sentence true: $5 = 5 \times \frac{6}{6}$, or $\frac{30}{6}$? Does $5\frac{1}{6}$ equal $\frac{30}{6} + \frac{1}{6}$, or $\frac{31}{6}$?

4. Copy the sentences and fill in the correct numerals.

a. $7\frac{1}{2} = \left(7 \times \frac{\square}{2}\right) + \frac{\triangle}{2}$, or $\frac{N}{2}$

b. $6\frac{2}{3} = \left(6 \times \frac{\square}{3}\right) + \frac{\triangle}{3}$, or $\frac{N}{3}$

5.a. Using the replacements you found for Exercise **4a**, find what $(7 \times \square) + \triangle$ equals. Is this number the numerator of the improper fraction $\frac{15}{2}$?

Is this sentence true: $7\frac{1}{2} = \frac{(7 \times 2) + 1}{2}$?

b. Using the replacements you found for Exercise **4b**, find what $(6 \times \square) + \triangle$ equals. Is this number the numerator of the improper fraction $\frac{20}{3}$?

Is this sentence true: $6\frac{2}{3} = \frac{(6 \times 3) + 2}{3}$?

6. Copy the sentences below and fill in the correct replacements.

a. $3\frac{5}{6} = \frac{(3 \times \square) + \triangle}{6}$, or $\frac{N}{6}$

b. $2\frac{5}{8} = \frac{(2 \times \square) + \triangle}{8}$, or $\frac{N}{8}$

c. $8\frac{4}{7} = \frac{(8 \times \square) + \triangle}{7}$, or $\frac{N}{7}$

d. $12\frac{1}{3} = \frac{(12 \times \square) + \triangle}{3}$, or $\frac{N}{3}$

7. In expressing a mixed fraction as an improper fraction, the work can be done mentally. Express each of the following as an improper fraction, doing as much of the work mentally as you can.

a. $1\frac{1}{10}$ **b.** $3\frac{1}{4}$ **c.** $4\frac{7}{8}$ **d.** $7\frac{2}{3}$ **e.** $4\frac{4}{5}$ **f.** $3\frac{4}{9}$

8. Copy and multiply. Express each mixed fraction as an improper fraction. Write your answers as mixed fractions in simplest form or as numerals for whole numbers.

a. $2\frac{1}{3} \times 1\frac{1}{2} = \square$

b. $\frac{4}{5} \times 6\frac{1}{4} = \triangle$

c. $4\frac{2}{5} \times 1\frac{2}{3} = N$

d. $3\frac{1}{3} \times 2\frac{1}{2} = \square$

9. What is 1×3? 1×11? $1 \times \frac{2}{3}$? $1 \times \frac{7}{5}$? What is the product of 1 and any number?

Is $\frac{4}{4}$ another name for 1? Look at this sentence:

$$\frac{4}{4} \times \frac{1}{2} = \frac{4 \times 1}{4 \times 2} = \frac{4}{8}$$

Did we multiply $\frac{1}{2}$ by 1? Is the fraction $\frac{4}{8}$ equivalent to $\frac{1}{2}$? Look at the part in color. Does this remind you of sentences you wrote in your earlier work with equivalent fractions?

You remember that when you multiplied both numerator and denominator by the same number, you got an equivalent fraction. Do you see now that you can think of this as multiplication using some fraction equivalent to 1?

10. By what fraction equivalent to 1 can $\frac{2}{3}$ be multiplied in order to obtain $\frac{10}{15}$? Show the work.

11. When we find that $\frac{14}{16} = \frac{2 \times 7}{2 \times 8}$, do we know that $\frac{14}{16}$ names the same number as $\frac{7}{8}$? If we think of the multiplication pattern "backward," can we think of $\frac{2 \times 7}{2 \times 8}$ as $1 \times \frac{7}{8}$? Explain.

1. Are there other ways in which we can use the idea of fractions that name the number 1? Let us see: $\frac{6}{5} \times \frac{3}{4} = \underline{?}$

a. In Step **a** of this example, how did we get the $\frac{18}{20}$?

b. Can we think of 18 as 2×9? Similarly, 20 can be thought of as $2 \times \underline{?}$.

c. Now, using the multiplication pattern "backward," we write Step **c**.

a. $\frac{6}{5} \times \frac{3}{4} = \frac{18}{20}$

b. $= \frac{2 \times 9}{2 \times 10}$

c. $= \frac{2}{2} \times \frac{9}{10}$

Since $\frac{2}{2}$ is another name for 1, what is the product? Is $\frac{9}{10}$ the simplest form?

Now let us try to find out *why* we were able to form $\frac{2}{2}$ as a name for 1 in this example. Look at the original fractions. Do you see that 2 is a factor of one of the numerators? Which numerator? Is 2 also a factor of one of the denominators? Which denominator? Were we able to use this *common factor* to form the $\frac{2}{2}$ in the model?

2. Use the method of Exercise **1** to find the products. Show all your steps.

a. $\frac{3}{4} \times \frac{5}{1} = \square$ **b.** $\frac{4}{3} \times \frac{7}{8} = \square$ **c.** $\frac{3}{7} \times \frac{5}{9} = \triangle$

3. We can make our own work easier if we think about factors *before* we multiply. Then we can mentally find the common factors in a numerator and a denominator and think of them as a fraction representing 1. Look at these examples.

a. $\frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{\underset{8}{\cancel{16}}} = \frac{1 \times 5}{3 \times 8}$

b. $\frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} = \frac{5 \times 1}{1 \times 3}$

In **a**, is 2 a factor of the 2 and the 16? Then again we can think of the common factor as representing the fraction $\frac{2}{2}$, which will not change the product. To help us remember what we are doing, we divide the 2 and the 16 by 2. What is $2 \div 2$? $16 \div 2$? We draw slash marks(/) and write the 1 and the 8 that are the remaining factors. What is the product?

In **b**, did we use $\frac{4}{4}$ as a name for 1? What other name for 1 was used? How did we get the 5 above the 15 and the 3 below the 9? What is the product?

Copy and multiply. Express mixed fractions as improper fractions before multiplying. Use the short cut if you can.

	a	b	c	d
1.	$1\frac{1}{2} \times 1\frac{3}{5} = \square$	$\frac{9}{7} \times \frac{7}{3} = \triangle$	$\frac{2}{9} \times 1\frac{4}{5} = \square$	$3\frac{1}{6} \times 1\frac{5}{7} = \mathrm{N}$
2.	$5\frac{1}{3} \times 1\frac{1}{16} = \square$	$1\frac{1}{9} \times 2\frac{1}{10} = \square$	$2\frac{2}{5} \times 1\frac{5}{12} = \mathrm{N}$	$\frac{5}{9} \times \frac{3}{10} = \triangle$

In the four examples below, the short cut is used. The questions in Exercises **3–6** are about these examples.

A. $\frac{7}{\cancel{6}_{1}} \times \frac{\cancel{6}^{1}}{5} = \triangle$

B. $\frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{2}_{1}} = \square$

C. $\frac{5}{\cancel{8}_{2}} \times \cancel{12}^{3} = \mathrm{N}$

D. $\frac{\cancel{4}^{2}}{9} \times \frac{7}{\cancel{8}_{4}} = \triangle$

3. What fraction or fractions that represent 1 are being used in Example **A**? Example **B**? Example **C**? Example **D**?

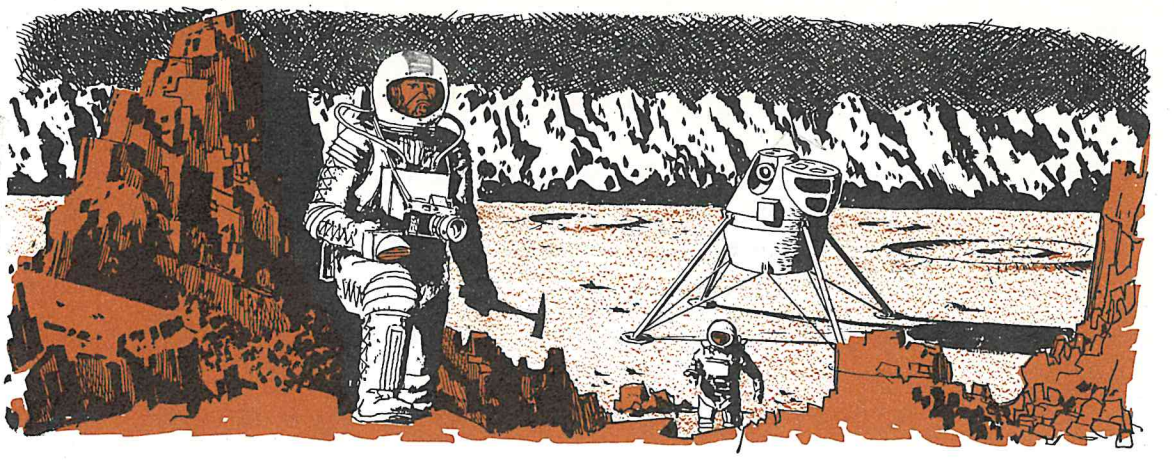
4. How do we get the numerals above and below the fractions in **A**? in **B**? in **C**?

5. Copy Examples **A–C** (without the work that has been done) and find the products. Use the short cut if you feel it is helpful. Write each answer in simplest form.

6. Look at Example **D**. Can you think of another fraction instead of $\frac{2}{2}$ that might be used to make the work easier? (That is, do 4 and 8 have a larger common factor than 2?) Copy the example without the work that has been done. Then do the work yourself in the way that you think best.

Find the products.

	a	b	c
7.	$7\frac{1}{2} \times 2\frac{2}{5} = \square$	$5\frac{1}{3} \times 1\frac{1}{16} = \triangle$	$2\frac{5}{8} \times 1\frac{1}{3} = \square$
8.	$1\frac{1}{9} \times 2\frac{1}{10} = \square$	$2\frac{1}{12} \times 1\frac{4}{5} = \triangle$	$1\frac{2}{3} \times 5\frac{1}{5} = \square$
9.	$\frac{5}{16} \times 8 = \square$	$5\frac{3}{5} \times 2\frac{1}{2} = \triangle$	$\frac{2}{3} \times \frac{1}{2} \times \frac{1}{4} = \square$
10.	$3\frac{1}{6} \times 1\frac{1}{5} = \square$	$\frac{5}{6} \times \frac{4}{5} \times \frac{1}{2} = \triangle$	$3\frac{7}{8} \times 4 = \square$



1. Gravity on the Moon is such that the weight of an object on the Moon is $\frac{4}{25}$ its weight on Earth. What would be the weight of an astronaut on the Moon if he weighs $177\frac{1}{2}$ pounds on Earth?

First, ask yourself questions about the problem. Would the astronaut's weight be more, or less, on the Moon than on Earth? There are two numbers in the problem. Will you use both of them? How?

Next, write a number sentence to help solve the problem. Then find the replacement that will make the sentence true.

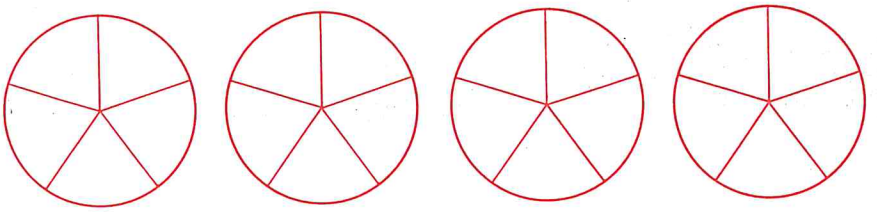
What answer do you get for the weight of the astronaut on the Moon? Is it less than his weight on Earth? Is that what you expected? Check your work.

2. How much less would the astronaut weigh on the Moon than he does on Earth?

3. Suppose that on Earth the helmet of the astronaut's "moon-suit" weighs $3\frac{3}{4}$ pounds. The suit itself with the attached boots weighs $7\frac{5}{8}$ pounds. Various useful tools that are hooked to his belt weigh $2\frac{1}{2}$ pounds. What is the total weight of this equipment?

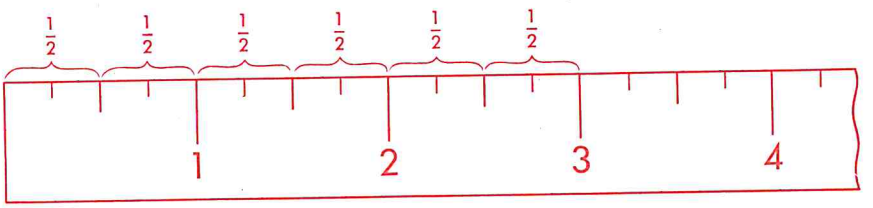
4. When the astronaut steps out on the surface of the Moon in his suit and with his equipment, what will his total weight be to the nearest pound?

1. In division with whole numbers, does an example such as $12 \div 2$ ask us to find the number of twos in 12? Then let us think of the division example $4 \div \frac{4}{5}$ as asking us to find the number of $\frac{4}{5}$'s in 4.



Above are four diagrams divided into fifths. Since each diagram represents $\frac{5}{5}$, or 1, do the four diagrams represent 4? Draw similar diagrams. Then shade $\frac{4}{5}$ of each diagram. Do you now have four shaded areas, each representing $\frac{4}{5}$ of a diagram? In each diagram, how many fifths are *not* shaded? If you shade each of these remaining parts, will you have shaded an area equal to another $\frac{4}{5}$ of a diagram? Then is this sentence true: $4 \div \frac{4}{5} = 5$?

2. Using this diagram of a ruler, let us work the example $3 \div \frac{1}{2} = \text{N}$. We wish to know the number of $\frac{1}{2}$'s in 3. On the diagram, distances of $\frac{1}{2}$ inch have been marked off. How many $\frac{1}{2}$'s are there in 3? Then is this sentence true: $3 \div \frac{1}{2} = 6$?



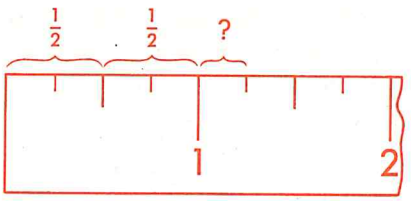
3. Using a ruler with inches divided into fourths, find the quotient for each of these examples. Write a sentence for each example.

a. $2 \div \frac{1}{4}$ **b.** $2 \div \frac{1}{2}$

c. $\frac{5}{2} \div \frac{1}{2}$ **d.** $3 \div \frac{3}{4}$

4. What does $1 \div \frac{1}{2}$ equal? $1\frac{1}{2} \div \frac{1}{2}$?

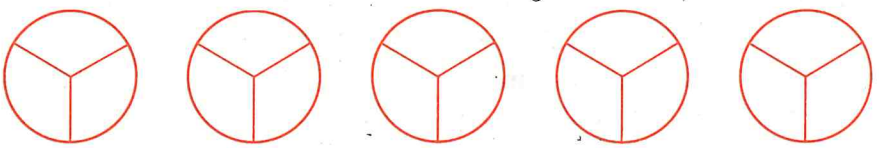
5. Now let us find what $1\frac{1}{4} \div \frac{1}{2}$ equals. Are there at least two $\frac{1}{2}$'s in $1\frac{1}{4}$? What do the two $\frac{1}{2}$'s equal? You can see there is $\frac{1}{4}$ inch remaining. Does this $\frac{1}{4}$ equal half of another $\frac{1}{2}$? That is, does $\frac{1}{4}$ equal "$\frac{1}{2}$ of $\frac{1}{2}$"?



Then $1\frac{1}{4} \div \frac{1}{2} = 2\frac{1}{2}$.

6. Draw four diagrams like those you drew for Exercise **1**. Find what $4 \div \frac{2}{5}$ equals by counting the number of $\frac{2}{5}$'s in 4.

7. Below are five diagrams divided into thirds. Since each diagram represents $\frac{3}{3}$, or 1, do the five diagrams represent 5? How many $\frac{1}{3}$'s are there in 5? Copy the sentence and replace the frame with the correct numeral: $5 \div \frac{1}{3} = \square$



8. Draw five diagrams like those in Exercise **7**. Now find what $5 \div \frac{2}{3}$ equals by finding the number of $\frac{2}{3}$'s in 5.

Begin by shading $\frac{2}{3}$ of each diagram. Do you now have five shaded areas, each representing $\frac{2}{3}$ of a diagram? How many thirds are still unshaded? Can you shade another area representing $\frac{2}{3}$ of a diagram? Can you shade still another $\frac{2}{3}$? Do it. How many $\frac{2}{3}$'s have you shaded altogether? What fractional part of a diagram is still unshaded? Does $\frac{1}{3}$ equal "$\frac{1}{2}$ of $\frac{2}{3}$"? Then is this sentence true: $5 \div \frac{2}{3} = 7\frac{1}{2}$?

9. To find what $5 \div \frac{3}{3}$ equals, can you count the number of $\frac{3}{3}$'s in the diagrams in Exercise **7**? Do it. What is the correct replacement for the frame: $5 \div \frac{3}{3} = \square$?

What is the simplest form for $\frac{3}{3}$? What does $5 \div 1$ equal? Is this what you found when you counted the $\frac{3}{3}$'s in 5?

10. Find the correct replacement for the frame in this sentence: $5 \div \frac{5}{3} = \triangle$. (Draw diagrams like those in Exercise **7** and find the number of $\frac{5}{3}$'s in 5.)

You know that division and multiplication are related. For every multiplication sentence such as $7 \times 8 = 56$, there is a related division sentence: $56 \div 8 = 7$.

1. Now, using what we know about fractions, let us try to discover more about multiplication and division. What is the quotient for **a**? the product for **b**?

a. $20 \div 4 = \square$ **b.** $\frac{1}{4} \times 20 = \square$

Do you get the same answer when you find "$\frac{1}{4}$ of 20," or $\frac{1}{4} \times 20$, as when you divide 20 by 4?

2. Try another pair of examples.

a. $27 \div 3 = \triangle$ **b.** $\frac{1}{3} \times 27 = \triangle$

Do you get the same answer for both examples?

3. Look at the division example: $56 \div 8 = 7$. Write a multiplication example that gives the same result: $\square \times 56 = 7$.

4. What is the divisor in the division example $20 \div 4 = 5$? What is the multiplier in the multiplication example $\frac{1}{4} \times 20 = 5$?

5. Name the divisor and multiplier in each pair of examples.

a. $27 \div 3 = 9$; $\frac{1}{3} \times 27 = 9$ **b.** $56 \div 8 = 7$; $\frac{1}{8} \times 56 = 7$

6. There is a relationship between the divisor and the multiplier in the pairs of examples in Exercises **4** and **5**. Try to discover the relationship as you work these examples.

a. $4 \times \frac{1}{4} = \square$ **b.** $3 \times \frac{1}{3} = \square$ **c.** $8 \times \frac{1}{8} = \square$

In each case, is the product of the two numbers 1?

Pairs of numbers whose product is equal to 1 are called **reciprocals.** The reciprocal of 3 is $\frac{1}{3}$. The reciprocal of $\frac{1}{3}$ is 3. What is the reciprocal of $\frac{1}{4}$? the reciprocal of 4?

7. In the sentence $5 \times N = 1$, N represents the reciprocal of 5. What is the correct replacement for N?

Find the missing reciprocals.

a. $2 \times N = 1$ **b.** $N \times 9 = 1$ **c.** $N \times \frac{1}{6} = 1$

d. $\frac{1}{8} \times N = 1$ **e.** $N \times 14 = 1$ **f.** $1 \times N = 1$

8. What is $80 \div 4$? $\frac{1}{4} \times 80$? Is this sentence true: $80 \div 4 = \frac{1}{4} \times 80$? Is the multiplier, $\frac{1}{4}$, the reciprocal of the divisor, 4?

Do you think you can rewrite the division example $90 \div 6$ as a multiplication example? Try it: $90 \div 6 = \square \times 90$. What does $90 \div 6$ equal? $\frac{1}{6} \times 90$? Is the multiplier, $\frac{1}{6}$, the reciprocal of the divisor, 6?

9.a. Is this sentence true: $8 \times \frac{1}{2} = 4$? Then, since dividing is the inverse of multiplying, $4 \div \frac{1}{2} = 8$.

b. Now let us see whether the example $4 \div \frac{1}{2}$ could have been done by using the reciprocal of the divisor. What is the reciprocal of $\frac{1}{2}$? What is 2×4? Then is it true that $4 \div \frac{1}{2} = 2 \times 4 = 8$?

10.a. Is this sentence true: $21 \times \frac{1}{3} = 7$? Then $7 \div \frac{1}{3} = 21$.

b. Now see whether reciprocals will work for the example $7 \div \frac{1}{3}$. What is the reciprocal of $\frac{1}{3}$? What is 3×7? Then is it true that $7 \div \frac{1}{3} = 3 \times 7 = 21$?

11. Do you see a pattern? Then copy this sentence and put in the correct numerals.

$$\frac{5}{4} \div \frac{1}{2} = \square \times \frac{5}{4} = \frac{N}{4}$$

Now express $\frac{10}{4}$ as an improper fraction in simplest form.

Have you found that $\frac{5}{4} \div \frac{1}{2} = \frac{5}{2}$? We can check the work in the way division is usually checked—by multiplying the divisor, $\frac{1}{2}$, by the quotient, $\frac{5}{2}$: $\frac{5}{2} \times \frac{1}{2} = \triangle$. Do you get the dividend, $\frac{5}{4}$? Then your answer is correct.

Using reciprocals, rewrite each division example as a multiplication example and find the answer. Use the sentences as guides.

	a	b
12.	$72 \div 8 = \square \times 72 = ?$	$12 \div \frac{1}{2} = \triangle \times 12 = ?$
13.	$\frac{9}{4} \div \frac{1}{2} = \triangle \times \frac{9}{4} = ?$	$36 \div 4 = \square \times 36 = ?$
14.	$48 \div 6 = \triangle \times 48 = ?$	$9 \div \frac{1}{4} = \square \times 9 = ?$
15.	$42 \div 7 = \triangle \times 42 = ?$	$42 \div \frac{1}{7} = \square \times 42 = ?$

On pages 86–87, you found quotients for the following division examples with fractions.

a. $3 \div \frac{1}{2} = 6$ b.. $2 \div \frac{1}{4} = 8$ c. $5 \div \frac{1}{3} = 15$

Let us use each of these examples to test the pattern involving reciprocals.

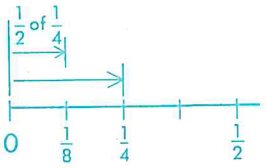
1. What is the reciprocal of $\frac{1}{2}$? What is 2×3? Then is this sentence true: $3 \div \frac{1}{2} = 2 \times 3$, or 6?

2. What is the reciprocal of $\frac{1}{4}$? What is 4×2? Then is this sentence true: $2 \div \frac{1}{4} = 4 \times 2$, or 8?

3. What is the reciprocal of $\frac{1}{3}$? What is 3×5? Then is this sentence true: $5 \div \frac{1}{3} = 3 \times 5$, or 15?

Does the pattern with reciprocals work in these cases, too?

4.a. Let us try another example, using a diagram first.



Does $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{2} \times \frac{1}{4}$, equal $\frac{1}{8}$? Then suppose you are given this division sentence: $\frac{1}{8} \div \frac{1}{4} = \square$. This sentence asks: "How many $\frac{1}{4}$'s in $\frac{1}{8}$?" Is there $\frac{1}{2}$ of $\frac{1}{4}$ in $\frac{1}{8}$? Then what is the correct replacement for the division sentence?

b. Now use the pattern with reciprocals. Copy this sentence and put in the correct numerals: $\frac{1}{8} \div \frac{1}{4} = \triangle \times \frac{1}{8} = \underline{?}$.

Do you again get $\frac{1}{2}$ as the answer?

▶ Can a division example be rewritten as a multiplication example? To divide by a number, can you multiply by the reciprocal of the number?

Using reciprocals, rewrite each division example as a multiplication example and find the answer.

	a	b
5.	$6 \div \frac{1}{4} = \square \times 6 = \underline{?}$	$4 \div 3 = \triangle \times 4 = \underline{?}$
6.	$\frac{5}{8} \div \frac{1}{8} = \triangle \times \frac{5}{8} = \underline{?}$	$6 \div 4 = \square \times 6 = \underline{?}$
7.	$11 \div 5 = \square \times 11 = \underline{?}$	$\frac{2}{3} \div \frac{1}{4} = \square \times \frac{2}{3} = \underline{?}$
8.	$\frac{1}{16} \div \frac{1}{4} = \square \times \frac{1}{16} = \underline{?}$	$\frac{1}{8} \div 3 = \triangle \times \frac{1}{8} = \underline{?}$

9. At the right is a division example that has been rewritten as a multiplication example. Where did we get the $\frac{1}{7}$ in Sentence **a**? the $\frac{21}{5}$ in **b**? Copy the example and complete the work. Express your answer in simplest form.

a. $4\frac{1}{5} \div 7 = \frac{1}{7} \times 4\frac{1}{5}$

b. $= \frac{1}{7} \times \frac{21}{5}$

10. Have you found that $4\frac{1}{5} \div 7 = \frac{3}{5}$? To check the results, can you multiply the divisor, 7, by the quotient, $\frac{3}{5}$? $\frac{3}{5} \times 7 = \cdot\square$. Express the product as a mixed fraction. Was your work for Exercise **9** correct?

Rewrite each division example as a multiplication example and find the answer.

	a	b	c	d	e
11.	$7 \div \frac{1}{10}$	$\frac{2}{3} \div \frac{1}{12}$	$\frac{7}{8} \div \frac{1}{2}$	$\frac{3}{4} \div \frac{1}{12}$	$\frac{1}{12} \div 2$
12.	$\frac{1}{4} \div \frac{1}{2}$	$\frac{1}{6} \div \frac{1}{4}$	$4 \div \frac{1}{16}$	$\frac{1}{5} \div 2$	$\frac{1}{20} \div \frac{1}{4}$
13.	$5\frac{5}{6} \div 7$	$3\frac{1}{4} \div 3$	$2\frac{1}{10} \div 7$	$\frac{1}{2} \div \frac{1}{3}$	$\frac{5}{8} \div \frac{1}{16}$
14.	$\frac{7}{8} \div 3$	$12\frac{1}{3} \div 3$	$3\frac{3}{8} \div 3$	$5\frac{1}{3} \div 2$	$8\frac{2}{5} \div 7$
15.	$\frac{5}{6} \div \frac{1}{8}$	$\frac{5}{12} \div \frac{1}{6}$	$2\frac{5}{8} \div 3$	$2\frac{7}{10} \div 9$	$2\frac{1}{4} \div 6$

16. Roy wants to cut a board that is $3\frac{1}{2}$ feet long into four pieces of equal length. How long will each piece be?

17. How many pieces of ribbon $\frac{1}{2}$ yard in length can Mary get from a piece that is $2\frac{1}{2}$ yards long?

18. Using reciprocals, find the quotients. Express each quotient as a proper fraction or an improper fraction.

a. $1 \div 12$ **b.** $5 \div 3$ **c.** $9 \div 2$ **d.** $1 \div 5$ **e.** $2 \div 7$

19. On page 53, you found that $\frac{1}{12}$ can be thought of as the result of the division example $1 \div 12$; $\frac{5}{3}$ can be thought of as the result of the division example $5 \div 3$; and so on. Does your work for Exercise **18** show that this is so?

20. Can you quickly name the quotient for each of these examples? Try it and see. Then check your answers by dividing.

a. $7 \div 6$ **b.** $1 \div 8$ **c.** $3 \div 5$ **d.** $9 \div 4$ **e.** $2 \div 15$

1. Rewrite this division example as a multiplication example and find the answer. $6 \div \frac{1}{3}$

2. Now look at the model example below.

$$6 \div \frac{1}{3} = 6 \times 3, \text{ or } ?$$

Is this the same example as the one you were asked to do in Exercise **1**? When you rewrote the division example as a multiplication example, did you write it as 3×6? Here, as you can see, we have written 6×3. That is, the numeral for the dividend appears first in the rewritten example. Then, following the "$\times$" sign, we have written a 3. What does this 3 represent?

When we write a multiplication sentence, does it matter in which order we write the numerals? Do we get the same result for 6×3 as for 3×6?

In rewriting division examples as multiplication examples, do you see that you can leave the numeral for the dividend in its original position? What do you substitute for the sign "$\div$"? What do you substitute for the divisor?

3. In this example, what numeral will you use to replace the frame? $\frac{9}{2} \div 5 = \frac{9}{2} \times \square$

Copy the example and do the work.

Copy and divide. In rewriting each example as a multiplication example, leave the numeral for the dividend in its original position.

	a	b	c	d	e
4.	$\frac{2}{3} \div 8$	$2\frac{1}{4} \div 3$	$\frac{4}{5} \div 6$	$6\frac{2}{3} \div 4$	$\frac{1}{5} \div \frac{1}{15}$
5.	$\frac{3}{16} \div \frac{1}{8}$	$\frac{3}{8} \div \frac{1}{16}$	$1\frac{1}{5} \div 8$	$\frac{9}{2} \div \frac{1}{3}$	$\frac{3}{10} \div \frac{1}{5}$
6.	$\frac{5}{6} \div 10$	$6 \div \frac{1}{5}$	$\frac{5}{4} \div \frac{1}{16}$	$8\frac{1}{3} \div \frac{1}{3}$	$1\frac{1}{6} \div 2$
7.	$1\frac{1}{5} \div \frac{1}{8}$	$\frac{8}{3} \div \frac{1}{9}$	$\frac{9}{2} \div 3$	$1\frac{1}{6} \div \frac{1}{2}$	$\frac{2}{5} \div 12$
8.	$3\frac{1}{4} \div \frac{1}{8}$	$\frac{1}{6} \div 6$	$2\frac{2}{3} \div 8$	$1\frac{5}{6} \div \frac{1}{3}$	$0 \div \frac{1}{9}$

1. In each sentence, find the missing numeral.

a	b	c
$(2 \times \frac{1}{2}) \times 36 = \square$	$(\frac{1}{3} \times 3) \times 15 = \triangle$	$(7 \times \frac{1}{7}) \times \frac{4}{3} = N$

In **a** you multiplied 36 by 1. What did you do in **b**? in **c**?

2. Look at this sentence: $5 \times (\frac{1}{5} \times 7) = \square$. Is the product the same when you rewrite the sentence as $(5 \times \frac{1}{5}) \times 7 = \square$? Can you now find the missing numeral mentally? Explain.

Using reciprocals, find the missing numerals.

	a	b	c
3.	$6 \times (\frac{1}{6} \times 8) = \square$	$\frac{1}{4} \times (4 \times 8) = \triangle$	$\frac{1}{3} \times (3 \times \frac{11}{5}) = \square$
4.	$5 \times (\frac{1}{5} \times \square) = 6$	$\frac{1}{8} \times (8 \times \triangle) = 5$	$2 \times (\square \times 7) = 7$

5.a. $\frac{1}{3} \times (\square \times 10) = 10$ **b.** $6 \times (\square \times 12) = 12$

6. Suppose you want to find the missing numeral in Sentence **a**. You can do this by using reciprocals and an expression in parentheses.

a. $\frac{1}{12} \times N = 3$

b. $\frac{1}{12} \times (\square \times \triangle) = 3$

In Sentence **b**, we have used $(\square \times \triangle)$ instead of N. Is this sentence like those in Exercises **3–5**? What numeral can you use to replace the $\square$? the $\triangle$? Since we used $(\square \times \triangle)$ to replace N, have we found that N is 12×3, or 36? Check. Does $\frac{1}{12} \times 36$ equal 3?

7. To find the missing numeral in the sentence $7 \times N = 5$, replace N with an expression in parentheses as shown in the model example. What is a correct replacement for the $\square$? for the $\triangle$? Since we used $(\square \times \triangle)$ to replace N, have we found that N is $\frac{1}{7} \times 5$, or $\frac{5}{7}$? Check the answer. Does $7 \times \frac{5}{7}$ equal 5?

$7 \times (\square \times \triangle) = 5$

8. Using the method of Exercises **6** and **7**, find the single numeral that will make each sentence true.

a. $\frac{1}{15} \times N = 4$ **b.** $6 \times N = 7$ **c.** $\frac{1}{8} \times N = 2\frac{1}{2}$

You have found the reciprocals of numbers such as 3, 7, or 15. You have also found the reciprocals of numbers that are named by fractions with a numerator of 1. For instance, what is the reciprocal of $\frac{1}{5}$? of 7? of $\frac{1}{3}$? of 9?

1. Now let us see if a number such as $\frac{2}{3}$ has a reciprocal. That is, let us look for a replacement that will make Sentence **a** true. To do this, let us use expressions in parentheses again.

a. $\frac{2}{3} \times N = 1$

b. $(2 \times \frac{1}{3}) \times N = 1$

c. $(2 \times \frac{1}{3}) \times (\frac{1}{2} \times 3) = 1$

d. $\frac{2}{3} \times \frac{3}{2} = 1$

Does $\frac{2}{3}$ equal $2 \times \frac{1}{3}$? Then may we write Sentence **b**, using the expression $2 \times \frac{1}{3}$ as a replacement for $\frac{2}{3}$? Look at the expression $2 \times \frac{1}{3}$ in Sentence **b**. What is the reciprocal of 2? of $\frac{1}{3}$? Can we think of $\frac{1}{2} \times 3$ as a name for a number? In Sentence **c**, we have used $\frac{1}{2} \times 3$ as a replacement for the N. Is Sentence **c** true?

Now look at Sentence **d**. How did we get the $\frac{2}{3}$? the $\frac{3}{2}$? Is Sentence **d** true? Have we found that $\frac{3}{2}$ is a correct replacement for the N in the original sentence? Then what is the reciprocal of $\frac{2}{3}$? What is the reciprocal of $\frac{3}{2}$?

2. By renaming $\frac{4}{5}$ as $4 \times \frac{1}{5}$, let us find the reciprocal of $\frac{4}{5}$.

$(4 \times \frac{1}{5}) \times (\square \times \triangle) = 1$

a. What is the reciprocal of 4? the reciprocal of $\frac{1}{5}$? Copy the model sentence and replace the frames.

b. What is $\frac{1}{4} \times 5$ expressed as an improper fraction? Does $\frac{4}{5} \times \frac{5}{4}$ equal 1? What is the reciprocal of $\frac{4}{5}$? of $\frac{5}{4}$?

3. Find the reciprocal of $\frac{8}{5}$; the reciprocal of $\frac{7}{9}$.

4. The four pairs of reciprocals you have found are listed at the right. Study each pair. Do you see a relationship that makes it easy to write the reciprocals?

$\frac{2}{3}$ and $\frac{3}{2}$ $\quad$ $\frac{4}{5}$ and $\frac{5}{4}$

$\frac{8}{5}$ and $\frac{5}{8}$ $\quad$ $\frac{7}{9}$ and $\frac{9}{7}$

Then try another number: $\frac{2}{5}$. In the fraction that represents the reciprocal of $\frac{2}{5}$, what is the numerator? the denominator? What is the reciprocal of $\frac{2}{5}$?

5. Find the reciprocal of each of the following fractional numbers. Then check to see that the product is 1.

a. $\frac{3}{4}$ **b.** $\frac{5}{6}$ **c.** $\frac{9}{2}$ **d.** $\frac{9}{10}$ **e.** $\frac{7}{12}$ **f.** $\frac{11}{8}$ **g.** $\frac{5}{32}$

6. Suppose you are asked to find the reciprocal of $7\frac{1}{2}$. Can you express $7\frac{1}{2}$ as an improper fraction? Do it. What is the reciprocal of $\frac{15}{2}$? Check by multiplying: $7\frac{1}{2} \times \frac{2}{15} = \triangle$.

7. Find the reciprocal of each of the following. Check to see that the product is 1.

a. $2\frac{1}{2}$ **b.** $4\frac{1}{3}$ **c.** $\frac{3}{2}$ **d.** $15\frac{1}{4}$ **e.** $1\frac{5}{6}$ **f.** $\frac{13}{24}$
g. $4\frac{5}{8}$ **h.** $2\frac{11}{16}$ **i.** $12\frac{2}{3}$ **j.** $4\frac{4}{5}$ **k.** $\frac{19}{12}$ **l.** $2\frac{7}{8}$

8. At the right is a division example being rewritten as a multiplication example. Using what you know about reciprocals, decide what to use as a replacement for the frame. Then copy the example and find the quotient.

$$\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \square$$

9. Look at Sentence **a**. Where did we get the $\frac{17}{3}$? the $\frac{7}{6}$? In **b**, the example is being rewritten as a multiplication example. What will you use as a replacement for the frame? Copy the example and find the quotient.

a. $5\frac{2}{3} \div 1\frac{1}{6} = \frac{17}{3} \div \frac{7}{6}$

b. $= \frac{17}{3} \times \square$

Copy and divide. Express answers in simplest form, as proper fractions, mixed fractions, or numerals for whole numbers.

	a	b	c	d	e
10.	$\frac{7}{12} \div \frac{7}{8}$	$\frac{2}{3} \div \frac{3}{4}$	$\frac{3}{5} \div \frac{2}{3}$	$\frac{5}{8} \div \frac{2}{3}$	$\frac{3}{10} \div \frac{2}{5}$
11.	$6 \div \frac{5}{6}$	$\frac{5}{16} \div 10$	$\frac{3}{4} \div \frac{5}{8}$	$\frac{5}{6} \div \frac{5}{24}$	$\frac{4}{5} \div \frac{3}{10}$
12.	$3\frac{1}{4} \div \frac{1}{12}$	$1\frac{3}{4} \div \frac{1}{6}$	$2\frac{1}{10} \div \frac{4}{5}$	$5\frac{1}{3} \div \frac{5}{6}$	$3\frac{1}{3} \div \frac{5}{9}$
13.	$\frac{9}{16} \div 1\frac{1}{8}$	$\frac{1}{2} \div 1\frac{1}{2}$	$\frac{7}{10} \div 2\frac{4}{5}$	$\frac{2}{3} \div 1\frac{1}{9}$	$\frac{7}{12} \div 1\frac{1}{3}$
14.	$4\frac{3}{8} \div \frac{5}{16}$	$\frac{3}{5} \div 1\frac{1}{5}$	$\frac{5}{12} \div 1\frac{2}{3}$	$\frac{5}{6} \div 2\frac{1}{3}$	$10\frac{1}{2} \div \frac{3}{8}$
15.	$\frac{7}{16} \div 2\frac{1}{3}$	$2\frac{1}{6} \div \frac{1}{8}$	$\frac{1}{2} \div 2\frac{1}{4}$	$8\frac{1}{2} \div \frac{4}{5}$	$\frac{7}{12} \div 1\frac{1}{6}$

1. The Boy Scouts went on a $6\frac{1}{4}$-mile hike. The Cub Scouts took a hike of $2\frac{1}{2}$ miles. The Boy Scouts hiked how many times as far as the Cub Scouts?

You are asked to compare the distance the Boy Scouts hiked with the distance the Cub Scouts hiked. To make the comparison, will you subtract or will you divide?

Do the two number sentences below describe the problem?

$$6\frac{1}{4} \div 2\frac{1}{2} = \square \qquad 6\frac{1}{4} = \square \times 2\frac{1}{2}$$

In each case, do you divide $6\frac{1}{4}$ by $2\frac{1}{2}$ to find the answer?

Express each mixed fraction as an improper fraction and do the work. Answer the question asked in the problem.

2. Mrs. Kay took $19\frac{1}{2}$ pounds of clothes to the self-service laundry. She used three machines, loading each with the same amount. How many pounds of laundry were in each machine?

Copy and divide.

	a	b	c	d	e
3.	$8\frac{1}{4} \div 4\frac{1}{8}$	$7\frac{1}{3} \div 2\frac{1}{5}$	$4\frac{1}{4} \div 1\frac{5}{12}$	$2\frac{7}{10} \div 4\frac{1}{2}$	$1\frac{3}{4} \div 1\frac{1}{2}$
4.	$\frac{3}{5} \div \frac{9}{10}$	$\frac{5}{6} \div \frac{3}{4}$	$1\frac{3}{4} \div \frac{7}{12}$	$\frac{5}{16} \div \frac{3}{8}$	$\frac{3}{5} \div \frac{3}{10}$
5.	$9\frac{3}{4} \div 3\frac{1}{4}$	$2\frac{3}{16} \div 1\frac{1}{6}$	$2\frac{1}{10} \div 1\frac{2}{5}$	$1\frac{5}{16} \div 3\frac{1}{2}$	$2\frac{1}{12} \div 1\frac{2}{3}$
6.	$\frac{5}{16} \div 2\frac{3}{4}$	$2\frac{1}{4} \div 4\frac{1}{2}$	$7\frac{1}{2} \div 1\frac{2}{3}$	$3\frac{1}{2} \div 1\frac{3}{4}$	$\frac{3}{10} \div 1\frac{4}{5}$
7.	$2 \div \frac{3}{7}$	$5 \div 1\frac{2}{3}$	$\frac{3}{16} \div \frac{3}{8}$	$\frac{9}{16} \div 3$	$1\frac{5}{9} \div 10$

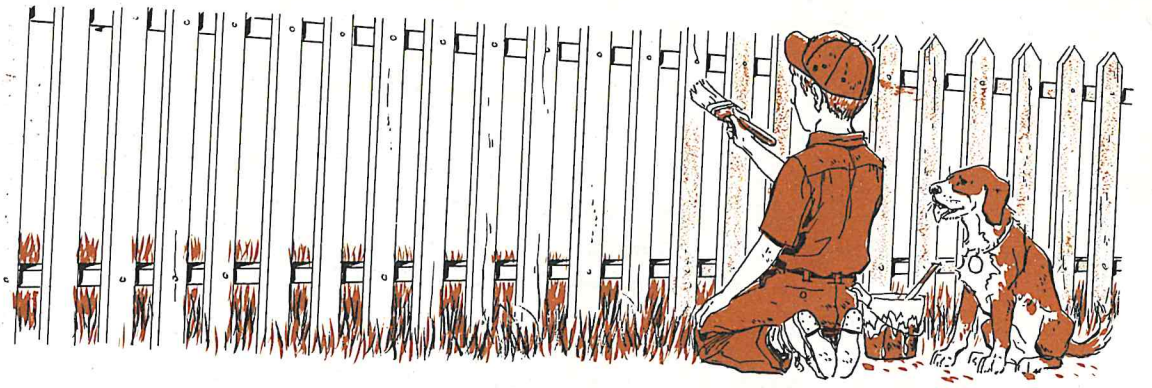
8. Find the missing numeral in each sentence.

a. $\frac{1}{3} \times (\square \times 5) = 5$

b. $\frac{8}{3} \times (\frac{3}{8} \times 12) = \square$

c. $\frac{5}{7} \times (\frac{7}{5} \times \triangle) = \frac{2}{3}$

d. $\frac{2}{5} \times (\frac{5}{2} \times 1\frac{1}{3}) = \triangle$



1. Jack is planning to paint a picket fence 125 feet long. On the basis of his past experience, he estimates that he can paint $12\frac{1}{2}$ feet per hour. At this rate, how many hours will it take him to complete the task?

2. Since Jack can work only $1\frac{1}{4}$ hours per day, how many days will it take to complete the painting job?

3. The pickets are $1\frac{3}{4}$ inches wide. How many such pickets can be cut from a board 12 inches wide?

4. If 6 pickets are cut from such a board, what is the width of the strip that is left?

5. The pickets have a height of $3\frac{2}{3}$ feet. How many can be cut from a strip 12 feet long?

6. What will be the length of the piece that is left over?

7. Alice lives $1\frac{1}{10}$ miles from school, and her friend Joan lives $5\frac{1}{2}$ miles from school. Joan lives how many times as far from school as Alice?

8. "I am thinking of a number," John said. "When I multiply it by 4 and then multiply the product by $\frac{1}{4}$, I get 10 as the answer. Can anyone tell what my original number is?"
What is the number?

9. If a certain number is multiplied by $\frac{2}{3}$ and then the product is multiplied by $\frac{3}{2}$, the result is 5. What is the original number?

1. Do you remember the short cut for multiplication with fractions (page 83)? Since division with fractions is done by rewriting the example as a multiplication example, do you think you can use the same short cut? Perhaps you have already been using it.

Two ways of working an example are shown below. See whether you can answer the questions that are asked.

a. $\frac{5}{6} \div \frac{4}{3} = \frac{5}{6} \times \frac{3}{4}$

$= \frac{15}{24}$

$= \frac{3 \times 5}{3 \times 8}$

$= \frac{3}{3} \times \frac{5}{8}$

b. $\frac{5}{6} \div \frac{4}{3} = \frac{5}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{4}$

$= \frac{5}{8}$

In **a**, how did we know that 15 and 24 have 3 as a common factor? When we write $\frac{3}{3} \times \frac{5}{8}$, do we know that the answer must be $\frac{5}{8}$? Explain.

In **b**, did we divide by the common factor, 3, *before* multiplying? How did we get the 1 above the 3 and the 2 below the 6?

Check the answer: $\frac{5}{8} \times \frac{4}{3} = \square$.

2. Study this example. Are we using the $\frac{5}{5}$ as a name for 1? Explain.

What other fraction representing 1 is being used?

Copy the example without the work that has been done. Then do the work yourself and find the answer.

$3\frac{3}{5} \div \frac{14}{15} = \frac{18}{5} \div \frac{14}{15}$

$= \frac{\cancel{18}^{9}}{\cancel{5}_{1}} \times \frac{\cancel{15}^{3}}{\cancel{14}_{7}}$

Copy and divide. Use the short cut if you can. Write answers in simplest form.

	a	b	c	d	e
3.	$\frac{3}{10} \div \frac{4}{5}$	$\frac{7}{12} \div \frac{5}{6}$	$\frac{2}{3} \div \frac{1}{3}$	$\frac{3}{4} \div \frac{5}{6}$	$\frac{7}{8} \div \frac{5}{16}$
4.	$9 \div 2\frac{2}{5}$	$4\frac{1}{2} \div 3$	$4\frac{1}{6} \div \frac{1}{12}$	$6\frac{4}{5} \div \frac{1}{10}$	$3\frac{2}{3} \div \frac{1}{6}$
5.	$4\frac{1}{6} \div \frac{1}{2}$	$4\frac{1}{5} \div \frac{3}{10}$	$\frac{5}{8} \div 2\frac{1}{4}$	$3\frac{3}{5} \div \frac{4}{5}$	$\frac{5}{6} \div 1\frac{1}{4}$
6.	$6\frac{1}{8} \div 1\frac{3}{4}$	$12\frac{1}{2} \div \frac{5}{6}$	$10\frac{1}{2} \div \frac{1}{3}$	$7 \div \frac{7}{16}$	$\frac{5}{8} \div \frac{5}{12}$

Copy, add, and check.

	a	b	c	d	e
1.	\$396.85	\$36.42	8,275	33,806	2,468
	190.76	85.09	907	959	4,659
	748.40	76.38	726	47,472	3,032
		50.46	6,505		5,892

Copy, subtract, and check.

2.	\$41.40	50,080	\$24.25	\$2,450.37	875,607
	19.78	9,582	6.09	649.48	8,678

Copy, multiply, and check.

3.	963	846	\$3.49	907	537
	67	372	58	309	508
4.	392	\$2.84	329	500	579
	700	850	319	973	700

Copy, divide, and check.

5.	$15\overline{)\$144.15}$	$58\overline{)37{,}199}$	$55\overline{)3{,}371}$	$13\overline{)4{,}015}$	$63\overline{)3{,}881}$
6.	$39\overline{)2{,}900}$	$44\overline{)42{,}271}$	$57\overline{)1{,}000}$	$26\overline{)1{,}258}$	$57\overline{)2{,}067}$

Find the missing numerals.

	a	b	c
7.	$15 \times \square = 240$	$4.000 \div \square = 25$	$285 + N + 372 = 703$
8.	$N \div 36 = 52$	$N = 3{,}697 + 408$	$1{,}267 + \triangle = 2{,}304$
9.	$\triangle \times 96 = 7{,}200$	$N \times 1 = 36{,}572$	$\square + 809 = 600 + 209$
10.	$937 - \square = 658$	$12 \times \square = 13 \times 12$	$4 \times 3 \times 50 = 3 \times \square$

11. Can 248 be divided evenly by 8? Then is 8 a factor of 248? Is 31 a factor of 248?

12. Find the average of 250, 275, 310, and 289.

13. The divisor is 14, the quotient is 367, and the remainder is 2. What is the dividend?

1. Copy and complete each chart so that it shows fractions equivalent to the fraction in the box.

a. $\boxed{\frac{3}{8}}$ $\frac{?}{16}$ $\frac{?}{24}$ $\frac{?}{32}$ $\frac{?}{40}$ $\frac{?}{64}$ $\frac{?}{88}$ **b.** $\boxed{\frac{9}{5}}$ $\frac{?}{10}$ $\frac{?}{15}$ $\frac{?}{20}$ $\frac{?}{25}$ $\frac{?}{30}$ $\frac{?}{50}$ $\frac{?}{75}$

Express each of the following in simplest form, as a proper fraction, a mixed fraction, or a numeral for a whole number.

2. a. $\frac{45}{9}$ **b.** $\frac{16}{10}$ **c.** $\frac{6}{9}$ **d.** $\frac{36}{24}$ **e.** $\frac{60}{48}$ **f.** $\frac{48}{60}$

3. a. $\frac{14}{18}$ **b.** $\frac{55}{70}$ **c.** $\frac{70}{55}$ **d.** $\frac{12}{42}$ **e.** $\frac{96}{8}$ **f.** $\frac{105}{9}$

Copy, and replace the frames.

	a	b	c	d
4.	$11\frac{1}{4} = 10\frac{\square}{4}$	$10\frac{3}{5} = 9\frac{\square}{5}$	$2\frac{1}{6} = 1\frac{\square}{6}$	$7\frac{1}{5} = 6\frac{\square}{5}$
5.	$1 + \frac{7}{4} = 2\frac{\triangle}{4}$	$8\frac{4}{3} = 9\frac{\triangle}{3}$	$5\frac{7}{9} = 4\frac{\triangle}{9}$	$25\frac{13}{6} = \square + \frac{1}{6}$

Add or subtract as the signs tell you. Check your work.

	a	b	c	d	e	f
6.	$12\frac{5}{12}$ $+ 6\frac{11}{12}$	$12\frac{5}{12}$ $- 6\frac{11}{12}$	$37\frac{9}{16}$ $+ 25\frac{9}{16}$	$14\frac{5}{6}$ $- 6\frac{1}{6}$	$18\frac{7}{9}$ $- 15\frac{8}{9}$	$16\frac{1}{3}$ $- 8$
7.	$8\frac{5}{6}$ $+ 7\frac{1}{12}$	$15\frac{1}{4}$ $- 6\frac{5}{8}$	$9\frac{5}{16}$ $+ 5\frac{3}{4}$	$9\frac{5}{16}$ $- 5\frac{3}{4}$	$3\frac{1}{6}$ $+ 2\frac{1}{18}$	14 $- 6\frac{5}{12}$
8.	$8\frac{1}{4}$ $+ 12\frac{4}{5}$	$9\frac{1}{6}$ $+ 3\frac{7}{8}$	$17\frac{1}{3}$ $- 8\frac{3}{4}$	$3\frac{1}{2}$ $5\frac{3}{8}$ $+ 6\frac{1}{10}$	$34\frac{1}{6}$ $- 27\frac{1}{8}$	$8\frac{1}{3}$ $4\frac{5}{6}$ $+ 3\frac{4}{5}$

9. Which represents the greater number, $\frac{5}{8}$ or $\frac{7}{8}$? $\frac{99}{5}$ or $\frac{98}{5}$? How can you tell?

10. Can you tell at a glance which represents the greater number, $\frac{3}{4}$ or $\frac{4}{5}$? Do you think you will be able to tell if you express $\frac{3}{4}$ and $\frac{4}{5}$ as fractions with a common denominator? Do it: $\frac{3}{4} = \frac{\square}{20}$; $\frac{4}{5} = \frac{\triangle}{20}$. Which is greater, $\frac{15}{20}$ or $\frac{16}{20}$? $\frac{3}{4}$ or $\frac{4}{5}$?

11. Which represents the greater number?

a. $\frac{3}{5}$ or $\frac{4}{7}$ **b.** $\frac{5}{6}$ or $\frac{7}{8}$ **c.** $\frac{14}{5}$ or $\frac{11}{4}$ **d.** $\frac{5}{9}$ or $\frac{4}{7}$

1.a. Read this numeral: 2,897,364. Copy and complete this chart to show what each digit in the numeral represents.

2, 8 9 7, 3 6 4

4 is in ones' place and represents 4
6 is in ? place and represents 60
3 is in ? place and represents ?
7 is in ? place and represents ?
9 is in ? place and represents ?
8 is in ? place and represents ?
2 is in ? place and represents ?

b. We can also think of the 7 in thousands' place as 70 hundreds, or ? tens. Similarly, we can think of the 8 in hundred-thousands' place as ? ten-thousands.

2. Find the number named by each of the following.

a. 10^3 **b.** 6^2 **c.** 4^3 **d.** 10^4 **e.** 2^4 **f.** 10^6

3. A number such as 342,908 can be expressed in these ways:

$(3 \times 100{,}000) + (4 \times 10{,}000) + (2 \times 1{,}000) + (9 \times 100) + 8$

$(3 \times 10^5) + (4 \times 10^4) + (2 \times 10^3) + (9 \times 10^2) + 8$

Express each of these numbers in the same ways.

a. 927,846 **b.** 98,123 **c.** 4,932,078

Find a single numeral to replace N.

4. $(9 \times 10^5) + (7 \times 10^3) + (3 \times 10^2) + 4 = N$

5. $(8 \times 10^6) + (3 \times 10^5) + (9 \times 10) + 3 = N$

6. $(7 \times 10^8) + (7 \times 10^7) + (6 \times 10^4) + (5 \times 10^3) + 9 = N$

7. Is it true that $\frac{17}{3} < 6$? Now compare these numbers, using ">," the sign for "is greater than."

8. Write a sentence comparing the numbers in each of the pairs. Use the sign "<" for the pairs in the first row and the sign ">" for the pairs in the second row.

a. $\frac{19}{9}$; 2 **b.** 15; $\frac{74}{5}$ **c.** 12; $\frac{145}{12}$ **d.** $\frac{37}{6}$; $\frac{19}{3}$

e. $\frac{27}{4}$; 7 **f.** 2; $\frac{29}{15}$ **g.** 12; $\frac{83}{7}$ **h.** 21; $\frac{421}{20}$

1. One basket of apples weighs $6\frac{7}{8}$ pounds; a larger basket weighs $14\frac{1}{2}$ pounds. How much more does the larger basket of apples weigh?

2. Ray helped his father clean the yard on two afternoons. He worked $2\frac{3}{4}$ hours on one day and $1\frac{2}{3}$ hours on the other day. How many hours did he work?

3. A jet airliner took off from a New York City airport at 4 o'clock in the afternoon for San Francisco. The flying time was $5\frac{1}{2}$ hours. What time would a passenger's wristwatch show when the jet landed at San Francisco if the passenger had not reset his watch?

4. San Francisco is in the third time zone west of the zone in which New York is located. At any given time, it is earlier in San Francisco than it is in New York. What time was it in San Francisco when the jet arrived?

5. The flying time on a return trip was 5 hours. A California passenger took off from San Francisco in a jet at 9 o'clock in the morning. What time was it in New York when the jet arrived?

6. After landing, the passenger waited $\frac{7}{12}$ of an hour before he was able to get his baggage. Then, because of the heavy rush-hour traffic, it took him $1\frac{1}{2}$ hours to drive to his home. How long was it from the time he arrived in New York until he reached his home?

7. John weighs $87\frac{7}{8}$ pounds, Bob weighs $91\frac{5}{8}$ pounds, and Harold weighs $95\frac{1}{2}$ pounds. John weighs how many pounds less than Bob? Bob weighs how many pounds less than Harold?

8. What is the average weight of the three boys?

9. John ran the 50-yard dash in $9\frac{1}{5}$ seconds one day. The school record is $9\frac{1}{10}$ seconds. By how much does John have to improve to equal the school record?

You have learned to divide with fractional numbers by rewriting the example as a multiplication example, using the reciprocal of the divisor. Do these examples in this way.

1. a. $\frac{16}{5} \div \frac{2}{5}$ **b.** $\frac{16}{7} \div \frac{2}{7}$ **c.** $\frac{16}{9} \div \frac{2}{9}$

2. a. $\frac{20}{3} \div \frac{4}{3}$ **b.** $\frac{20}{9} \div \frac{4}{9}$ **c.** $\frac{20}{7} \div \frac{4}{7}$

In each example above, are the fractions for the dividend and the divisor *like fractions*? Do you see a pattern to your answers? If a division example has like fractions, do you think that you can work it without rewriting the example as a multiplication example? Try it with the examples below.

Copy the sentences and replace the frames.

3. a. $\frac{14}{3} \div \frac{2}{3} = \triangle \div 2$, or ? **b.** $\frac{28}{5} \div \frac{7}{5} = 28 \div \square$, or ?

4. a. $\frac{15}{16} \div \frac{3}{16} = \triangle \div \square$, or ? **b.** $\frac{12}{25} \div \frac{4}{25} = \triangle \div \square$, or ?

5. Does $1 \div 4$ equal $\frac{1}{4}$? Then what is the quotient for the example at the right? Check by multiplying: $\frac{1}{4} \times \frac{4}{5} = N$.

$\frac{1}{5} \div \frac{4}{5} = ?$

6. Does $7 \div 3$ equal $\frac{7}{3}$? What is the mixed fraction for $\frac{7}{3}$? Copy the model sentence, replace the frames, and find the quotient.

$$\frac{7}{8} \div \frac{3}{8} = 7 \div \square, \text{ or } \frac{7}{\square}, \text{ or } ?$$

7. Copy, replace the frames, and find the quotients.

a. $\frac{3}{4} \div \frac{7}{4} = 3 \div \square$, or $\frac{3}{\square}$ **b.** $\frac{5}{3} \div \frac{2}{3} = \triangle \div 2$, or $\frac{\triangle}{2}$, or ?

c. $\frac{1}{8} \div \frac{3}{8} = \triangle \div \square$, or $\frac{\triangle}{\square}$ **d.** $\frac{8}{9} \div \frac{5}{9} = \triangle \div \square$, or $\frac{\triangle}{\square}$, or ?

Do you see a way to shorten the work? Suppose there are two fractions with a common denominator; for example, $\frac{\triangle}{3}$ and $\frac{\square}{3}$. Is this sentence true: $\frac{\triangle}{3} \div \frac{\square}{3} = \frac{\triangle}{\square}$? Suppose the fractions are represented as $\frac{\triangle}{N}$ and $\frac{\square}{N}$. How would you represent the quotient for the example $\frac{\triangle}{N} \div \frac{\square}{N}$?

8. Look at this example: $3\frac{2}{5} \div \frac{4}{5} = N$. How can you express $3\frac{2}{5}$ so that you can use the pattern? Do this. Express the quotient as an improper fraction; as a mixed fraction.

Divide, using the pattern you discovered on page 103.

	a	b	c	d
1.	$\frac{3}{7} \div \frac{4}{7}$	$2\frac{1}{3} \div \frac{2}{3}$	$3\frac{1}{8} \div \frac{5}{8}$	$\frac{5}{8} \div 3\frac{1}{8}$
2.	$\frac{1}{3} \div 3\frac{2}{3}$	$\frac{8}{9} \div \frac{4}{9}$	$1\frac{4}{5} \div \frac{3}{5}$	$2\frac{5}{6} \div \frac{5}{6}$

When two fractions have a common denominator and you divide without writing a multiplication example, you are using a method called the **common denominator method.**

3. In the example $\frac{5}{8} \div \frac{3}{4} = ?$, do the fractions have a common denominator? Then do you think that $\frac{5}{8} \div \frac{3}{4} = \frac{5}{3}$?

Can you rewrite the example so that you can use the common denominator method? What is a common denominator for $\frac{5}{8}$ and $\frac{3}{4}$? Rewrite the example and divide: $\frac{5}{8} \div \frac{\square}{8} = \frac{\triangle}{\square}$.

4. Where did we get the $\frac{3}{2}$ in this sentence? Can 6 be expressed as an improper fraction with a denominator of 2? Then can you use the common denominator method to find the quotient? Try it and see. Express your answer in simplest form.

$$1\frac{1}{2} \div 6 = \frac{3}{2} \div \frac{\square}{2}$$

5. Copy and divide, using the common denominator method.

a. $\frac{5}{6} \div \frac{1}{12}$	**b.** $\frac{1}{2} \div \frac{5}{16}$	**c.** $5 \div 1\frac{1}{4}$	**d.** $\frac{7}{10} \div \frac{3}{5}$
e. $1\frac{4}{5} \div \frac{3}{5}$	**f.** $2\frac{1}{2} \div 4$	**g.** $2\frac{1}{4} \div 1\frac{1}{8}$	**h.** $6 \div \frac{5}{6}$

6.a. Can you use the common denominator method for this example? What is a common denominator for $\frac{11}{4}$ and $\frac{5}{3}$? Then $\frac{11}{4} = \frac{\triangle}{12}$; $\frac{5}{3} = \frac{\square}{12}$. Now work the example.

$$2\frac{3}{4} \div 1\frac{2}{3} = \frac{11}{4} \div \frac{5}{3}$$

b. Would it be easier to find what $\frac{11}{4} \div \frac{5}{3}$ equals by using multiplication with the reciprocal of the divisor? Try it and see.

7. Which do you think will be easier to use with these examples—the common denominator method or the reciprocal of the divisor? Use whichever method you wish.

a. $\frac{5}{16} \div \frac{5}{6}$	**b.** $5\frac{1}{4} \div \frac{7}{8}$	**c.** $6\frac{1}{2} \div 4$	**d.** $3\frac{1}{3} \div 1\frac{1}{4}$

1. Lynn had 69 duplicate stamps. She traded $\frac{2}{3}$ of them to her friend Nancy. How many stamps did Nancy get?

Does the number sentence at the right describe the problem?

$\frac{2}{3} \times 69 = N$

What is $\frac{2}{3} \times 69$? Then how many stamps did Nancy get in this trade?

2. Nancy received a total of 72 stamps in her trading that day. She mounted $\frac{3}{4}$ of them in her album before the afternoon was over. How many did she mount in her album?

Copy, and find the missing numeral.

	a	b	c
3.	$\frac{3}{10}$ of $100 = \square$	$\frac{7}{12}$ of $60 = \triangle$	$\frac{3}{5} \times 45 = N$
4.	$\frac{3}{4}$ of $36 = \square$	$\frac{8}{9}$ of $63 = \triangle$	$\frac{5}{6}$ of $30 = \square$
5.	$\frac{5}{24} \times 24 = \square$	$\frac{1}{8} \times 56 = N$	$\frac{3}{16} \times 32 = \triangle$

6. Fifteen of the 30 pupils in Mrs. Martin's class are more than twelve years old. What fractional part of the class is more than twelve years old?

Did you say "$\frac{15}{30}$"? Copy the model sentence and replace the frame with the simplest form of the fraction.

$\frac{15}{30} = \square$

7. Six of the 30 pupils in the class are more than 5 feet 4 inches tall. What fractional part of the class is that tall? In your work, use a sentence like that in Exercise **6**.

8. What fractional part of 80 is 24? Find the simplest form.

Find the fraction (in simplest form) that answers each question.

	a	b
9.	What part of 80 is 16?	100 is what part of 300?
10.	225 is what part of 300?	24 is what part of 84?
11.	18 is what part of 81?	What part of 240 is 60?
12.	100 is what part of 125?	48 is what part of 72?

Using what you know about reciprocals, find the missing numeral in each of these sentences.

	a	b
1.	$\frac{1}{4} \times (4 \times 6) = \square$	$\frac{1}{3} \times (\triangle \times 9) = 9$
2.	$\frac{5}{4} \times (\frac{4}{5} \times \square) = 15$	$4 \times (\frac{1}{4} \times 50) = N$
3.	$\frac{7}{9} \times (\triangle \times 45) = 45$	$\frac{8}{9} \times (\frac{9}{8} \times \triangle) = 63$

4. Now suppose you are asked to find the missing numeral in the sentence $\frac{5}{6} \times N = 20$. As you did on page 93, you can replace N with an expression in parentheses, $(\square \times \triangle)$, and write:

$$\frac{5}{6} \times (\square \times \triangle) = 20$$

Now use your work with reciprocals. What is a correct replacement for the $\square$? for the $\triangle$? Since you have used $(\square \times \triangle)$ to replace N, have you now found that N is $\frac{6}{5} \times 20$? What does $\frac{6}{5} \times 20$ equal? Then N is 24. Check to see that the answer is correct. Does $\frac{5}{6} \times 24$ equal 20?

5. Marcia was making presents for the coming holiday. She said she had made 12 presents, and this was $\frac{2}{3}$ of the number of presents she needed. How many presents did she need?

Does the following number sentence tell about the problem: $\frac{2}{3} \times N = 12$? Using reciprocals, solve the problem. Check your answer.

6. Marcia's parents were addressing holiday greeting cards. Her mother counted 108 that were finished. Her father said that was $\frac{3}{4}$ of the job done. How many cards in all were they going to address?

7. If $\frac{3}{8}$ of a certain number is 21, what is the number?

8. If $\frac{5}{7}$ of a certain number is 30, what is the number?

Copy, and find the missing numerals.

	a	b	c
9.	$\frac{3}{16} \times N = 21$	$\frac{4}{5} \times \underline{?} = 28$	$\frac{1}{9} \times N = \frac{1}{2}$
10.	$\frac{3}{2} \times N = 60$	$\frac{1}{8} \times N = 2\frac{1}{4}$	$\frac{2}{3} \times \underline{?} = 9$

In your work on pages 105–106, you were using fractions in three different ways. The model sentences at the right illustrate these ways.

a. $\frac{4}{5} \times 60 = \square$

b. 30 is what part of 90?

c. $\frac{5}{6} \times N = 80$

1. Make up a problem that Sentence **a** might be used to solve. Then find the answer to the question you asked.

2. Make up a problem that Sentence **b** might be used to solve. Then find the answer to the question.

3. Make up a problem that Sentence **c** might be used to solve. Then find the answer to the question.

For each of Problems **4–6,** begin by writing a sentence that is like one of the model sentences above. Then find the answer to the question asked in the problem.

4. There are 28 pupils in the school Garden Club. Three fourths of these 28 are in the sixth grade. How many members of the Garden Club are in the sixth grade?

5. The 28 members of the Garden Club represent $\frac{2}{9}$ of the total number of pupils who belong to school clubs of all kinds. How many pupils belong to school clubs?

6. Four of the 28 pupils in the Garden Club are in the fifth grade. The fifth-graders represent what fractional part of the number of members in the Garden Club?

7. For which problem was your sentence like Sentence **a** above? For which problem was your sentence like Sentence **b**? For which problem was your sentence like Sentence **c**?

8. Jane is twelve years old. Her cousin is eighteen. Jane's age is what fractional part of her cousin's age?

9. Twelve years from now, will Jane's age be $\frac{2}{3}$ of her cousin's age? If not, what fractional part of her cousin's age will Jane's age be?

10. Jerry has 160 stamps in his album. Frank said, "That is $\frac{4}{5}$ of the number of stamps that I have." How many stamps does Frank have?

1. At the right is a table of multiplication sentences. Copy the table and find the products.

Study the sentences you have written. Then answer the following questions.

a. If the multiplier is greater than 1, is the product greater than, or less than, the multiplicand?

b. If the multiplier is less than 1, is the product greater than, or less than, the multiplicand?

c. As the multiplier decreases—that is, as it becomes smaller—does the product increase or does it decrease?

$4 \times 60 = \underline{?}$
$3 \times 60 = \underline{?}$
$2 \times 60 = \underline{?}$
$1 \times 60 = \underline{?}$
$\frac{4}{5} \times 60 = \underline{?}$
$\frac{3}{5} \times 60 = \underline{?}$
$\frac{2}{5} \times 60 = \underline{?}$
$\frac{1}{5} \times 60 = \underline{?}$

2. Now copy this table of division sentences and find the quotients.

Study the sentences you have written. Then answer the following questions.

a. When the divisor is greater than 1, is the quotient greater than, or less than, the dividend?

b. When the divisor is less than 1, is the quotient greater than, or less than, the dividend?

c. As the divisor decreases, does the quotient increase or does it decrease?

$60 \div 4 = \underline{?}$
$60 \div 3 = \underline{?}$
$60 \div 2 = \underline{?}$
$60 \div 1 = \underline{?}$
$60 \div \frac{4}{5} = \underline{?}$
$60 \div \frac{3}{5} = \underline{?}$
$60 \div \frac{2}{5} = \underline{?}$
$60 \div \frac{1}{5} = \underline{?}$

Now see whether you understand the patterns in Exercises **1** and **2**. Without multiplying or dividing, tell whether each of these sentences is true or not true.

	a	b	c
3.	$1\frac{1}{5} \times 2 > 2$	$\frac{99}{100} \times 4 < 4$	$\frac{100}{99} \times 3 < 3$
4.	$8 \div 1\frac{1}{3} < 8$	$\frac{5}{5} \times 9 > 9$	$5 \div \frac{15}{14} > 5$
5.	$\frac{3}{3} \times 7 < 7$	$6 \div \frac{99}{100} < 6$	$12 \div \frac{4}{4} > 12$

6. In Exercise **1**, the multiplicand was a whole number. Now use this table to see whether the same pattern appears when the multiplicand is a fractional number.

Copy the table and find the products.

$3 \times \frac{1}{6} = ?$

$2 \times \frac{1}{6} = ?$

$1 \times \frac{1}{6} = ?$

$\frac{3}{5} \times \frac{1}{6} = ?$

$\frac{2}{5} \times \frac{1}{6} = ?$

$\frac{1}{5} \times \frac{1}{6} = ?$

a. If the multiplier is greater than 1, is the product greater than the multiplicand?
b. If the multiplier is less than 1, is the product less than the multiplicand?
c. As the multiplier decreases, does the product decrease or does it increase?
d. Is this the same pattern that appeared in Exercise **1**?

7. In Exercise **2**, the dividend was a whole number. Now use this table to see whether the same pattern appears when the dividend is a fractional number.

Copy the table and find the quotients.

$\frac{1}{10} \div 3 = ?$

$\frac{1}{10} \div 2 = ?$

$\frac{1}{10} \div 1 = ?$

$\frac{1}{10} \div \frac{3}{5} = ?$

$\frac{1}{10} \div \frac{2}{5} = ?$

$\frac{1}{10} \div \frac{1}{5} = ?$

a. If the divisor is greater than 1, is the quotient less than the dividend?
b. If the divisor is less than 1, is the quotient greater than the dividend?
c. As the divisor decreases, does the quotient decrease or does it increase?
d. Is this the same pattern as in Exercise **2**?

Use your conclusions for Exercises **6** and **7** to help estimate the answers to the problems below. Then find the actual answer.

8. A 12-foot strip of wood is cut into pieces, each $\frac{7}{12}$ foot long. Are there more than 12 pieces, or fewer than 12 pieces? Find the number of pieces.

9. John weighs 92 pounds. His sister weighs $\frac{7}{8}$ as much. Does she weigh more, or less, than 92 pounds? Find her weight.

10. Mr. Green owns a lot that has an area of $\frac{7}{8}$ acre. Mr. Gold owns a lot that is $\frac{1}{3}$ acre. How many times as large as Mr. Gold's lot is Mr. Green's lot?

Some of the problems below are related to each other. You will have to use the information in one problem and its answer in order to solve another problem. Other problems mention more numbers than are needed to find the answer to the question that is asked. Read each problem carefully, think about the numbers that are given, and then decide which numbers are needed for the solution. Check each answer by using the facts given in the problem.

1. Jim spent $\frac{3}{4}$ of his savings to buy a telescope that cost \$5.85. How much money had he saved?

After reading the problem, you know you need to find the total amount of money Jim had saved. If you represent this amount by N, does the following number sentence tell about the problem?

$$\frac{3}{4} \times N = \$5.85$$

What is the number of cents in \$5.85? Then can you work with 585 and use reciprocals to find the missing numeral? To give an answer to the problem, how will you have to express your result?

Do the work. What is the answer?

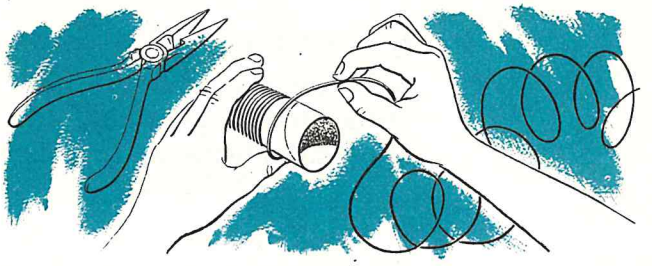
2. After Jim bought the telescope, how much did he have remaining in his savings? What fractional part remained?

3. Mr. Edwards has two boards, each 10 feet long and 4 inches wide. He needs 8 pieces, each $1\frac{5}{12}$ feet long. Will one of the boards be long enough for the 8 pieces? If not, how much of the second board will he have to use? Remember, each piece must be in one piece; it cannot be made from two shorter pieces.

4. On his hardware bill Mr. Edwards saw an item of $1\frac{1}{2}$ quarts of varnish for \$5.40. What was the cost per quart?

5. One day Mr. Edwards drove 125 miles; the next day he drove 240 miles. On the second day, he spent $5\frac{2}{3}$ hours driving. To the nearest mile per hour, what was his average speed that day?

6. For a science project, Tom needed a coil with 120 turns. The form for the coil was a small cardboard tube 4 inches long. What length of wire would he need if each turn used $4\frac{3}{4}$ inches of wire?



7. The same project required several connections, each $1\frac{7}{8}$ inches long. How many connections could Tom get from a piece of copper wire 19 inches long? How much wire would be left over?

8.a. A can of frozen lemonade concentrate contains 6 fluid ounces. The directions tell you to mix 4 cans of water with the contents. When this is done, how many fluid ounces will you have altogether?

b. There are 32 fluid ounces in a liquid quart. What part of a quart does one can of lemonade make?

9. When the Martin family left home for a weekend trip, Laura noticed the odometer reading. It was $10{,}057\frac{9}{10}$ miles. When they arrived at their destination, the odometer read $10{,}130\frac{9}{10}$ miles. The time taken for the trip was $1\frac{2}{3}$ hours. What was the average speed in miles per hour?

10. There were 700 students at a school basketball game. John heard the principal say that this was $\frac{5}{6}$ of the number of students in the school. How many students were there in the school altogether?

11. Tim's older brother is a member of the school's basketball team. In the previous season, he played in 15 games. He said: "I was lucky. I played in $\frac{5}{8}$ of the games the team played." How many games did the team play the previous season?

12. "I am thinking of a number," Sally said. "If I multiply it by $\frac{1}{5}$ and then multiply the product by 10, I get $2\frac{1}{2}$ as the answer."

Can you find the original number?

1. What is 8×15? Then what is $120 \div 15$? $120 \div 8$?

2. Is this sentence true: $11 \times 16 = 176$? Use what you know about multiplication and division, and tell what is the missing numeral in Sentence **a**; in Sentence **b**.

a. $\square = 176 \div 16$ **b.** $N = 176 \div 11$

3. Is this sentence true: $\frac{1}{2} \times 20 = 10$? Then what is the missing numeral in Sentence **a**? in Sentence **b**?

a. $\square = 10 \div 20$ **b.** $N = 10 \div \frac{1}{2}$

4. Is this sentence true: $\frac{3}{4} \times 16 = 12$? Then what is the missing numeral in Sentence **a**? in Sentence **b**?

a. $\square = 12 \div 16$ **b.** $N = 12 \div \frac{3}{4}$

5. Is this sentence true: $\frac{3}{5} \times 100 = 60$? Then what is the missing numeral in this sentence: $N = 60 \div \frac{3}{5}$?

6. Suppose you are asked to find the missing numeral in this sentence: $\frac{7}{8} \times N = 21$. Does N equal $21 \div \frac{7}{8}$? What is $21 \div \frac{7}{8}$? Have you now found that N is 24? Check the answer. Does $\frac{7}{8} \times 24$ equal 21?

7. Using the same method, find the missing numeral in this sentence: $\frac{2}{3} \times N = 100$. Check your answer.

8. Can you also find the missing numeral in $\frac{2}{3} \times N = 100$ (Exercise **7**) by using reciprocals? Do it.

9. Was part of your work for Exercise **7** similar to the work in **a**, below? Was part of your work for Exercise **8** similar to the work in **b**?

a.
$$\frac{2}{3} \times N = 100$$
$$\text{So } N = 100 \div \frac{2}{3}$$
$$= 100 \times \frac{3}{2}$$

b.
$$\frac{2}{3} \times N = 100$$
$$\frac{2}{3} \times (\triangle \times \square) = 100$$
$$\frac{2}{3} \times (\frac{3}{2} \times 100) = 100$$

Look at the expressions in color. Is each an expression for N? Do both methods give equivalent expressions for N?

10. Copy, and find the missing numerals.

a. $\frac{4}{5} \times N = 48$ **b.** $2\frac{1}{2} \times N = 30$ **c.** $N \times \frac{1}{3} = 90$

1. A garden 75 feet wide is to be divided into 12 strips of equal width. How wide will each strip be?

a. Can you find the answer, using the following number sentence: $75 \div 12 = 75 \times \frac{1}{12}$, or ? What is the answer?

b. The example can also be done in the division form used with whole numbers. Is the example at the right correct as far as it goes? How would the results of this example be expressed?

$$\begin{array}{r} 6 \\ 12\overline{)75} \\ \underline{72} \\ 3 \end{array}$$

c. With what you know about fractions, you can carry the division example further, as shown at the right. Divide the remainder, 3, by 12. What is $3 \div 12$ expressed as a fraction in simplest form? Then do you again get $6\frac{1}{4}$ feet as the answer to the problem?

$$\begin{array}{r} 6\frac{\triangle}{\square} \\ 12\overline{)75} \\ \underline{72} \\ 3 \end{array}$$

2. On a trip of 696 miles, Mr. Smith found that his car used 32 gallons of gasoline. What was the average number of miles his car traveled on a gallon? Express your answer as a mixed fraction.

Was his mileage closer to 21, or to 22, miles per gallon?

3. Mr. Foster raises chickens and sells their eggs. One morning he collected 1,052 eggs. This is how many dozen? Express your answer as a mixed fraction in simplest form.

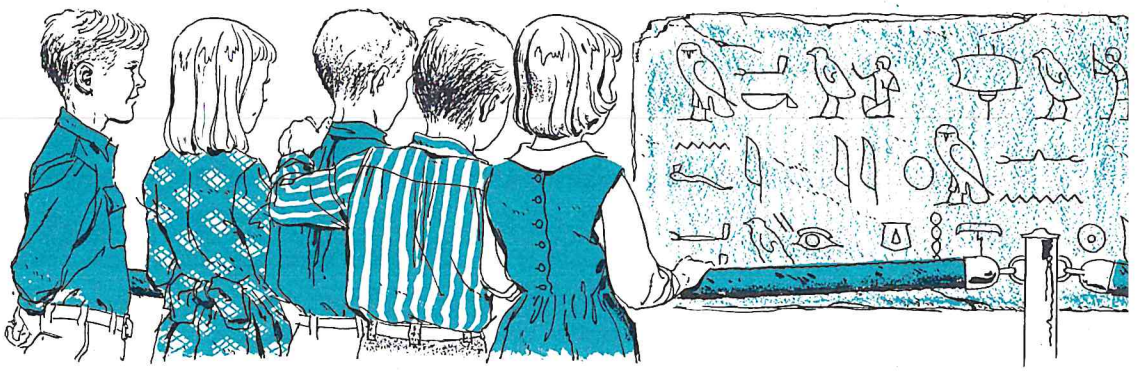
Divide, and express the quotient as a mixed fraction or a numeral for a whole number.

	a	b	c	d
4.	$20\overline{)584}$	$18\overline{)387}$	$13\overline{)2579}$	$16\overline{)610}$
5.	$12\overline{)866}$	$24\overline{)984}$	$88\overline{)5{,}058}$	$67\overline{)2{,}479}$

6. Did Exercise **5b** "come out even"? Can you check your answer by multiplying? Do it: $41 \times 24 = \square$

What was your answer to Exercise **4a**? Find the product of $29\frac{1}{5}$ and 20: $29\frac{1}{5} \times 20 = \frac{146}{5} \times 20$, or ?. Do you get the dividend?

Try checking Exercise **4b** in the same way.



For some of the problems on this page it is sensible to write a mixed fraction for the quotient. For others it is not sensible. Read a problem to decide what numbers to use, perform the division, and then decide how to express the quotient.

1. The sixth-grade classes, 148 students altogether, plan a trip to a museum that is 8 miles from the school. Each of the school buses will carry 44 students. How many buses will be needed for the trip?

As you see when you do the division example, there is a remainder. Would it be sensible to express the quotient as a mixed fraction in this problem? Could one use a fractional part of a bus? How many buses will have to be ordered?

2. One hundred fifty-two pounds of potatoes are to be put into 15-pound bags. How many bags of potatoes will there be? How many full bags? What fractional part of a bag?

3. During a 5-day trip, the Harrison family drove 1,548 miles. On the average, how many miles a day did they travel?

4. Frances cut a piece of material 45 inches long into pieces from which to make headbands that measured 17 inches. How many headbands did she get?

5. How many times can a tank that holds 75 gallons be filled from a tank that holds 500 gallons?

6. A family left home at 7 A.M. and reached their destination, 435 miles away, at 4 P.M. What was their average speed in miles per hour?

If you make any mistakes, study the pages given at the right of the exercises.

EXTRA PRACTICE 287–290

1. Copy, multiply, and express answers in simplest form.

a. $\frac{5}{6} \times \frac{2}{3}$ **b.** $\frac{4}{5} \times \frac{5}{8}$ **c.** $\frac{1}{12} \times \frac{3}{4}$ 74–75

d. $7 \times \frac{1}{16}$ **e.** $4 \times \frac{2}{3}$ **f.** $8 \times \frac{3}{4}$ 74–76

g. $3\frac{1}{6} \times 1\frac{1}{5}$ **h.** $2\frac{1}{2} \times 2\frac{2}{5}$ **i.** $\frac{1}{4} \times \frac{5}{12} \times \frac{3}{5}$ 84

2. Find the missing reciprocals. 88, 94

a. $3 \times N = 1$ **b.** $\frac{1}{8} \times \square = 1$ **c.** $\triangle \times \frac{1}{3} = 1$

d. $\frac{2}{3} \times \square = 1$ **e.** $12\frac{1}{3} \times \square = 1$ **f.** $\frac{8}{5} \times \triangle = 1$

3. Copy, divide, and express answers in simplest form. 89–92

a. $\frac{3}{4} \div 6 = \square$ **b.** $5\frac{5}{8} \div 5 = \triangle$ **c.** $\frac{1}{3} \div \frac{1}{8} = \square$

4. Use reciprocals to find the missing numerals. 93

a. $20 \times N = 480$ **b.** $\frac{1}{8} \times N = 32$ **c.** $\frac{1}{3} \times N = \frac{3}{4}$

5. Copy, divide, and express answers in simplest form. 95

a. $\frac{3}{4} \div 4\frac{1}{2}$ **b.** $10 \div 2\frac{1}{2}$ **c.** $7\frac{3}{4} \div \frac{7}{8}$

6. Copy and divide, using the common denominator method. 103–104

a. $\frac{5}{8} \div 1\frac{1}{4}$ **b.** $\frac{7}{12} \div \frac{2}{3}$ **c.** $6 \div 2\frac{1}{4}$

7. Find the fraction that answers each question. 105

a. What part of 75 is 25? **b.** 16 is what part of 64?

8. Find the missing numerals. 106

a. $\frac{3}{16} \times N = 27$ **b.** $\frac{4}{7} \times \square = 16$ **c.** $\frac{8}{5} \times N = 64$

9. Copy the letters **a–c.** Write *true* or *not true* after each. 108

a. $1\frac{1}{3} \times 2 > 2$ **b.** $8 \div \frac{1}{3} < 8$ **c.** $\frac{50}{49} \times 4 > 4$

10. Divide, and express each quotient as a mixed fraction. 113

a. $24\overline{)132}$ **b.** $27\overline{)333}$

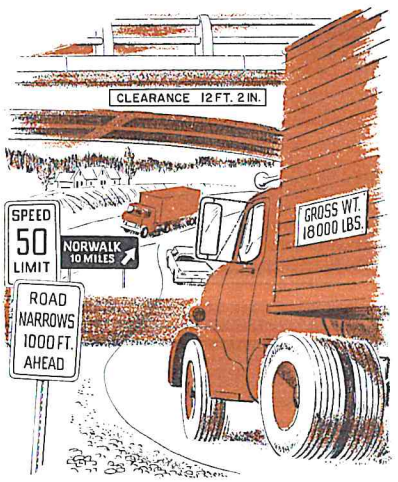
11. Tom withdrew $\frac{2}{3}$ of his \$15 savings from the bank. He spent $\frac{1}{5}$ of the amount he withdrew. How much did he spend?

12. One hundred twenty-five pounds of potatoes are to be put into 15-pound bags. How many full bags of potatoes will there be? What fractional part of another bag?

DISCOVERING MORE ABOUT NUMBERS: PAGES 319–321

4 Measurements Give Information

1. The Carters are furnishing a new home and are planning to buy a rug for the living room. Stores have rugs of many different sizes. How can the Carters find out what size rug to buy?



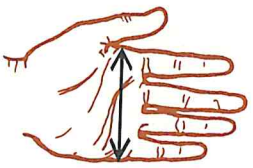
2. Look at the picture. What is the clearance for the underpass? Do you know what "clearance" means?

Truck drivers know the height of the trucks they drive. If the height of a truck is 11 feet 5 inches, can the driver use this underpass? How many inches will there be to spare?

a. Does the "clearance" sign give information about a measurement? How is this information valuable to drivers?

b. Do the other road signs tell about measurements of one kind or another? Describe some situations in which the information they give would be important.

3. This druggist is filling a prescription written by a doctor. The prescription gives measurements of the ingredients to use in making a certain medicine. Must the druggist understand these measurements and follow them closely? Why? Do you see that measurements are important in our lives?



Years ago people sometimes used parts of their arms or legs in measuring or describing various distances. Perhaps you have heard of a horse that is "15 hands high." Originally, the **hand** was the distance across the widest part of a man's hand. In some places, the word **palm** meant a measurement equal to a hand's width.

1. Use your hand to measure the height of this book. That is, find out how many hands it is from top to bottom. State any part of a hand as a fraction—for example, $\frac{1}{4}$ or $\frac{1}{3}$ hand.

2. Find the width of your desk in hands. Compare your answer with those of your classmates. Are all the answers the same?

3. You have been using your hand as a **unit of measure.** Why are the results not the same for the entire class?

Since hand widths differ, the hand is not the same distance for everyone. Would it be helpful if the class used a unit of measure that gave the same results for everyone? Would it also be helpful if the unit were marked off into fractional parts?

4. Do you and your classmates have notebook paper of the same size? If so, use the width of a sheet as a unit of measure. Call it a "blip." Fold the sheet into halves, fourths, and eighths. Do you now have fractional parts of a blip?

What is the width of your desk in blips? Does everyone get about the same answer?

▶When the same unit of measure is used by many people, and when it is described in such a way that people can tell just what it is, it is called a **standard unit of measure.**

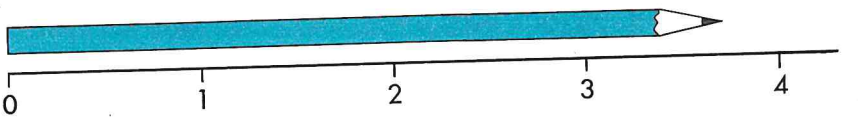
5. Is the inch a standard unit of measure? Could the blip be considered a standard unit of measure for your class? Is the blip a standard unit in the same sense as the inch?

6. Name four standard units for measuring distance, other than the inch. Name two standard units for measuring weight.

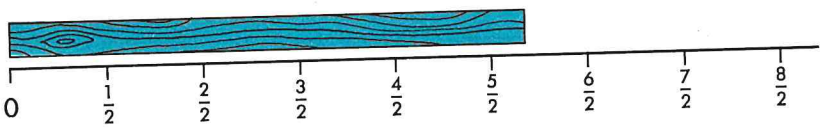
1. When you used your hand (or the sheet of paper) to measure your desk, you moved the unit of measure along the desk and counted the number of units wide the desk was. An easier way to measure is to use a **scale.**

The illustration below shows a scale marked off in inches. Do the numerals 1, 2, 3, and so on, show the number of inches each mark is from the 0 mark? When the scale is laid alongside an object as shown (with the 0 mark at one end of the object), we can read the length easily.

Is the length of the pencil closer to 3 inches or to 4 inches? We say the length is 4 inches to the **nearest inch.**

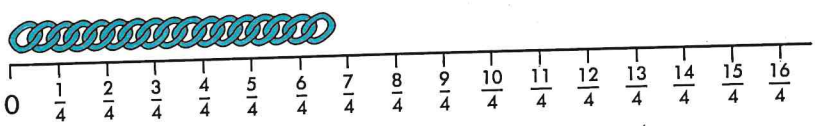


2.a. We can also make a scale in which distances are shown in half inches. The illustration below shows such a scale. Do the labels $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, and so on, show the number of half inches from 0? What is another name for $\frac{2}{2}$ inch? $\frac{3}{2}$ inches? $\frac{5}{2}$ inches? $\frac{8}{2}$ inches?



b. To the nearest half inch, what is the length of the piece of wood?

3.a. On the scale below, what do the marks indicate? What is another name for $\frac{5}{4}$ inches? $\frac{7}{4}$ inches? $\frac{8}{4}$ inches? $\frac{12}{4}$ inches?



b. To the nearest fourth inch, what is the length of the chain?

4. From a sheet of cardboard, cut three strips. On one strip, make a copy of the scale of inches (Exercise **1**); on the second, make a copy of the scale of half inches (Exercise **2**); on the third, make a copy of the scale that shows fourths of an inch (Exercise **3**).

5. Using your scale of inches, find, to the nearest inch, the length of each bar in the drawing below.

Is your answer in each case "2 inches to the nearest inch"? If you told someone who could not see the drawings that each bar is 2 inches to the nearest inch, would he have any idea which bar is actually longer?

6. Now use your scale of half inches and find, to the nearest half inch, the length of each bar.

What answers do you get? Would these measurements give someone else a better idea of the length of each bar than the measurements you stated in Exercise **5**?

Does measuring an object to the nearest half inch give a closer measurement of it than measuring to the nearest inch?

7. Using your scale of fourths of an inch, find the length of each bar to the nearest fourth of an inch. What answers do you get?

Do these measurements tell more about the actual length of the bars than the measurements in half inches?

▶A measurement can be more **precise** when made with a measuring instrument that is divided into small units of measure than when made with an instrument that has larger units. The smaller the divisions used, the more precise the measurement will be. We describe measurements as "precise to the nearest inch," "precise to the nearest half inch," and so on.

8. Do you have a ruler on which the scale is marked off in eighths of an inch? Measure the bars in Exercise **5** to the nearest eighth of an inch. Are these more precise measurements than those you made in Exercises **5–7**?

1. Find, to the nearest half inch, the length of this bar.

2. Find the length of the bar to the nearest fourth inch.

3. In your answers for Exercises **1** and **2**, did you use the phrases "to the nearest half inch" and "to the nearest fourth inch"? Sometimes, in reading, you may see measurements stated simply as "$1\frac{1}{2}$ inches" or "$1\frac{1}{4}$ inches." When a measurement is stated in this way, the denominator of the fraction tells us the precision of the measurement. Thus, a measurement given as $1\frac{1}{2}$ inches is considered precise to the nearest half inch. How precise is a measurement given as $1\frac{1}{4}$ inches? $2\frac{7}{8}$ inches? 15 inches?

4. Find the length of this bar to the nearest eighth inch.

5. Did you find that the length of the bar is $\frac{6}{8}$ inch? Did you then write $\frac{6}{8}$ in simplest form as "$\frac{3}{4}$ inch to the nearest eighth inch"?

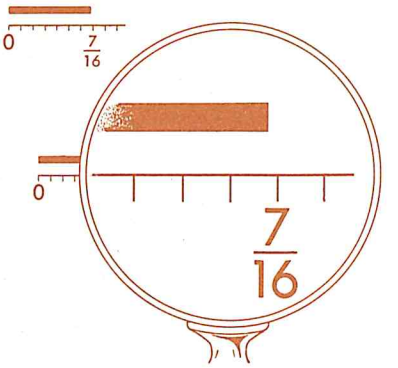
Sometimes, in giving measurements, the simplest form for a fractional number is not used. If we write "$\frac{6}{8}$ inch" as the answer, do you see that the denominator of the fraction tells how precisely we measured?

6. Find the length of this bar to the nearest half inch.

Did you write your answer as "2 inches"? A measurement stated as 2 inches is thought to be precise to the nearest inch. How precise is the measurement you made? Then, as you can see, it is better to write your answer as "2 inches to the nearest half inch." It may also be written as "$2\frac{0}{2}$ inches," although this is seldom used. In what way does this tell how precise the measurement is?

7. Jerry is $58\frac{1}{4}$ inches tall. Peter's height is 58 inches to the nearest half inch. Which measurement is more precise? Can you tell which boy is taller?

You have found that you can measure a distance more and more precisely by using scales with smaller and smaller divisions. Do you think that with a scale divided into sixteenths or thirty-seconds of an inch you could measure a distance exactly?



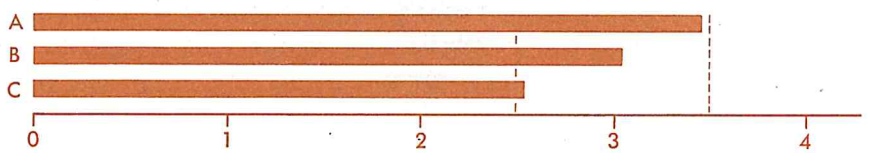
1. The scale below the bar in the top drawing has divisions of sixteenths of an inch. Do you think the length of the bar is exactly $\frac{7}{16}$ inch?

The second drawing shows how the bar and scale appear when magnified. Is the length of the bar exactly $\frac{7}{16}$ inch?

2. Do this experiment. With a ruler, try to draw a line segment exactly $\frac{9}{16}$ inch in length. Then measure the line under a magnifying glass. You will find that it is *not* exactly $\frac{9}{16}$ inch.

No measurement is exact. No matter how precisely we measure, there is a difference between the stated measure and the true measure. This difference is called **error of measurement.**

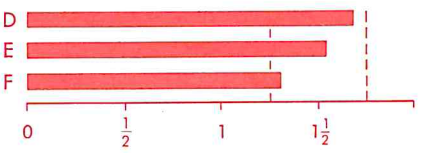
3. To the nearest inch, what is the length of bar *A*? *B*? *C*?



Does each bar measure 3 inches to the nearest inch? Would any measurement between $2\frac{1}{2}$ inches and 3 inches be stated as "3 inches to the nearest inch"? Would any measurement between 3 inches and $3\frac{1}{2}$ inches be stated the same way?

Then, if you read that a measurement is "3 inches to the nearest inch," you know that the true length must be between $2\frac{1}{2}$ inches and $3\frac{1}{2}$ inches.

Do you see that in a measurement stated to the nearest inch the true length of the object may be as much as $\frac{1}{2}$ inch less than, or $\frac{1}{2}$ inch more than, the stated measurement?

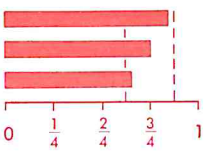


4. To the nearest half inch, what is the length of bar D? bar E? bar F?

Do you find that each bar measures $1\frac{1}{2}$ inches to the nearest half inch? Would any measurement between $1\frac{1}{4}$ inches and $1\frac{1}{2}$ inches be stated as "$1\frac{1}{2}$ inches to the nearest half inch"? Would any measurement between $1\frac{1}{2}$ inches and $1\frac{3}{4}$ inches be "$1\frac{1}{2}$ inches to the nearest half inch"?

5. If you read that a measurement is "$1\frac{1}{2}$ inches to the nearest half inch," then you know that the true length of the object must be between ? inches and ? inches.

Do you see that in a measurement stated to the nearest half inch the true length of the object may be as much as $\frac{1}{4}$ inch greater than, or $\frac{1}{4}$ inch less than, the stated measurement?



6. Does each of these bars measure $\frac{3}{4}$ inch to the nearest fourth inch?

If you read that a measurement is "$\frac{3}{4}$ inch to the nearest fourth inch," do you know that the true length must be between $\frac{5}{8}$ inch and $\frac{7}{8}$ inch? When a measurement is stated to the nearest fourth inch, may the true length of the object be as much as $\frac{1}{8}$ inch greater than, or $\frac{1}{8}$ inch less than, the stated measurement?

7. For a measurement made to the nearest inch, is there a $\frac{1}{2}$-inch difference possible between the true length and the stated measurement? For a measurement made to the nearest half inch, is there a $\frac{1}{4}$-inch difference possible between the stated measurement and the true length? What is the difference possible when a measurement is made to the nearest fourth inch? Do you see a pattern in the answers?

▶We say that the **greatest possible error** in a measurement is one half of the smallest unit used in making the measurement.

On page 117, when you were asked to name standard units of measure for weight, did you name the ounce? the pound? the ton?

1. How many ounces are equivalent to 1 pound? How many pounds are equivalent to 1 ton?

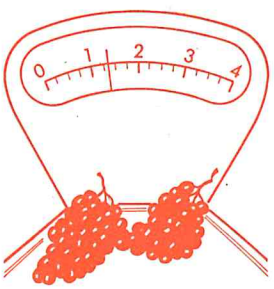
2. One hundred pounds is what fractional part of a ton? What fractional part of a ton is 200 pounds? 700 pounds? 1,250 pounds? 1,800 pounds?

3. The unit of measure to which we refer when we speak of "a ton"—that is, 2,000 pounds—is sometimes called the "short ton." There is another unit, called the "long ton," which is 2,240 pounds. What fractional part of a long ton is a short ton? Express your answer in simplest form.

4. What is the greatest possible error in a measurement given as $1\frac{1}{4}$ tons? Suppose the weight of a load of gravel is given as $13\frac{1}{2}$ tons. What is the least weight that the load can be? the greatest?

5. The clerk in a supermarket weighed some grapes and said, "$1\frac{1}{4}$ pounds." A clerk in a post office weighed a package and said, "1 pound 4 ounces." Do you think that both measurements were equally precise? If not, which measurement do you think was more precise?

6. If you think of a measurement of $1\frac{1}{4}$ pounds as precise to the nearest fourth of a pound, then the true weight must be between ? pounds and ? pounds. What is the greatest possible error?



7. Think of the measurement "1 pound 4 ounces" as precise to the nearest ounce. Then could the true weight be as little as 1 pound $3\frac{1}{4}$ ounces? Could it be as much as 1 pound $4\frac{3}{4}$ ounces? If your answer is "No" to each of these, then answer this question: Between which two weights must the true weight be?

1. Jim read in a science book that for each 5 pounds an object weighs on Earth it would have a weight of only 2 pounds on Mars.

a. If an object weighs 10 pounds on Earth, how much would it weigh on Mars?
b. If an object weighs 15 pounds on Earth, how much would it weigh on Mars?
c. If an object weighs 100 pounds on Earth, how much would it weigh on Mars?

2. Copy and complete the table below so that it shows for each weight on Earth the corresponding weight on Mars.

Weight on Earth (in pounds)	5	10	15	20	25	50	100	250
Weight on Mars (in pounds)	2	?	?	?	?	?	?	?

3. Now look at your completed table. For each number of pounds on Earth, is there a number of pounds on Mars? Can we think of these as *pairs of numbers:* 5 and 2, 10 and 4, and so on? Let the number of pounds on Earth be the first member of each pair.

a. What is the quotient when you divide 5 by 2? Express your answer as a mixed fraction in simplest form.
b. What is the quotient when you divide 10 by 4? Again express your answer as a mixed fraction in simplest form.
c. Did you get the same answer in each case?

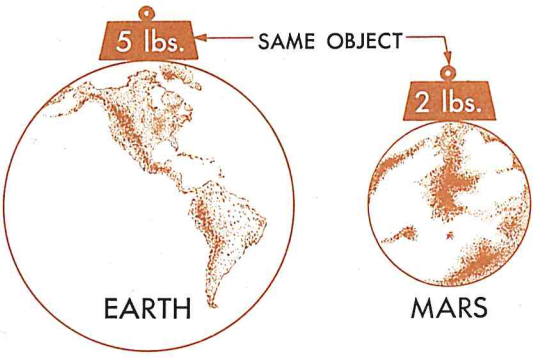
4. When you find that $5 \div 2 = 2\frac{1}{2}$, do you know that $5 = 2\frac{1}{2} \times 2$? Is it also true that $10 = 2\frac{1}{2} \times 4$?

5. Now investigate the quotients for each of the other pairs of numbers in your table. In each case, divide the first number by the second. Use the three sentences below as a guide.

a. $15 \div 6 = \square$ **b.** $20 \div 8 = \square$ **c.** $25 \div 10 = \square$

Do you get $2\frac{1}{2}$ as the quotient in each case? Does this quotient, $2\frac{1}{2}$, means that an object's weight on Earth is $2\frac{1}{2}$ times its weight on Mars?

On page 124, we compared weight on Earth to weight on Mars. We began by saying that for each 5 pounds an object weighs on Earth it weighs 2 pounds on Mars. There is another way to describe this relationship. We say that the **ratio** of the weight on Earth *to* the weight on Mars is **5 to 2.**



▶A ratio is a comparison of two numbers. For example, when two numbers have a ratio of 5 to 2, the first number is $\frac{5}{2}$, or $2\frac{1}{2}$, times the second.

1. Suppose, on page 124, we had begun by saying that for each 10 pounds an object weighs on Earth it weighs 4 pounds on Mars. We could say that the ratio of the weight on Earth to the weight on Mars is **10 to 4.**

a. From this information, could you figure out what an object that weighed 250 pounds on Earth would weigh on Mars? Do it. Explain how you got the answer. Did you get the same answer that you did when you first completed the table?
b. Is it true that 10 is $2\frac{1}{2}$ times 4? Is it true that 250 is $2\frac{1}{2}$ times 100?

We can say then that 10 and 4, or 250 and 100, have the ratio 5 to 2. The ratios "10 to 4" and "250 to 100" are **equivalent** to "5 to 2."

2. Mr. Holley, a house painter, found that to make a certain shade of light green paint, he had to mix 4 gallons of green paint with 1 gallon of white paint. Then is the ratio of the amount of green paint used *to* the amount of white paint used 4 to 1?

3. A few days later Mr. Holley wanted twice as much of the same shade of light green paint. How many gallons of green paint should he use? How many gallons of white paint?

4. Copy and complete this table comparing the number of gallons of green paint and the number of gallons of white paint to be used in making the shade of light green paint. Remember, the ratio of amounts in the mixture is 4 to 1.

Green	4	8	12	16	20	24
White	1	?	?	?	?	?

5. For each number of gallons of green paint, is there a certain number of gallons of white paint? Think of these as pairs of numbers, 4 and 1, 8 and 2, and so on. Which should be the first number in the pair—the number of gallons of green paint, or the number of gallons of white paint?

For each pair, what is the quotient of the first number divided by the second?

6. In each case, is the quotient 4? Does this quotient, 4, tell you that in each pair the first number is 4 times the second?

For any pair of numbers to have a ratio of 4 to 1, must the first number be 4 times the second?

7. Do the numbers in the pair 260 and 64 have the ratio of 4 to 1? Can you tell by doing this division example: $260 \div 64 = \square$? Work the example. Is the quotient 4? Then is the first number 4 times the second? Do 260 and 64 have the ratio of 4 to 1? What number and 64 do have the ratio of 4 to 1?

8. A syrup for canning fruit consists of 1 cup of sugar to each 3 cups of water. Or, stated differently, the ratio of the amount of sugar to the amount of water is 1 to 3. If 2 cups of sugar are used, how many cups of water must be used? If 5 cups of sugar are used, how many cups of water must be used?

9. Do you now have these number pairs: 1 and 3, 2 and 6, 5 and 15? What is $1 \div 3$? $2 \div 6$? $5 \div 15$? Is the quotient the same in each case? Does the quotient, $\frac{1}{3}$, mean that the first number is $\frac{1}{3}$ of the second number?

1. Bar A is 1 inch long. Bar B is 7 inches long. What is the ratio of the length of A to that of B?

What is the quotient for this example: $1 \div 7 = \square$? Does this quotient, $\frac{1}{7}$, mean that the number of inches in the length of A is $\frac{1}{7}$ of the number of inches in the length of B?

2. This diagram represents two bars, C and D. What is the ratio of the length of C to the length of D?

Now find the quotient for this example: $1\frac{1}{2} \div 10\frac{1}{2} = \square$. Express the quotient in simplest form.

Does this quotient, $\frac{1}{7}$, mean that the length of C is $\frac{1}{7}$ of the length of D? Can we also say that the ratio of the length of C to the length of D is 1 to 7?

3. What is the ratio of the length of bar E to the length of bar F?

Find the quotient for the example: $1\frac{3}{4} \div 12\frac{1}{4} = \square$. What does this quotient tell us? Can we say that the ratio of the length of E to the length of F is 1 to 7?

All pairs of numbers that have the same quotient have the same ratio.

4. If any pair of numbers has the quotient $\frac{1}{7}$, is the ratio of the first to the second 1 to 7? If a pair of numbers has the quotient $\frac{1}{9}$, is the ratio of the first to the second 1 to 9? What is the ratio if the quotient is $\frac{2}{3}$? if the quotient is $\frac{5}{3}$? if the quotient is $\frac{7}{2}$?

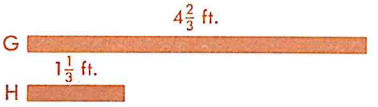
5. Suppose you are told that two numbers have the ratio of $4\frac{2}{3}$ to 7. What is the quotient for the example: $4\frac{2}{3} \div 7 = \triangle$? Then can we say that the two numbers have a ratio of 2 to 3? That is, is the first number $\frac{2}{3}$ of the second?

Which is clearer and easier to remember—the ratio 2 to 3 or the ratio $4\frac{2}{3}$ to 7?

6. One piece of shelving is 7 feet long; another is 2 feet long. What is the ratio of the length of the first piece to the length of the second?

Expressed as an improper fraction, what is the quotient for the example: $7 \div 2 = \triangle$?

7. Bar G is $4\frac{2}{3}$ feet long. Bar H is $1\frac{1}{3}$ feet long. What is the ratio of the length of G to the length of H?



Now find the quotient for this example: $4\frac{2}{3} \div 1\frac{1}{3} = \text{N}$. Express the quotient as an improper fraction in simplest form. Do you get $\frac{7}{2}$ as the quotient? Is this the same quotient you got for the numbers 7 and 2? Then do $4\frac{2}{3}$ and $1\frac{1}{3}$ have the ratio 7 to 2? Is 7 to 2 easier to remember than $4\frac{2}{3}$ to $1\frac{1}{3}$?

8. What is the ratio of 5 feet to 3 feet? the ratio of $3\frac{1}{3}$ feet to 2 feet? of $2\frac{1}{2}$ feet to $1\frac{1}{2}$ feet?

Now find the quotient for each of these examples. Express each quotient as an improper fraction in simplest form.

a. $5 \div 3 = \triangle$ **b.** $3\frac{1}{3} \div 2 = \triangle$ **c.** $2\frac{1}{2} \div 1\frac{1}{2} = \triangle$

Is the quotient for **b** and **c** the same as the quotient for **a**? Then does the pair of numbers $3\frac{1}{3}$ and 2 have the ratio 5 to 3? Does the pair of numbers $2\frac{1}{2}$ and $1\frac{1}{2}$ have the ratio 5 to 3? Is 5 to 3 easier to remember than $3\frac{1}{3}$ to 2? than $2\frac{1}{2}$ to $1\frac{1}{2}$?

▶For convenience, we generally express a ratio by using the smallest whole numbers possible.

9. Using the smallest whole numbers possible, express the ratio of:

a. $1\frac{1}{2}$ to $2\frac{1}{2}$ **b.** $1\frac{1}{3}$ to 4 **c.** $1\frac{1}{5}$ to $\frac{2}{5}$ **d.** $1\frac{3}{4}$ to $\frac{3}{4}$

10. Suppose we want to find the ratio of 1 inch to 2 feet. Do you think the ratio is 1 to 2? What is $1 \div 2$? Then, if you said the ratio is "1 to 2," would you be saying that 1 inch is $\frac{1}{2}$ of 2 feet? Is 1 inch equal to $\frac{1}{2}$ of 2 feet?

How many inches are 2 feet? Then is 1 to 24 the ratio of 1 inch to 2 feet? What is the ratio of 1 foot to 4 yards?

1. In the problems about the mixture of paints, you worked with gallons. Is a **gallon** (gal.) a standard unit of measure for liquids? Name three kinds of liquids that are sold by the gallon.

2. Is the **quart** (qt.) a standard unit of measure? How many quarts are equivalent to 1 gallon? Then a quart is what fractional part of a gallon?

3. The **pint** (pt.) is another standard unit of measure for liquids. How many pints are equivalent to 1 quart? How many pints are equivalent to 1 gallon?

4. Two boys used containers of different sizes to measure the milk in a can. Bill said the measurement was "$1\frac{1}{2}$ gallons." Harold said it was "1 gallon 2 quarts 1 pint." Does this mean that one of the boys made a mistake in measuring? If not, explain the difference in their measurements.

5. Read the label on the can of roof coating illustrated at the right. When you see such a label on a can in a hardware store, does it mean that the measurement is precise only to the nearest gallon? Do you think a 5-gallon can of roof coating may contain nearly $\frac{1}{2}$ gallon less than the 5 gallons?

Actually, the measurements given in such cases are much more precise than the label may seem to indicate. State governments have passed laws establishing the precision with which these measurements are to be made.

6. In arithmetic problems, the length of an object may be given as "7 feet"; and as you work the problem, you may have to express the 7 feet as ? inches. If the original measurement were really 7 feet to the nearest foot, would it be correct to express it as "84 inches"? But, just as the label on the can does not really mean that the measurement is precise only to the nearest gallon, let us agree that in our arithmetic problems "7 feet" may represent a measurement that is much more precise than "to the nearest foot."

1. One Saturday Jack worked 2 hours and 25 minutes cleaning up the yard. A week later he finished the job in 1 hour and 55 minutes. How long did he work altogether?

Copy the example. (Notice that we often use abbreviations in writing such examples.) What is 25 + 55? 2 + 1?

2 hr. 25 min.
1 hr. 55 min.
? hr. ? min. = 4 hr. ? min.

Since there are 60 minutes in an hour, 3 hours 80 minutes can be expressed as 4 hours and how many minutes? What is the answer to the problem?

2. In this example, did we regroup 13 ones as 1 ten and 3 ones, and add the 1 ten with the other tens?

69 = 6 tens, 9 ones
+ 24 = 2 tens, 4 ones
→ 8 tens, 13 ones, or 9 tens, 3 ones

Do you see that in Exercise **1** we also regrouped, using the fact that 60 minutes is equivalent to 1 hour?

In this example, will you use the fact that 12 inches is equivalent to 1 foot? Why? Work the example. Then explain your reasoning.

2 ft. 11 in.
9 ft. 5 in.
1 ft. 10 in.
? ft. ? in. = 14 ft. ? in.

3. Use regrouping to make each of these sentences true.

a. 9 yd. 5 ft. = 10 yd. ? ft.
b. 6 lb. 29 oz. = 7 lb. ? oz.
c. 11 gal. 6 qt. = 12 gal. ? qt.

Copy and add.

4.a. 9 ft. 5 in.
4 ft. 10 in.

b. 5 hr. 25 min.
4 hr. 40 min.

c. 4 gal. 1 qt.
3 gal. 2 qt.

5.a. 6 da. 12 hr.
5 da. 10 hr.
2 hr.

b. 10 qt. 1 pt.
2 qt. 1 pt.

c. 8 lb. 12 oz.
6 lb. 10 oz.
2 lb. 6 oz.

1. Roy wanted to find the weight of his dog. First he held the dog in his arms and stepped on the scale. He read the weight as 99 pounds 4 ounces. Then he weighed himself alone. His weight was 89 pounds 8 ounces. What was the weight of the dog?

The subtraction example for the problem is shown below. Can we subtract 8 ounces from 4 ounces?

$$\begin{array}{rcl} 99 \text{ lb. } 4 \text{ oz.} & = & 98 \text{ lb. } ? \text{ oz.} \\ -\ 89 \text{ lb. } 8 \text{ oz.} & = & 89 \text{ lb. } 8 \text{ oz.} \\ \longrightarrow & & ? \text{ lb. } ? \text{ oz.} \end{array}$$

Regroup the 99 pounds 4 ounces: 99 pounds 4 ounces is 98 pounds and how many ounces? If you said "98 pounds 16 ounces," recheck your work.

Copy the example and subtract. What is the answer?

2. Regroup so that each sentence is true.

a. 7 yd. 1 ft. = 6 yd. ? ft.
b. 9 gal. 0 qt. = 8 gal. ? qt.
c. 3 mi. 1,225 ft. = 2 mi. ? ft.

Copy and subtract.

	a	b	c
3.	10 yd. 2 ft. 6 yd. 1 ft.	21 da. 6 hr. 8 da. 12 hr.	5 mi. 1,000 ft. 2 mi. 2,875 ft.
4.	1 hr. 15 min. 45 min.	10 gal. 3 gal. 3 qt.	15 min. 20 sec. 6 min. 45 sec.
5.	7 yd. 1 ft. 3 yd. 2 ft.	10 lb. 6 oz. 2 lb. 8 oz.	8 gal. 1 qt. 3 gal. 3 qt.

6. In 1952 the steamship *United States* made a record eastbound crossing of the Atlantic Ocean in 3 days 10 hours 40 minutes. In the same year Horace C. Boren left New York and flew around the world in 4 days 19 hours 38 minutes. How much longer did it take Pilot Boren to fly around the world than for the *United States* to cross the Atlantic?

1. Nancy's father used 3 packages of ground meat for hamburgers for a cookout. Each package weighed 2 pounds 6 ounces. How much ground meat did he use?

Will you multiply to find the answer? Write a number sentence to describe the problem.

The multiplication example for the problem is shown here. What is 3×6? 3×2? Since 2 lb. 6 oz. means 2 lb. *and* 6 oz., does the 6 lb. 18 oz. mean 6 lb. *and* 18 oz.? Now regroup the 6 lb. 18 oz. and express the answer in the way that pounds and ounces are usually expressed.

2 lb. 6 oz.
× 3
? lb. ? oz.

2. There is another way by which you can find the answer for Exercise **1**. In simplest form, what fractional part of a pound is 6 oz?

Can 2 lb. 6 oz. be expressed as $2\frac{3}{8}$ lb.? Now you can write this number sentence for the problem: $3 \times 2\frac{3}{8} = \square$. Find the product and express it as a mixed fraction. Then give your answer, using this mixed fraction. Does this answer mean the same as the answer for Exercise **1**? Explain.

Copy and multiply, using the method of Exercise **1**. Check your results by using the method of Exercise **2**.

a	b
3. 3×4 hr. 25 min.	2×3 lb. 8 oz.
4. 5×9 qt. 1 pt.	5×7 yd. 2 ft.
5. 6×2 wk. 3 da.	5×8 ft. 4 in.
6. 4×1 lb. 14 oz.	3×6 gal. 3 qt.
7. 10×5 min. 30 sec.	4×2 T. 500 lb.
8. 7×4 yr. 90 da.	15×2 yr. 15 wk.

1. Bill's father let him use a piece of shelving 10 feet 8 inches long for some bookshelves. Bill wanted to have 4 shelves, so he cut the board into that many equal pieces. To find the length of each shelf, do you think you should add, subtract, multiply or divide?

a. What fractional part of a foot is 8 inches? Then 10 feet 8 inches can be expressed as $10\frac{?}{3}$ feet.

b. Does this number sentence describe the problem: $10\frac{2}{3} \div 4 = \square$? Find the quotient. Express your answer in terms of feet, using a mixed fraction; then express the answer in feet and inches.

Divide. Express the answer in two ways, as you did in Exercise **1**.

	a	b
2.	60 lb. $\div 1\frac{1}{2}$	5 lb. 8 oz. $\div$ 4
3.	5 yr. 3 mo. $\div$ 3	6 min. 15 sec. $\div$ 5
4.	10 gal. $\div 2\frac{1}{2}$	9 ft. 10 in. $\div$ 2
5.	23 yd. 1 ft. $\div$ 5	9 hr. $\div \frac{3}{4}$

6. The ages of the 6 girls in a singing group called the "Orioles" are as follows: 10 yr. 3 mo.; 10 yr. 6 mo.; 9 yr. 9 mo.; 10 yr. 0 mo.; 10 yr. 1 mo.; and 9 yr. 11 mo. What is the average age of the members of this singing group? Express your answer in years and months.

If a man runs a mile in 4 minutes, is his average speed more than 1,300 feet per minute? If a bus travels a distance of 135 miles in 3 hours, what is its average speed in miles per hour?

In stating a speed, do we use a measure of distance? a measure of time? You can see that measures of speed are given in units of distance and time.

As you can see from the first paragraph, the units of distance and time can vary. Sometimes we use miles per hour, sometimes miles per minute, sometimes feet per second. When no special directions are given, you should express speed in the units given in a problem.

1. A jet airliner flew from London to New York, a distance of 3,455 miles, in 8 hours. To the nearest mile, what was its average speed?

2. How many feet per minute is a car traveling when it travels 30 miles per hour?

a. One way to solve this problem is to begin by finding the number of *feet per hour* equivalent to 30 miles per hour. How many feet is 1 mile? Then 30 miles is how many feet?

Do you now know that 30 miles per hour is 158,400 feet per hour? Then how can you find the speed in feet per *minute*? Do the work. What answer do you get?

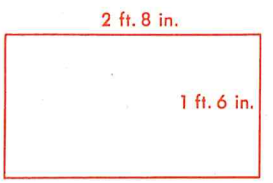
b. There is another way to solve the problem. If the car is traveling 30 miles per hour, what fractional part of a mile does it travel in 1 minute? What is the simplest form for this fractional number? Then, since the car travels $\frac{1}{2}$ mile per minute, how many feet per minute does it travel? Do you get the same answer as before?

3. Express 30 miles per hour as feet per second.

4. A pitcher can throw a fast ball at a speed of 96 miles per hour. About what is the speed in feet per minute? (Round the result to the nearest ten feet.)

5. To break free of the earth's gravity, a space vehicle must attain a speed of 18,000 miles per hour. What is this speed in miles per minute? in miles per second?

1. Mr. Martin wanted to put weather stripping around a basement window. He measured to see what length of weather stripping to buy. How many sides of the window did he need to measure? Using the measurements in the diagram, find the distance around the window.

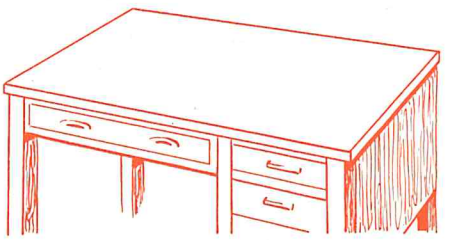


Does the outline of the window have the shape of a rectangle? The distance around a rectangle, a square, or a triangle (that is, the sum of the lengths of its sides) is called its **perimeter.** We can speak also of the perimeter of a field or a lot or a picture frame.

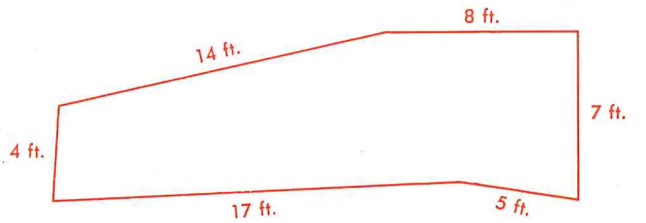
2. To find the answer to Exercise **1**, did you add the four measurements? Or did you write a number sentence of this kind: $(2 \times 2 \text{ ft. } 8 \text{ in.}) + (2 \times 1 \text{ ft. } 6 \text{ in.}) = \text{N}$?

Could you have expressed the measurements in terms of feet, using mixed fractions, and then solved the problem in that form? Check your work for Exercise **1**, using a method different from the one you have used.

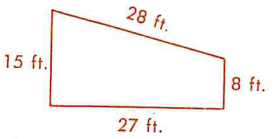
3. The edge of a plywood desk top is to be finished with matching strips. The top has the shape of a rectangle. Two strips, each 60 inches long, and two strips, each 32 inches long, are needed. In feet and inches, what is the total length of the stripping that is needed?



4. The diagram shows the shape of a rock garden. What is the perimeter of the figure?



5. The shorter sides of a picture frame are each $13\frac{3}{4}$ inches long, and the longer sides are each $16\frac{3}{4}$ inches long. What is the perimeter?



6. Because of the back-yard space available, a swimming pool has the shape shown at the left. What is the perimeter of the top of the pool?

7. The perimeter of a square is 19 feet 4 inches. What is the length of each side?

8. The sides of a triangle have the same length. The perimeter is 4 feet 3 inches. What is the length of a side?

9. Each side of a square has a length of 5 feet 6 inches. What is its perimeter?

10. A Little League baseball diamond is a square with a distance of 60 feet between bases. What is the least distance a boy runs if he hits a home run?

11. A plot of grass has the shape of a triangle. The lengths of its sides are 79 feet 5 inches, 48 feet 6 inches, and 52 feet 9 inches. Find its perimeter. What reason might there be for finding the perimeter?

12.a. What is the perimeter of a rectangle if the shorter sides are 4 inches long and the longer sides are 6 inches long?
b. What is the perimeter of a rectangle if the shorter sides are $2\frac{1}{2}$ inches long and the longer sides are $7\frac{1}{2}$ inches long?
c. Give the measurements of at least three other rectangles that have a perimeter of 20 inches.

Find the perimeters of the rectangles with the following lengths of sides.

	a	b
13.	1 ft. 2 in.; 3 ft. 6 in.	$3\frac{1}{2}$ in.; $5\frac{1}{2}$ in.
14.	$7\frac{1}{4}$ ft.; $3\frac{3}{4}$ ft.	74 ft.; 42 ft.

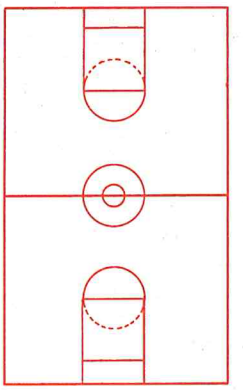
15. Find the perimeters of squares whose sides have the following lengths.

a. 160 ft. **b.** $5\frac{3}{4}$ ft. **c.** 2 ft. 10 in. **d.** $7\frac{2}{3}$ ft.

When you think of a globe, do you think of a flat surface, or of a curved surface? When you think of an ordinary sheet of glass, do you think of a flat surface? Name some other things that you think of as flat surfaces.

When we draw a rectangle, a square, or a triangle, we think of it as lying along a flat surface.

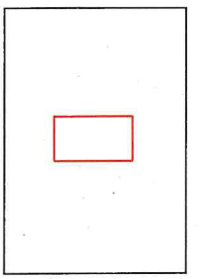
1. At the right is a diagram of a basketball court. Do the boundary lines of the court have the shape of a rectangle?



If you measured the boundary lines and found the distance around the court, what would you call this distance?

What do we mean when we speak of the "playing area" of the basketball court? Does the playing area include all of the surface of the court inside the lines?

2. This illustration shows a sheet of paper with a shipping label on it. Can we think of each as a flat surface? What shape is formed by the bounding edges of the paper? the label? Which has more surface on which you can write? Can we say that the sheet of paper has more writing area?



Now think of the illustration as two rectangles. Can you think of the writing surface as space enclosed by each rectangle?

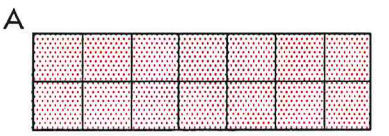
▶The amount of enclosed space in a figure such as a square, a rectangle, or a triangle is called its **area.**

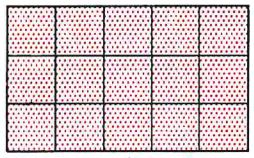
3. Without measuring, name the rectangle that you think has the greater area—*A* or *B*. Are you sure of your answer?

A

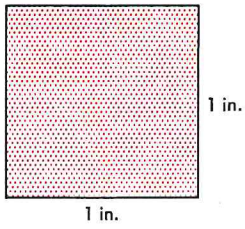
B

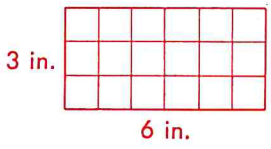
4. When we measure distance, we use a unit of length. We can use the **unit of area** at the right to measure the areas of the rectangles from Exercise **3**. Shown below are rectangles *A* and *B* with the units of area marked off. How many units fit inside *A*? *B*? Which rectangle has the greater area?





▶A standard unit of area is shown at the right. It is the area enclosed by a square each side of which is 1 inch long. This standard unit is the **square inch.**





5. This drawing represents a rectangle that is 3 inches by 6 inches. Since the length of the rectangle is 6 inches, how many 1-inch squares must be in the bottom row? Since the width is 3 inches, how many rows of 1-inch squares fit inside the rectangle? What is the total number of 1-inch squares inside the rectangle? Then is the area 18 square inches?

6. Draw a rectangle 4″ by 5″. (The sign ″ means inches.) Then mark off 1-inch squares as was done in Exercise **5**.

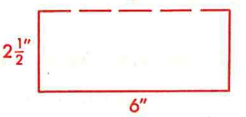
How many 1-inch squares fit in each row? How many rows are there? What is the area in square inches?

7. Draw a rectangle 2″ by 8″. How many 1-inch squares fit in the rectangle? What is the area in square inches?

8. Do you see a pattern to the results for Exercises **5–7**? Then what is the area in square inches of a rectangle 5″ by 7″? a square 4″ by 4″? a rectangle 1″ by 8″? Explain your reasoning.

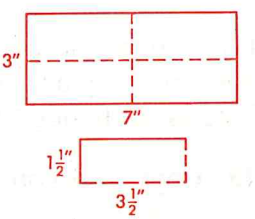
1. This drawing represents a sheet of paper 5″ by 6″. What is its area in square inches?

2. Now imagine that we cut the sheet of paper in half as shown. Should the area of the smaller piece be $\frac{1}{2}$ of the area of the original sheet? What is $\frac{1}{2}$ of 30 square inches?



3. To find the area of the original sheet of paper, did you think: "$5 \times 6 = 30$; so the area is 30 square inches"? Suppose you had used a similar method to find the area of the half sheet. What is $2\frac{1}{2} \times 6$? Is 15 square inches the correct answer?

4.a. What is the area of a sheet of paper 3″ by 7″? Suppose you folded the paper in fourths as indicated. Does each piece have an area of $\frac{1}{4}$ of 21 square inches? How many square inches is this area?

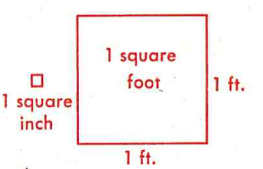


b. Is each fourth of the paper $1\frac{1}{2}$″ by $3\frac{1}{2}$″? What does $1\frac{1}{2} \times 3\frac{1}{2}$ equal? Do you again get $5\frac{1}{4}$ square inches?

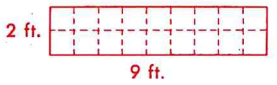
5. What is the area of a rectangle $1\frac{1}{2}$″ by 5″? a rectangle $1\frac{1}{2}$″ by $2\frac{1}{2}$″? If necessary, you may make a drawing of a rectangle 3″ by 5″ and follow the method used in Exercises **1–4**.

6. Imagine a square each side of which is 12 inches long. What is its area in square inches?

7. The **square foot,** a standard unit of area, is the area inside a square that is 1 foot by 1 foot. This drawing suggests the difference between a square inch and a square foot.



How many square inches fit inside a square foot? Is the area of the square described in Exercise **6** equivalent to 1 square foot?



8. This drawing represents a rectangle 2 feet wide and 9 feet long. What is its area in square feet?

What is the area of a rectangle 3′ by 8′? (The sign ′ means feet.) What is the area of a rectangle 11′ by 12′?

Find the area of the rectangle or square with sides having these measurements.

	a	b	c
9.	$6\frac{1}{2}''$ by $7''$	$3'$ by $5\frac{1}{2}'$	$9'$ by $12\frac{1}{4}'$
10.	$8\frac{1}{2}'$ by $8\frac{1}{2}'$	$9'$ by $13'$	$2\frac{3}{4}''$ by $2\frac{3}{4}''$
11.	$\frac{1}{2}'$ by $\frac{1}{2}'$	$\frac{1}{4}''$ by $1''$	$1\frac{2}{3}'$ by $3\frac{2}{3}'$

12. Another standard unit of area is the **square yard.** It is the area of a square with each side 1 yard long. How many feet long is each side? How many square feet in a square yard?

13. Copy and complete the chart below.

? square inches (sq. in.)	is 1 square foot (sq. ft.)
? square feet	is 1 square yard (sq. yd.)

14. A square of linoleum tile is $9''$ by $9''$. How many square inches is this? What fractional part of a square foot is it?

15. Ralph said he had measured his room and found it was "10 feet by 4 yards." Do you think this means that he measured one side to the nearest foot and the other to the nearest yard? Imagine that in both cases he has measured very precisely—more precisely than "to the nearest foot."

What is the area of his room in square feet? Is it 40 square feet? Why not?

What is 4 yards expressed as feet? Then what is the area of the room in square feet? in square yards?

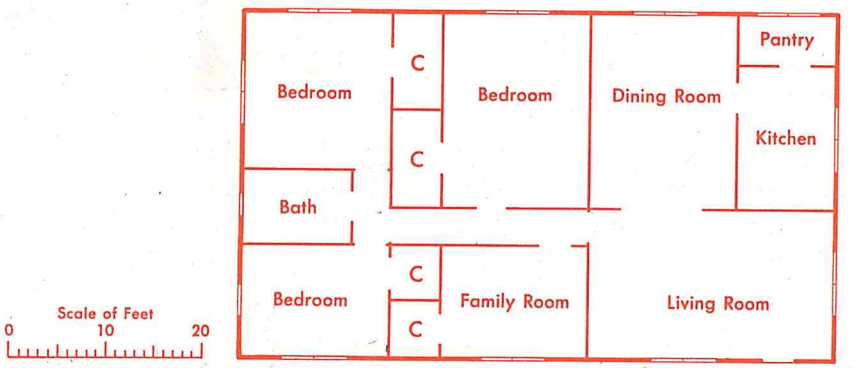
16. Find the areas of these rectangles.

a. 36 in. by 8 ft. **b.** 5 ft. by 9 yd. **c.** $1\frac{1}{2}$ yd. by 17 in.

Have you ever seen the floor plans for a house? Are such plans the same size as the actual house? Why not? Floor plans must give an accurate idea of the true shape, size, and arrangement of the rooms. Below are floor plans for a house.

Measure the distance from the 0 to the 20 on the scale beside the plans. Is this distance 1 inch? The scale means that 1 inch on the drawing represents 20 feet of actual distance. We call a drawing of this kind a **scale drawing.**

Measure the distance from the 0 to the 10 on the scale. If a distance on the drawing is $\frac{1}{2}$ inch, what is the actual distance it represents? Could you have told that without measuring? How?

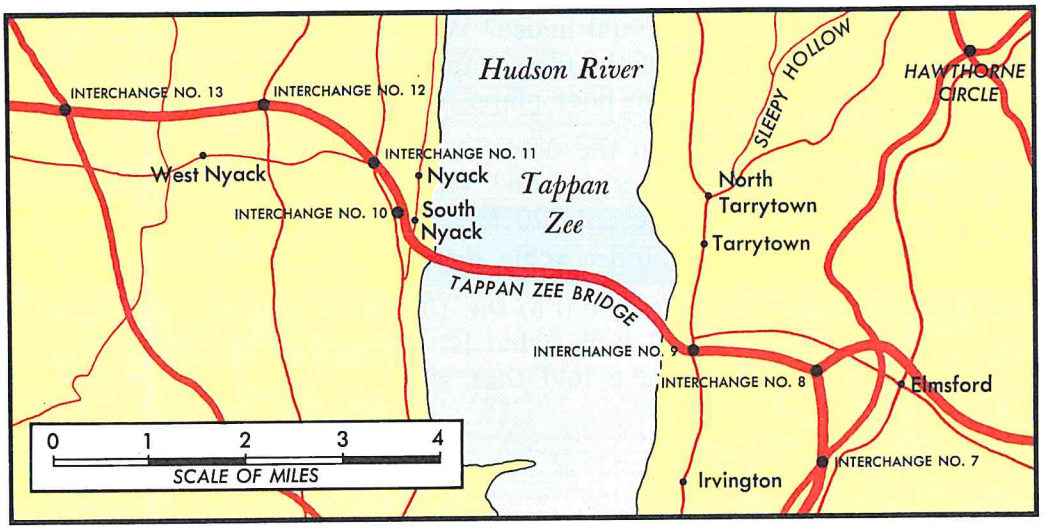


1. What is the actual distance if the distance on the drawing measures 2 inches? if the distance on the drawing measures $\frac{1}{4}$ inch? $\frac{3}{4}$ inch? $1\frac{1}{4}$ inches? $2\frac{1}{2}$ inches?

2. Measure the length and width of the living room as it is shown in the drawing. What are its actual width and length?

3. Express 20 feet in inches. Then does the scale for the floor plans mean that for each 1 inch of distance on the drawing the actual distance is 240 inches? Can we express this relationship as a ratio and say that the ratio is 1 to 240?

If this ratio is correct, should 2 inches on the drawing represent 480 inches of actual distance? Does it? Explain.



1. A map is a scale drawing, in greatly reduced size, of a section of country. Shown above is a map of an area near New York City. Measure the distance from the 0 to the 2 on the "Scale of Miles." What is this distance? Then 1 inch on the map represents how many miles?

2. How many miles are represented by $\frac{1}{2}$ inch on the map? by $\frac{1}{4}$ inch? by $\frac{3}{4}$ inch?

3. Use your ruler to measure to the nearest quarter inch the distance along a straight line between Nyack and Tarrytown. Is it $1\frac{1}{2}$ inches? How many miles is this?

4. About how far is it from Irvington, where Washington Irving lived, to Sleepy Hollow, where the headless horseman made his legendary ride?

5. How far would a helicopter travel in going from Hawthorne Circle to Thruway Interchange No. 7 and then to Interchange No. 13?

6. About what is the length of the Tappan Zee Bridge?

7. What is the ratio of 1 inch to 2 miles?

If you make any mistakes, study the pages given at the right of the exercises.

EXTRA PRACTICE 291–296

1. Is a measurement given as $14\frac{1}{4}$ feet of the same precision, or less precision, or greater precision than a measurement given as $14\frac{2}{8}$ feet? 118–120

2. Find the greatest possible error in each measurement. 122

a. 4 ft. **b.** $3\frac{1}{2}$ in. **c.** $17\frac{2}{3}$ yd. **d.** 13 oz.

3. Express each ratio with the smallest whole numbers possible. 124–128

a. $\frac{5}{2}$ to 1 **b.** $2\frac{1}{2}$ to $3\frac{1}{2}$ **c.** $1\frac{1}{3}$ to 5 **d.** $1\frac{1}{8}$ to 1

4. Express 40 miles per hour as feet per second. 134

5. How many feet of fence are needed to enclose a rectangular garden that is 15 feet long and 9 feet wide? 135–136

6. A grass plot has the shape of a triangle and all of its sides are the same length. If the perimeter is 24 feet, what is the length of each side? 133, 135–136

7. Find the areas of the rectangles with sides having these measurements. 137–140

a. $5\frac{1}{2}''$ by $8''$ **b.** $3'$ by $5\frac{1}{3}'$ **c.** 7 yd. by $8\frac{1}{4}$ yd.

8. Add, subtract, multiply, or divide as indicated. 132–133

a. 5 gal. 3 qt. + 4 gal. 2 qt.

b. 12 yd. 1 ft. − 8 yd. 2 ft.

c. 7 lb. 10 oz. + 4 lb. 8 oz.

d. 5 yd. 2 ft. ÷ 2 **e.** 9 × 2 qt. 1 pt. **f.** 2 lb. 1 oz. ÷ 3

9. One inch on a map represents 3 miles. Find the number of miles represented by each of these measurements. 142

a. 2 inches **b.** 3 inches **c.** $\frac{1}{2}$ inch **d.** $\frac{1}{4}$ inch

10. The ratio of the distance on a map to the actual distance is 1 to 25,000. One inch on the map represents how many inches on the ground? One inch represents how many feet (to the nearest foot)? 141–142

DISCOVERING MORE ABOUT NUMBERS: PAGES 322–325

1. Round each of these to the nearest ten thousand. 2–3

a. 3,208,516 **b.** 7,876,347 **c.** 4,392,673

2. Tell what the figure 9 means in each numeral. 5–6

a. 2,438,902 **b.** 29,560 **c.** 9,350,423 **d.** 905,875

3. What does the 0 mean in each numeral in Exercise **2**? 5–6

4. Round each of these to the nearest thousand. 2–3

a. 47,580 **b.** 12,162 **c.** 96,056 **d.** 68,649

5. You know that 534 equals $(5 \times 100) + (3 \times 10) + 4$. Express each of the following numbers in a similar way. 8

a. 8,467 **b.** 35,906 **c.** 248,125 **d.** 52,007

6. You also know that exponents can be used to show place value in a numeral: $534 = (5 \times 10^2) + (3 \times 10) + 4$. Express each of the numbers in Exercise **5** in a similar way. 8

7. Copy, and replace each frame with a numeral that has no exponents. 8

a. $10^3 \times 10^2 = \square$ **b.** $10 \times 10^2 = \square$ **c.** $10^2 \times 10^2 = \square$
d. $10^2 = \triangle$ **e.** $10^3 = \triangle$ **f.** $10^4 = \triangle$

8. For each of your sentences for Exercise **7 d-f**, what is the relationship between the exponent of 10 and the number of zeros in the replacement for the frame? 7

9. Copy, and replace each $\triangle$ with a power of 10. Use exponents.

a. $10{,}000 = \triangle$ **b.** $100{,}000 = \triangle$ **c.** $1{,}000{,}000 = \triangle$ 7

10. Copy, and replace each $\diamond$ with a base ten numeral. 13

a. $10_{six} = \diamond$ **b.** $25_{six} = \diamond$

11. Copy, and replace each N with a base six numeral. 12

a. $10 = N$ **b.** $12 = N$ **c.** $36 = N$

12. Copy, and replace each $\square$ with a Roman numeral. 14

a. $25 = \square$ **b.** $155 = \square$ **c.** $1{,}000 = \square$

Add and check. 16–18

	a	b	c	d	e
1.	7,086 5,865 9,065	4,908 499 48	268 238 456	65,408 67,795	2,454 7,987
2.	\$5.20 .97 .08 7.40 1.70	94 64 29 40 73	\$4.85 6.70 3.58 3.62	\$2.50 6.76 .15 .10 .70	\$13.75 28.69 48.30 36.14

Subtract and check. 16–18

3.	9,235 3,947	717 259	48,004 8,609	900 258	82,465 7,696
4.	21,503 2,775	\$943.46 249.87	400 183	8,742 6,383	5,512 2,645

Multiply and check. 24–29

5.	\$.97 40	296 38	4,500 93	3,067 65	49 75
6.	\$5.68 807	\$6.40 400	800 748	512 568	900 910

Divide and check. 30–37

7.	$50\overline{)3,291}$	$79\overline{)2,105}$	$65\overline{)2,000}$	$46\overline{)1,220}$	$22\overline{)1,200}$
8.	$97\overline{)6,801}$	$31\overline{)\$28.24}$	$68\overline{)2,418}$	$73\overline{)51,103}$	$34\overline{)1,012}$

Find the missing numerals. 19–20

	a	b	c
9.	$\square \times 78 = 5{,}070$	$5{,}070 \div \square = 78$	$\triangle + 1{,}763 = 4{,}947$
10.	$5{,}874 - \square = 1{,}789$	$\square - 859 = 7{,}482$	$85 \times 64 = N$
11.	$\square \div 28 = 47$	$8{,}741 \times \square = 26{,}223$	$31{,}460 \div 65 = \square$

1. Use the "<" sign and write a true sentence involving each of the pairs of fractions. 42

a. $\frac{1}{5}, \frac{1}{3}$ **b.** $\frac{1}{3}, \frac{1}{4}$ **c.** $\frac{1}{4}, \frac{1}{8}$ **d.** $\frac{1}{5}, \frac{1}{8}$ **e.** $\frac{1}{5}, \frac{1}{4}$

2. Copy and replace each frame with a mixed fraction in lowest terms or a numeral for a whole number. 46–47

a. $\frac{32}{4} = \square$ **b.** $\frac{18}{15} = \triangle$ **c.** $\frac{31}{2} = \Diamond$ **d.** $\frac{63}{8} = \square$

3. Copy and replace N with a numeral that makes the sentence true. 76

a. $N = \frac{1}{8} \times 48$ **b.** $\frac{7}{8} \times 56 = N$

4. Copy and replace each frame with the fraction from the following set of fractions which makes the sentence true: 44

$\frac{2}{14}, \frac{50}{100}, \frac{4}{6}, \frac{75}{100}, \frac{10}{100}, \frac{6}{24}, \frac{10}{16}, \frac{63}{81}$.

a. $\frac{1}{2} = \square$ **b.** $\frac{3}{4} = \triangle$ **c.** $\frac{5}{8} = \Diamond$ **d.** $\frac{1}{4} = \square$

e. $\frac{2}{3} = \square$ **f.** $\frac{1}{7} = \triangle$ **g.** $\frac{7}{9} = \square$ **h.** $\frac{1}{10} = \Diamond$

Copy and add or subtract as the signs tell you. 59–61, 68

	a	b	c	d	e
5.	$7\frac{5}{8}$ $+ 4\frac{3}{8}$	$9\frac{2}{3}$ $+ 9\frac{3}{5}$	$6\frac{1}{5}$ $+ 2\frac{2}{5}$	$6\frac{2}{5}$ $7\frac{1}{6}$ $+ 4\frac{1}{2}$	$8\frac{3}{10}$ $9\frac{1}{5}$ $+ 3\frac{3}{4}$
6.	$12\frac{5}{16}$ $- 9\frac{1}{16}$	$17\frac{7}{10}$ $- 8\frac{3}{10}$	$22\frac{5}{6}$ $- 18\frac{5}{12}$	$13\frac{1}{3}$ $- 5\frac{1}{3}$	10 $- \frac{7}{10}$

Find the answers for these examples. 68, 88–89

	a	b	c
7.	$1\frac{5}{8} + 2\frac{2}{3}$	$4\frac{1}{2} + 5\frac{1}{4}$	$8\frac{1}{3} - 3\frac{1}{8}$
8.	$6\frac{3}{4} - 4\frac{3}{10}$	$6\frac{3}{4} + 7\frac{4}{5}$	$5\frac{1}{5} - 3\frac{5}{16}$
9.	$\frac{1}{12}$ of 24	$3\frac{1}{3} \times 3$	$1\frac{1}{3} \times 4$
10.	$6\frac{3}{4} \div 9$	$8 \div \frac{2}{3}$	$\frac{2}{3} \div \frac{1}{6}$

Ask yourself questions such as the following as you solve the problems below. What does the problem tell me? Can I write a number sentence that tells about the problem? Are two number sentences needed to answer the question? Is there a number in the problem that I do not need to use?

1. One afternoon Harold delivered 76 papers in $1\frac{1}{2}$ hr. This was $\frac{2}{3}$ of the papers he had to deliver. How many papers did he have to deliver?

2. Jerry took seven friends to see a movie. If the total cost of the tickets was ten dollars, what was the price of one admission?

3. Laura spent 2 hr. 15 min. writing and rewriting a poem for the class paper. Susan spent 1 hr. 45 min. writing a story. Laura spent how much more time than Susan?

4. El Paso, Texas, averages 292 days of sunshine per year and Minneapolis, Minnesota, averages 204 days. How many fewer days of sunshine does Minneapolis average than El Paso?

5. Mrs. Davis bought milk as follows: Monday, 1 gal. 1 qt.; Tuesday, 3 qt.; Wednesday, 2 gal. 3 qt. How much milk did Mrs. Davis buy? If milk is 30 cents a quart, how much did this milk cost?

6. Mr. Green's rectangular lot is 150 feet long and 100 feet wide. How many bags of fertilizer must he buy if one bag will fertilize 5,000 square feet? If fertilizer is \$4.75 a bag, how much will this cost him?

7. How many $\frac{3}{4}$-inch pieces of ribbon can Jane cut from a ribbon that is $6\frac{3}{4}$ inches long?

8. You know that there are 5,280 feet in a mile. How many square feet are there in one square mile?

9. Seven eighths of the 832 pupils in the Fairview School go home for lunch. How many pupils go home for lunch? How many do not go home for lunch?

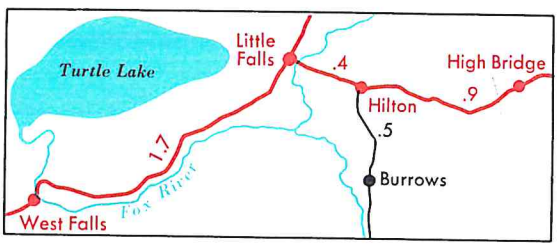
5 Decimals and Common Fractions

1. On a toothbrush, the width of a tuft of bristles may be only .1 inch. How do you read the numeral .1? Does .1 mean the same as $\frac{1}{10}$?

2. At a high school track meet, Tom ran the 100-yard dash in $10\frac{7}{10}$ seconds. Later he read in a record book: "In 1958 Marlene Mathews set a world's record for women by running 100 yards in 10.3 seconds."

Does 10.3 mean $10\frac{3}{10}$? Then who ran the 100 yards faster—Tom or Marlene Mathews?

3. On many road maps there are numerals that tell the distance in miles from one place to another.



Is the distance from Hilton to Little Falls 4 miles, or is it $\frac{4}{10}$ mile? How do you know? Are .4 and $\frac{4}{10}$ names for the same number?

4. Hilton is how far from Highbridge? from Burrows?

▶A numeral such as .4 is a **decimal fraction.** The dot in the numeral is called a **decimal point.** As you have seen, .4 names the same number as $\frac{4}{10}$. The numeral $\frac{4}{10}$ is a **common fraction.**

5. What is the correct replacement for the frame in this sentence: $1.7 = 1\frac{\triangle}{10}$? How do you read $1\frac{7}{10}$? The numeral 1.7 is read in the same way—"one *and* seven tenths." How do you read 2.7? 2.5? 3.8?

You remember that $1\frac{7}{10}$ is a mixed fraction. A numeral such as 1.7 is a **decimal mixed fraction.** For convenience, decimal fractions such as .4 or decimal mixed fractions such as 1.7 are both called **decimals.**

hundreds	tens	ones	tenths	hundredths
		.	0	1
		.	0	8
		.	1	2

1. From work in an earlier grade, you may remember how we read decimal numerals such as those in the chart.

Does the first numeral in the chart mean $\frac{1}{10}$, or $\frac{1}{100}$? What does the numeral .02 mean? .03? .08?

2. Whenever a common fraction has a denominator of 100—for example, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, and so on—we can write it as a decimal. To write $\frac{4}{100}$ as a decimal, would you write .4, or would you write .04?

What does .4 mean?

3.a. Write each of these as a decimal: $\frac{5}{100}$; $\frac{7}{100}$; $\frac{9}{100}$.

b. In each case in **a**, does the decimal have two places at the right of the decimal point? What figure is in tenths' place in each of the decimals?

4. Now look at the third numeral in the place-value chart. Does this mean $\frac{12}{100}$? What does .11 mean? .27? .36?

5. Read these numerals.

a. .43 **b.** .54 **c.** .72 **d.** .60 **e.** .06 **f.** .10

6. Write each of the numerals in Exercise **5** as a common fraction with a denominator of 100.

7. How do you read this mixed fraction: $1\frac{3}{100}$? Do we read the following decimal in the same way: 1.03? How do you read each of the following?

a. 4.07 **b.** 15.15 **c.** 8.01 **d.** 8.1 **e.** 8.21

Did you use the word "and" as you read each decimal?

8. Write the decimal for each of the following.

a. $1\frac{5}{100}$ **b.** $1\frac{5}{10}$ **c.** $1\frac{56}{100}$ **d.** $45\frac{7}{10}$ **e.** $45\frac{73}{100}$

9. You know that a numeral such as 342 can be thought of as ? hundreds + ? tens + ? ones. Now look at the decimal .56. You know that it means 56 hundredths. But it can also be described in another way. What figure is in tenths' place? in hundredths' place? Then .56 can be described as 5 tenths + 6 hundredths.

hundreds	tens	ones	tenths	hundredths
		.	5	6
		.	7	8
		.	9	3

10. We can use common fractions to check the description of .56. Express $\frac{5}{10}$ as an equivalent fraction with a denominator of 100. Then $\frac{5}{10} + \frac{6}{100} = \frac{\square}{100} + \frac{6}{100}$, or ?. Is your answer $\frac{56}{100}$?

11. Now describe the decimal .78 in two ways.

a. .78 means ? hundredths.

b. .78 also means ? tenths + ? hundredths.

c. Check your answer for **b** by using common fractions. First, express $\frac{7}{10}$ as a fraction with a denominator of 100. Then use the sentence at the right to complete the check.

$\frac{7}{10} + \frac{8}{100} = \frac{\Delta}{100} + \frac{8}{100}$, or ?

12. Describe .93 in the two ways shown in Exercise **11**. Check your second description as in Exercise **11c**.

13. Copy and complete these sentences.

a. .42 = ? tenths + 2 hundredths

b. .60 = 6 tenths + ? hundredths

c. .08 = ? tenths + ? hundredths

d. 54.35 = ? tens + ? ones + ? tenths + 5 hundredths

e. 86.09 = ? tens + ? ones + ? tenths + ? hundredths

f. 27.70 = ? tens + ? ones + 7 tenths + ? hundredths

14. Write each as a decimal.

a. $3\frac{1}{10}$ **b.** $\frac{9}{100}$ **c.** $365\frac{35}{100}$ **d.** $8{,}355\frac{7}{10}$ **e.** $207\frac{60}{100}$

1. The thickness of a human hair is about $\frac{3}{1000}$ of an inch. Do you think we can write $\frac{3}{1000}$ as a decimal? Would it be correct to write $\frac{3}{1000}$ as .3? as .03? as .003?

2. This chart shows the names of the places from thousands to thousandths. Look at the first numeral in the chart. In what place is the 1? Then what does .001 mean? What does the numeral .008 mean? .006? Write each of these decimals as a common fraction.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
				. 0	0	1
				. 0	1	4
				. 3	5	8
2 ,	5	1	2	. 2	0	6

3. Look at the second numeral in the chart. Does it mean $\frac{14}{1000}$? What does the numeral .036 mean? .085? .099?

4. Read the third numeral in the chart. Write it as a common fraction with a denominator of 1000.

Read the last numeral in the chart. Write it as a mixed fraction.

5. Suppose you are asked to write $\frac{19}{1000}$ as a decimal. Would you write it as .19, or would you write it as .019? What does .19 mean?

6. Suppose you are asked to write $\frac{70}{1000}$ as a decimal. Would it be correct to write it as .70? If not, why not? Write $\frac{70}{1000}$ as a decimal.

7. Write each of the following as a decimal.

a. $\frac{5}{1000}$ **b.** $3\frac{5}{1000}$ **c.** $\frac{36}{1000}$ **d.** $\frac{853}{1000}$ **e.** $2\frac{23}{1000}$

8. Does each of the numerals you wrote in Exercise **7** have three places at the right of the decimal point?

9. You know that .256 means 256 thousandths. We can also describe it as ? tenths + ? hundredths + ? thousandths.

10. Copy and complete the following sentences.

a. .486 = ? tenths + ? hundredths + ? thousandths
b. .209 = ? tenths + ? hundredths + ? thousandths

1. Let us experiment with a set of numbers.

a. 700 **b.** 70 **c.** 7 **d.** $\frac{7}{10}$ **e.** $\frac{7}{100}$ **f.** $\frac{7}{1000}$

Is each of these sentences true, or is it false?

$700 = 10 \times 70$ $70 = 10 \times 7$ $7 = 10 \times \frac{7}{10}$

Is it also true that $\frac{7}{10} = 10 \times \frac{7}{100}$? Do the multiplication and show your work.

Is this sentence true: $\frac{7}{100} = 10 \times \frac{7}{1000}$? Do the multiplication and show your work.

2. Now study the numeral at the right: 7 7 7.7 7 7

The first "7" means 700 ←
The next "7" means ? ←
The next "7" means ? ←
The next "7" means $\frac{7}{10}$ ←
The next "7" means ? ←
The next "7" means ? ←

You have already found that 700 is ten times 70; 70 is ten times 7; 7 is ten times $\frac{7}{10}$; and so on. Then does each digit 7 have ten times the value of the 7 at its right?

Is it also true that each 7 has one tenth of the value of the 7 at its left?

3. Now try another experiment with a set of numbers.

a. 300 **b.** 30 **c.** 3 **d.** $\frac{3}{10}$ **e.** $\frac{3}{100}$ **f.** $\frac{3}{1000}$

Is each of the following sentences true?

$300 = 10 \times 30$ $30 = 10 \times 3$ $3 = 10 \times \frac{3}{10}$

$\frac{3}{10} = 10 \times \frac{3}{100}$ $\frac{3}{100} = 10 \times \frac{3}{1000}$

4. Show what 333.333 means, using the form given in Exercise **2**.

Does each 3 have ten times the value of the 3 at its right? Then does each 3 have one tenth of the value of the 3 at its *left*? If you are not sure, find what $\frac{1}{10}$ of 300 is; $\frac{1}{10}$ of 30; $\frac{1}{10}$ of 3; $\frac{1}{10}$ of $\frac{3}{10}$; and $\frac{1}{10}$ of $\frac{3}{100}$.

In the numeral 945, the 9 in hundreds' place means 900. Does it also mean 9×100? The 4 is in tens' place, so it means $4 \times ?$. Does the 5 mean 5×1?

1. We can think of places in a decimal in a similar way. We can think of place value for tenths' place as $\frac{1}{10}$; for hundredths' place, $\frac{1}{100}$; and for thousandths' place, $\frac{1}{1000}$.

Does .8 equal $8 \times \frac{1}{10}$? Does .09 equal $9 \times \frac{1}{100}$? What is the correct replacement for the frame: $.002 = 2 \times \frac{1}{\square}$?

2. Copy, and replace the frames.

a. $.6 = 6 \times \frac{1}{\square}$ **b.** $.005 = 5 \times \frac{1}{\triangle}$ **c.** $.03 = 3 \times \frac{1}{\diamond}$

3. You have seen that in a decimal each place has a value one tenth of the value of the place at its left. Let us use common fractions to study this pattern further.

a. The first place at the right of the decimal point is tenths' place. You have written its place value as $\frac{1}{10}$. Now find $\frac{1}{10}$ of $\frac{1}{10}$.

$$\frac{1}{10} \text{ of } \frac{1}{10} \text{ means } \frac{1}{10} \times \frac{1}{10}, \text{ or } \frac{1}{\square}$$

Is $\frac{1}{10} \times \frac{1}{10}$ another name for $\frac{1}{100}$? Is $\frac{1}{100}$ one tenth of $\frac{1}{10}$?

b. Now find $\frac{1}{10}$ of $\frac{1}{100}$.

$$\frac{1}{10} \text{ of } \frac{1}{100} \text{ means } \frac{1}{10} \times \frac{1}{100}, \text{ or } \frac{1}{\triangle}$$

Do you find that $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$? Since "$\frac{1}{10} \times \frac{1}{10}$" is another name for $\frac{1}{100}$, is it true that $\frac{1}{10} \times (\frac{1}{10} \times \frac{1}{10}) = \frac{1}{1000}$?

4. Is $\frac{1}{1000}$ the place value for thousandths' place? What is $\frac{1}{10}$ of $\frac{1}{1000}$? $\frac{1}{10} \times \frac{1}{1000} = \frac{1}{\triangle}$.

5. What is $\frac{1}{10}$ of $\frac{1}{10{,}000}$? Multiply and find out.

▶You have now discovered that $\frac{1}{10{,}000}$ and $\frac{1}{100{,}000}$ are the values of the next two places after the thousandths' place. How do we write $\frac{1}{10}$ as a decimal? $\frac{1}{100}$? $\frac{1}{1000}$? Then how do you think we write $\frac{1}{10{,}000}$ as a decimal? $\frac{1}{100{,}000}$?

6. This chart shows the place values of decimals to hundred thousandths.

Look at the first numeral in the chart. The 3 is in ten-thousandths' place. How do you read .0003? Write .0003 as a common fraction.

	tenths	hundredths	thousandths	ten thousandths	hundred thousandths
.0	0	0	3		
.0	0	0	0	5	
.0	0	9			
.0	0	9	1		
.0	0	9	1	5	

7. Read this decimal: .0007. Write it as a common fraction.

8. Now look at the second numeral in the chart. In what place is the 5? How do you read .00005? Write .00005 as a common fraction.

9.a. Read each of these decimals: .08; .97. In each numeral, in which place is the last digit at the right? Did you read each numeral as *hundredths*?

b. Read these numerals: .004; .043; .465. In which place is the last digit in each of the numerals? Did you read each numeral as *thousandths*?

c. Read each of these numerals: .0006; .0064; .0672. In which place is the last digit in each of the numerals? Did you read each numeral as *ten thousandths*? Does the name for the place value of the last digit help you read the decimal?

d. Read the third, fourth, and fifth numerals in the chart. Write each as a common fraction.

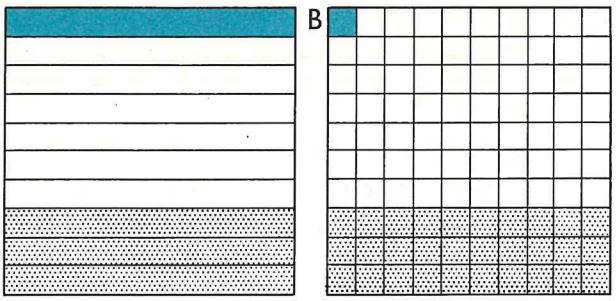
10. Write each of these common fractions as a decimal.

a. $\frac{6}{100}$ **b.** $\frac{99}{100}$ **c.** $\frac{253}{1000}$ **d.** $\frac{66}{1000}$ **e.** $\frac{11}{10{,}000}$ **f.** $\frac{8526}{10{,}000}$

Look at the numerals for the denominators of the fractions in **a–f**. How many zeros are there in 100? in 1000? in 10,000? How many places are there in each corresponding decimal: .06, .99, and so on? In each case, does the number of zeros equal the number of places that are at the right of the decimal point in the decimal? Is the same relationship true for $\frac{9}{10}$ and .9? $\frac{3}{10}$ and .3? $\frac{418}{100{,}000}$ and .00418?

1. Diagram *A* is divided into tenths. What fractional part of it is blue? Write the decimal fraction.

In *B*, each tenth is divided into ten equal parts. What fractional part of *B* is blue? Do the diagrams show that $\frac{1}{100}$ is one tenth of $\frac{1}{10}$?



2. What part of *A* is gray? Write the decimal fraction. Are the gray parts of *A* and *B* equal in size? How many hundredths of *B* are gray? Then 3 tenths = ? hundredths.

3. Suppose each hundredth in *B* were divided into ten equal parts. How many equal parts would there be? Then would each of these parts represent .001 of the diagram? How many thousandths would be gray?

Is this sentence true: .3 = .30 = .300?

4. Suppose .9 of Diagram *A* were shaded. How many hundredths would be shaded? How many thousandths? Copy this sentence and replace the question marks: .9 = .?? = .???.

Use common fractions to check: $\frac{9}{10} = \frac{\triangle}{100}$; $\frac{9}{10} = \frac{\square}{1000}$.

5. Are $\frac{10}{10}$ and $1\frac{0}{10}$ two names for the number 1? What number is named by the decimal 1.0?

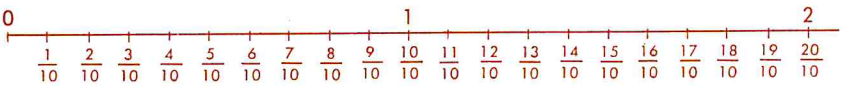
What number is named by $\frac{20}{10}$? $\frac{50}{10}$? $\frac{90}{10}$? $\frac{100}{10}$? Express each number by a decimal that has a tenths' place.

6. Are $\frac{100}{100}$ and $1\frac{0}{100}$ two names for the number 1? What number is named by the decimal 1.00?

What number is named by $\frac{200}{100}$? $\frac{800}{100}$? $\frac{1200}{100}$? Express each number by a decimal with two places at the right of the decimal point.

7. Express each of the following as a decimal. If you have difficulty, first express each as a mixed fraction.

a. $\frac{21}{10}$ **b.** $\frac{201}{100}$ **c.** $\frac{718}{100}$ **d.** $\frac{87}{10}$ **e.** $\frac{105}{10}$ **f.** $\frac{216}{10}$ **g.** $\frac{1234}{100}$

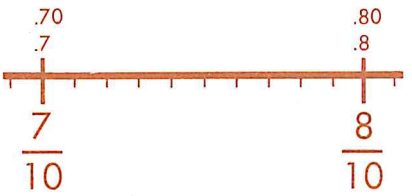


1. This drawing shows a number line with the space from 0 to 1 and from 1 to 2 divided into tenths. As you can see, common fractions have been used to name the tenths. Can we also use decimals to name the tenths? What decimal should you write under $\frac{9}{10}$? $\frac{10}{10}$? $\frac{13}{10}$? $\frac{15}{10}$? $\frac{19}{10}$?

2. Make a drawing of a number line with the space from 0 to 1 and from 1 to 2 divided into tenths. Label the points with decimals, each of which has a tenths' place.

3. Imagine that you have divided each of the tenths into ten equal spaces. How many of the smaller spaces would there be from 0 to 1? What fractional part of a unit space would each of the smaller spaces be?

Look at the number line you drew for Exercise **2**. What is another decimal that you can write at the point labeled .1? at the point labeled .2? Where will you write .30? .70? .80? 1.90? Write these decimals below the decimals that are other names for the same numbers.



4. At the left is a magnified drawing of part of the number line with the space from .7 to .8 divided into ten equal parts. Is each of the smaller spaces one hundredth of the space from 0 to 1? Then is .71 the correct numeral to write at the first division after .7 or .70? What decimal would you write at the next division? the next?

Find the point that is halfway between .70 and .80. What numeral would you write at this point?

5. Imagine that this drawing is of the space from .2 to .3. What numeral would be at the point halfway between .2 and .3?

1. The decimal at the right is read "eight thousand, five hundred twenty-seven and three hundred forty-six thousandths." 8,527.346

a. What word do we say when we come to the decimal point?
b. The 8 in 8,527.346 means $8 \times 1{,}000$. Tell what each of the other digits means.

2. Express each of the following as a mixed fraction and as a decimal.

a. sixteen and seven tenths
b. three hundred twenty and nineteen thousandths
c. two thousand four and seven hundredths
d. eight thousand sixty and two hundred ten-thousandths

3. Copy, and write three more decimal numerals that fit the pattern of the first three.

a. .67, .68, .69, ?, ?, ?
b. 1.5, 1.7, 1.9, ?, ?, ?
c. .08, .10, .12, ?, ?, ?
d. .998, .999, 1.000, ?, ?, ?

4. For each of the following, tell whether the number is less than 1, equal to 1, or greater than 1. Use the symbols "<," "=," or ">" in your answers. For example: $.109 < 1$.

a. 1.000	**b.** 2.111	**c.** .999	**d.** .111
e. .001	**f.** 1	**g.** 1.111	**h.** 1.001

Copy, and replace the frames.

	a	b	c	d
5.	$.7 = \frac{\triangle}{10}$	$1.1 = \frac{\square}{10}$	$1.9 = \frac{\diamond}{10}$	$2.4 = \frac{\square}{10}$
6.	$.98 = \frac{\square}{100}$	$1.05 = \frac{\triangle}{100}$	$1.92 = \frac{\diamond}{100}$	$2.36 = \frac{\square}{100}$
7.	$.835 = \frac{\triangle}{1000}$	$1.001 = \frac{\square}{1000}$	$2.001 = \frac{\diamond}{1000}$	$3.200 = \frac{\square}{1000}$

Express each as a decimal.

8.	$\frac{9}{10}$	$\frac{13}{10}$	$\frac{17}{10}$	$\frac{20}{10}$	$\frac{26}{10}$	$\frac{36}{10}$	$\frac{50}{10}$	$\frac{61}{10}$
9.	$\frac{7}{100}$	$\frac{29}{100}$	$\frac{129}{100}$	$\frac{229}{100}$	$\frac{502}{100}$	$\frac{723}{100}$	$\frac{999}{100}$	$\frac{1001}{100}$
10.	$\frac{3}{1000}$	$\frac{10}{1000}$	$\frac{87}{1000}$	$\frac{101}{1000}$	$\frac{956}{1000}$	$\frac{1000}{1000}$	$\frac{1002}{1000}$	$\frac{2005}{1000}$

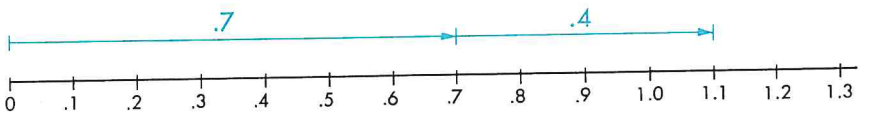
1. Jim's house is .5 mile from the library. The post office is .3 mile farther along the same road. Jim walked from his house to the library and then to the post office.

If you are asked how far Jim walked, do you add to find the answer? Do these number sentences tell about the problem?

$$.5 + .3 = \square \qquad \frac{5}{10} + \frac{3}{10} = \square$$

How far did Jim walk?

2. The blue arrows above this number line show a move of .7 followed by a move of .4. What numeral is at the end of the second move? Then what is the correct decimal to replace the frame in this sentence: $.7 + .4 = \triangle$?



Check your answer by renaming .7 and .4 as common fractions, and adding.

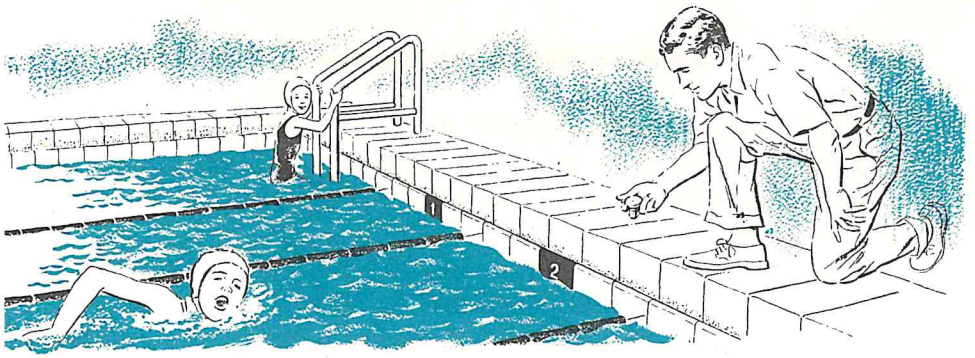
3. Since any decimal can be expressed as a common fraction, can you use common fractions to find the sum for any addition example involving decimals? Try it with these examples. Express your answers in decimal form.

a. $.1 + .8 = \square$ **b.** $.4 + .3 = \triangle$ **c.** $.13 + .12 = \square$
d. $.03 + .16 = \diamond$ **e.** $.81 + .02 = \square$ **f.** $.05 + .04 = \triangle$

Look at your work for Exercise **d**. Do the common fractions for .03 and .16 have the same denominator—that is, are they *like fractions*? The decimals .03 and .16 are called **like decimals** because each has the same number of places at the right of the decimal point. Did each of the examples in Exercise **3** involve like decimals?

4. Can you find the missing decimals in these examples without writing any common fractions? Try it and see.

a. $.08 + .04 = \square$ **b.** $.32 + .19 = \triangle$ **c.** $.28 + .71 = \diamond$
d. $.28 + .72 = \square$ **e.** $.28 + .73 = \triangle$ **f.** $1.04 + .19 = \diamond$



1. Mary swam from one end of the pool to the other in 7.8 seconds. Then she swam back to her starting place in 9.5 seconds. How long did it take her to swim both ways?

a. Using common fractions, you can find the answer. Copy the example and do the work.

$$\begin{array}{r} 7\frac{8}{10} \\ 9\frac{5}{10} \\ \hline \end{array}$$

b. Since decimals are simply another way of expressing numbers, let us try to do the addition example with decimals, working column by column.

$$\begin{array}{r} 7.8 \\ 9.5 \\ \hline \end{array}$$

What is the sum of 8 tenths and 5 tenths? Is it .13, or is it 1.3? In the step shown here, we have written a .3 directly below the tenths' column. Now, just as you did in **a**, add the 1 with the 7 and the 9. Do you get the same answer as in **a**?

$$\begin{array}{r} 7.8 \\ 9.5 \\ \hline ??.3 \end{array}$$

2. Here are three addition examples, each given in two forms. Can you work the example involving decimals without first working the one with common fractions? Try it and see.

a.
$$\begin{array}{r} 3\frac{3}{10} \\ 2\frac{8}{10} \\ \hline \end{array} \qquad \begin{array}{r} 3.3 \\ 2.8 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 5\frac{1}{10} \\ 3\frac{6}{10} \\ \hline \end{array} \qquad \begin{array}{r} 5.1 \\ 3.6 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 4\frac{9}{10} \\ 2\frac{8}{10} \\ \hline \end{array} \qquad \begin{array}{r} 4.9 \\ 2.8 \\ \hline \end{array}$$

3. Can 1.8 be thought of as 18 tenths? Can 2.1 be thought of as 21 tenths? In the same way, 3.2 can be thought of as ? tenths. Copy the example, and add as if you were adding 18, 21, and 32.

$$\begin{array}{r} 1.8 \\ 2.1 \\ 3.2 \\ \hline \end{array}$$

Does the 71 you obtained mean 71 tenths? What is 71 tenths expressed as a decimal mixed fraction? Be sure you place the decimal point correctly in your answer.

4. Can we think of 1.03 as 103 hundredths? In the same way, we can think of 1.99 as ? hundredths. Copy the example, and add as if you were adding 103 and 199.

$$\begin{array}{r} 1.03 \\ +1.99 \\ \hline \end{array}$$

Does the 302 you obtained mean 302 hundredths? Then where will you put the decimal point? Do it. What is the sum?

5. At the right is a subtraction example with decimals. Do you think you can subtract just as you did with whole numbers? Try it and see.

$$\begin{array}{r} .87 \\ -.69 \\ \hline \end{array}$$

What is the remainder? Did you remember to use a decimal point in your written answer?

Check, using common fractions: $\frac{87}{100} - \frac{69}{100} = \frac{\triangle}{100}$, or . ??.

6. Copy this example, and subtract as if you were subtracting 141 from 432. Then put the decimal point in the correct place.

$$\begin{array}{r} 4.32 \\ -1.41 \\ \hline \end{array}$$

Check, using improper fractions: $\frac{432}{100} - \frac{141}{100} = \frac{\triangle}{100}$, or ?. ??.

▶Can we add and subtract with decimals just as we add and subtract with whole numbers?

Copy and add or subtract as the signs tell you. To avoid mistakes, write the decimal points in a straight column.

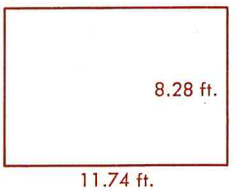
	a	b	c	d	e
7.	1.6 +1.8	3.5 −1.7	9.75 −3.23	8.63 −4.81	11.25 +6.76
8.	5.35 −2.76	12.96 +11.07	10.03 −8.23	156.3 54.8 +8.9	.88 5.10 +18.56
9.	86.32 −8.25	467.58 −56.19	36.61 −15.72	36.005 +18.756	57.346 +33.579
10.	46.563 −24.275	125.145 −33.268	526.663 +193.478	24.402 −5.285	72.427 −43.634

1. A gallon of milk weighs about 8.60 pounds. A gallon of water weighs about 8.33 pounds. About how much more does a gallon of milk weigh than a gallon of water?

2. A gallon of kerosene weighs about 6.68 pounds. A gallon of vinegar weighs about 9.01 pounds. About how much less does a gallon of kerosene weigh than a gallon of vinegar?

3. About how much more does a gallon of vinegar weigh than a gallon of water?

4. On a mechanic's ruler, each foot is divided into tenths and hundredths. Harold used a mechanic's ruler to measure the length and width of his room as shown on a floor plan. The width was 8.28 feet; the length was 11.74 feet.



a. What is the perimeter of the room?
b. The length of the room is how much greater than the width?

5. The amount of rainfall in a month is often expressed to hundredths of an inch. Newville had 3.24 inches of rainfall in January, 3.14 inches in February, and 3.36 inches in March. Newville had a total of how many inches of rainfall in these three months?

6. The money system used in the United States is related to the decimal numeral system. A cent, for example, is $\frac{1}{100}$ of a dollar, or \$.01. A dime is $\frac{1}{10}$ of a dollar, or ? hundredths of a dollar; that is, \$.10.

Copy and complete this sentence: Thirty-five cents, or \$.35, can be thought of as ? tenths of a dollar + ? hundredths of a dollar. It can also be thought of as ? hundredths of a dollar.

7. During a two-day trip, Mr. Jessup spent the following amounts for meals: \$1.15, \$2.65, \$3.75, \$.85, \$2.35, and \$3.50. What was the total amount he spent for meals?

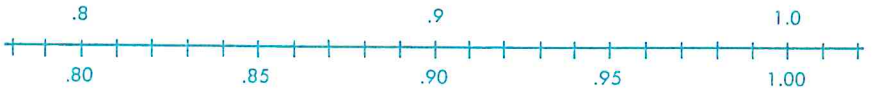
1. Paul and Helen saw a magazine article about a certain metal part that was .28 inch thick. "That's just about .2 inch," Helen said.

"No, it's nearly .3 inch," Paul said.

Which one of them gave a better description of the thickness? If you need help in answering, think of .2 as .20 and .3 as .30. Is 28 hundredths closer to 20 hundredths or to 30 hundredths? Then is .28 closer to .2 or to .3?

▶ In your work with whole numbers, you have rounded numbers. We can also round numbers named by decimals. For example: .28, rounded to the nearest tenth, is .3.

2. This drawing shows the part of a number line from .8 to 1.0 marked off in hundredths. Not all of the points have been labeled.



a. Find the point that you would label .83. Is it closer to .80 or to .90? Then .83, rounded to the nearest tenth, is .8. In rounding to tenths, digits at the right of tenths' place are dropped.

b. Find the point that you would label .87. Is it closer to .80 or .90? Then what is .87 rounded to the nearest tenth?

c. Round to the nearest tenth: .88; .82.

3. Is .97 closer to .90 or to 1.00? Then, rounded to the nearest tenth, .97 is 1.0.

What is .93 rounded to the nearest tenth?

4. Suppose you are asked to round .95 to the nearest tenth. Is .95 halfway between .9 and 1.0?

By agreement, in such cases we "round upward." So .95 would be rounded to 1.0.

5. Round each of the following to the nearest tenth.

a. .81 **b.** .86 **c.** .85 **d.** .78 **e.** .92 **f.** .99

6. Think back over the work you have done in rounding to the nearest tenth. When 1, 2, 3, or 4 is in hundredths' place, do you round downward? When 6, 7, 8, or 9 is in hundredths' place, do you round upward? What do you do when 5 is in hundredths' place?

▶Is rounding with decimals similar to rounding with numerals for whole numbers?

7. Suppose you are asked to round .369 to the nearest hundredth. In rounding to the nearest hundredth, you will drop the digit at the right of hundredths' place.

Look at the digit in thousandths' place. Since it is 9, will you round upward, or will you round downward? Then what is .369 rounded to the nearest hundredth? Is it .36, or is it .37?

To check your answer, think of .36 as .360 and .37 as .370. Is 369 thousandths closer to 360 thousandths or to 370 thousandths? Then were you correct in your rounding?

8. Round to the nearest hundredth. First look at the digit in thousandths' place; then round. Check in the same way that you checked Exercise **7**.

a. .218 **b.** .321 **c.** .476 **d.** .635 **e.** 2.148 **f.** 4.499

9. Round to the nearest thousandth.

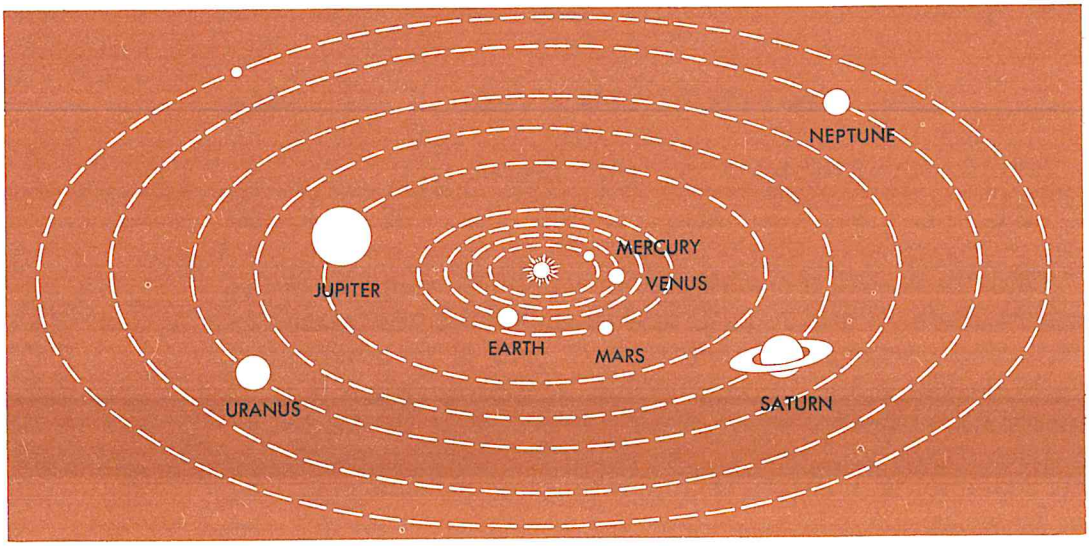
a. .5127 **b.** .6368 **c.** .8414 **d.** .4615 **e.** .9111 **f.** 1.2188

10. Is 7.1 closer to 7 or to 8? Then 7.1, rounded to the nearest whole number, is 7. What is 7.3 rounded to the nearest whole number? 7.8? 7.9? 7.5?

11. Round to the nearest whole number.

a. 6.3 **b.** 11.5 **c.** 12.2 **d.** 41.4 **e.** 82.7

f. 114.6 **g.** 349.5 **h.** 9.8 **i.** 10.6 **j.** 109.8



1. Look at the table below. At what speed does the planet Mercury travel in its orbit? Suppose you are asked to round 29.73 to the nearest whole number. Can you find the answer just by looking at the digit in tenths' place? Explain.

What is 29.73 rounded to the nearest whole number?

2.a. Jupiter travels in its orbit at 8.11 miles per second. By rounding 8.11 to the nearest whole number, can you quickly tell the approximate difference between Mercury's speed and Jupiter's speed? What is the approximate difference?

b. Now find the actual difference. Does this sentence tell about the problem: $29.73 - 8.11 = \square$? What is the answer?

3. Use rounded whole numbers in the following work.

PLANET	ORBITAL SPEED *Miles per second*
Mercury	29.73
Venus	21.75
Earth	18.50
Mars	14.98
Jupiter	8.11
Saturn	5.99
Uranus	4.22
Neptune	3.40
Pluto	3.00

a. The speed of Mercury is about how many times the speed of Mars?

b. The speed of Mars is about how many times the speed of Pluto?

c. Is the speed of Earth more than 3 times the speed of Saturn?

4.a. Tell the approximate difference in the speeds of Jupiter and Uranus; of Venus and Saturn.

b. Find the actual differences in the speeds.

Express as mixed fractions or proper fractions.

	a	b	c	d	e	f
1.	2.5	18.2	.6	4.01	.29	8.75

Express each of the following in decimal form.

2. $3\frac{7}{10}$ $\frac{325}{1000}$ $25\frac{25}{1000}$ $14\frac{3}{10}$ $\frac{37}{100}$ $\frac{5}{10,000}$

3. $9\frac{9}{10}$ $\frac{1}{10,000}$ $\frac{7}{100}$ $\frac{70}{100}$ $386\frac{3}{1000}$ $20\frac{20}{10,000}$

4. Arrange the numbers in each set in order of size, beginning with the largest.

a. 1.11; 11.1; 111; .111 **b.** .035; 3.05; 3.50; .350

5. Name a fraction that shows something has been divided into 6 equal parts and you are talking about 5 of the parts.

6. Which represents the largest number: $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{8}$ or $\frac{1}{5}$? Which represents the smallest number?

7. Express as mixed fractions or as whole numbers.

$\frac{12}{3}$ $\frac{7}{4}$ $\frac{16}{8}$ $\frac{14}{6}$ $\frac{72}{24}$ $\frac{74}{15}$

8. Copy the numerals that are other names for $3\frac{1}{2}$.

a. 3.50 **b.** $\frac{7}{2}$ **c.** $3\frac{5}{100}$ **d.** 3.05 **e.** $3\frac{5}{10}$ **f.** $3\frac{50}{100}$

9. Copy, and replace N with the single numeral that makes the sentence true.

a. $(5 \times 1{,}000) + (3 \times 100) + 4 + (3 \times \frac{1}{10}) = N$

b. $(9 \times 10{,}000) + (4 \times 1{,}000) + (5 \times \frac{1}{100}) = N$

c. $8 + (7 \times \frac{1}{10}) + (6 \times \frac{1}{100}) + (9 \times \frac{1}{1000}) = N$

10. Copy, and replace the frames with the correct numerals.

a. $.768 = (\triangle \times \frac{1}{10}) + (6 \times \frac{1}{100}) + (\square \times \frac{1}{1000})$

b. $.095 = (9 \times \frac{1}{\square}) + (5 \times \frac{1}{\triangle})$

c. $.3204 = (3 \times \frac{1}{\diamond}) + (2 \times \frac{1}{\square}) + (4 \times \frac{1}{\triangle})$

11. What does 10^2 equal? 3^2? 5^4? 3^5?

12. Tell which sentences are true and which are not true.

$2^5 < 6^2$; $3^2 < 2^3$; $8^2 = 4^3$; $10^2 > 5^3$; $10^3 < 5^5$; $9^2 = 3^4$

Copy and add or subtract as the signs tell you.

	a	b	c	d	e
1.	$\begin{array}{r} \frac{3}{5} \\ \frac{1}{2} \\ +\frac{1}{10} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{8} \\ \frac{2}{3} \\ +\frac{1}{8} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{16} \\ \frac{3}{8} \\ +\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{1}{3} \\ -3\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 6\frac{1}{6} \\ -5\frac{4}{5} \\ \hline \end{array}$
2.	$\begin{array}{r} 4\frac{1}{4} \\ 7\frac{3}{8} \\ +3\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{1}{3} \\ -\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 1\frac{3}{5} \\ -1\frac{3}{10} \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 1\frac{1}{6} \\ +\frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} 44 \\ -17\frac{3}{16} \\ \hline \end{array}$

Using what you know about addition and subtraction, find the missing numerals.

	a	b	c
3.	$10\frac{1}{6} - 7\frac{1}{2} = \square$	$4\frac{5}{8} + 2\frac{1}{3} = \triangle$	$\diamond - 3\frac{1}{5} = 2\frac{3}{4}$
4.	$8\frac{2}{5} - \mathrm{N} = 3\frac{2}{3}$	$\square - 3\frac{5}{9} = 4\frac{1}{3}$	$1\frac{5}{6} + \frac{3}{5} = \triangle$
5.	$2\frac{3}{7} + \triangle = 2\frac{3}{7}$	$\diamond + \frac{3}{5} = 2\frac{2}{5}$	$(\triangle - \frac{5}{9}) + \frac{5}{9} = \frac{5}{9}$

6. What is the reciprocal of $\frac{1}{2}$? the reciprocal of 7? the reciprocal of $\frac{2}{3}$? the reciprocal of $\frac{3}{5}$? the reciprocal of $1\frac{1}{3}$?

Copy, and find the missing numerals.

7.	$\frac{1}{8}$ of $72 = \square$	$\frac{5}{12}$ of $72 = \triangle$	$\frac{7}{6} \times 72 = \diamond$
8.	$\frac{1}{5} \times (\triangle \times 7) = 7$	$\frac{3}{4} \times (\square \times 12) = 12$	$\frac{5}{2} \times (\frac{2}{5} \times \triangle) = 40$
9.	$\frac{1}{3} \times \mathrm{N} = 2$	$\frac{5}{6} \times \mathrm{N} = 25$	$\frac{9}{8} \times \mathrm{N} = 27$

10. What fractional part of 30 is 18? Give your answer in simplest form.

Copy and multiply.

11.	$\frac{5}{12} \times \frac{3}{5} \times \frac{2}{3}$	$6\frac{1}{2} \times 4$	$\frac{7}{2} \times \frac{2}{9}$
12.	$2 \times 4\frac{1}{3}$	$4\frac{1}{5} \times 4\frac{1}{6}$	$3\frac{1}{4} \times 3\frac{1}{3}$
13.	$2\frac{3}{16} \times 4$	$\frac{1}{2} \times \frac{3}{10}$	$1\frac{1}{8} \times 6$

Copy and divide.

14.	$9 \div \frac{5}{6}$	$\frac{5}{12} \div \frac{1}{4}$	$6 \div 2\frac{1}{2}$
15.	$4 \div 1\frac{1}{2}$	$7\frac{1}{2} \div 2\frac{1}{2}$	$\frac{3}{16} \div 4$
16.	$3\frac{1}{3} \div \frac{3}{5}$	$8\frac{1}{2} \div \frac{3}{8}$	$\frac{7}{9} \div 1\frac{7}{9}$

1. The perimeter of a square is 288 inches. What is the length of a side in inches? in feet? in yards?

2. What is the area of a tennis court that is 36 feet wide and 78 feet long?

3. Fourteen ounces is what fractional part of a pound?

4.a. Suppose the weight of an object is stated as "1 pound 3 ounces." This indicates the measurement is precise to the nearest ?.
b. Which of the following is the greatest possible error for the measurement—1 ounce, $\frac{1}{2}$ ounce, or $\frac{1}{4}$ ounce?
c. The true weight of the object must be between 1 pound ? ounces and 1 pound ? ounces.

Copy and complete each of the following.

a	b
5. 2,800 lb. = ? T. ? lb.	52 hr. = ? da. ? hr.
6. 5,283 ft. = ? mi. ? ft.	4 lb. 6 oz. = ? oz.
7. 8 min. 70 sec. = ? min. ? sec.	4 mi. = ? ft.
8. 23 hr. 6 min. = 22 hr. ? min.	4 ft. 3 in. = ? in.

9. Copy and add.

a.
3 ft. 6 in.
2 ft. 7 in.

b.
4 da. 23 hr.
4 da. 21 hr.

c.
11 lb. 13 oz.
8 lb. 7 oz.

10. Copy and subtract.

a.
4 T. 600 lb.
1 T. 800 lb.

b.
10 yd.
3 yd. 2 ft.

c.
16 gal.
$14\frac{1}{4}$ gal.

11. Copy and multiply.

a. 4×4 hr. 45 min.

b. $7 \times 3\frac{1}{2}$ gal.

12.a. What fractional part of a foot is 9 inches? Then 8 feet 9 inches can be expressed as $8\frac{3}{4}$ feet. Use this fact to help find the answer to this example: 8 feet 9 inches $\div$ 5.
b. What does 1 mi. 1,760 ft. $\div$ 8 equal?

The table below gives recent data concerning the areas of the six largest and the six smallest states in the United States.

I. SIX LARGEST STATES		II. SIX SMALLEST STATES	
State	Area (sq. mi.)	State	Area (sq. mi.)
Alaska	586,400	Connecticut	5,009
Arizona	113,909	Delaware	2,057
California	158,693	Hawaii	6,424
Montana	147,138	Massachusetts	8,257
New Mexico	121,666	New Jersey	7,836
Texas	267,339	Rhode Island	1,214

1. In each section of the table, the names of the states are arranged alphabetically. Copy Section *I* of the table, listing the states according to area, with the largest state first.

2. Copy Section *II* of the table, listing the six states in order of increasing size. That is, the smallest state should head this column.

3. What is the difference in area between the largest of the large states and the smallest of the small states?

4. Is the sum of the areas of the second and third largest states greater, or less, than the area of the largest state? How much greater or less?

5. What is the difference in area between the largest of the small states and the smallest of the small states?

6. Take the largest states in order of size, beginning with the second largest, and see how many must be taken before the sum of the areas will be greater than the area of the largest state.

7. Repeat Exercise **6**, but this time work with the six smallest states. See how many must be taken so that the sum of their areas will be greater than the area of the largest of the small states.

If you make any mistakes, study the pages given at the right of the exercises.

EXTRA PRACTICE 297–298

1. Express each of the following as a decimal. 148–154

a. $\frac{38}{100}$ **b.** $3\frac{95}{1000}$ **c.** $\frac{5}{10{,}000}$ **d.** $4\frac{705}{1000}$ **e.** $5\frac{693}{10{,}000}$

2. Express as mixed or common fractions in simplest form.

a. 7.7 **b.** .09 **c.** .061 **d.** .06099 148–154

3. Copy and complete. 150–151

a. .73 = ? tenths + ? hundredths

b. .085 = ? tenths + ? hundredths + ? thousandths

4. Give the products in decimal form. 153

a. $7 \times \frac{1}{10}$ **b.** $3 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$ **c.** $9 \times \frac{1}{10} \times \frac{1}{10}$

5. Copy, and replace each frame. 148, 153, 155

a. $1.05 = \frac{\triangle}{100}$ **b.** $2.17 = \frac{\square}{100}$ **c.** $1.845 = \frac{1845}{\diamond}$

6. Copy, and replace each frame with a decimal. 154

a. $\frac{305}{100} = \square$ **b.** $\frac{1010}{1000} = \triangle$ **c.** $\frac{2125}{1000} = \diamond$

7. Find the sum or difference. 158–160

a. $2.3 + 6.5 = \square$ **b.** $.225 + .220 = \triangle$ **c.** $.999 - .334 = \triangle$

Copy and add or subtract. 158–160

	a	b	c	d
8.	4.37 +2.15	8.6 −3.9	33.94 −5.05	6.759 −3.526
9.	.65132 .21004 .18521	126.34 52.12 9.31 8.02	4.536 3.340 2.822	.25 9.75 13.09 .66

10. Round each of the following first to the nearest hundredth, then to the nearest tenth. 162–163

a. .629 **b.** .175 **c.** .064 **d.** .987

DISCOVERING MORE ABOUT NUMBERS: PAGES 326–328

6 Multiplication with Decimals

1. On the Saturday before the school play, Helen's father took her to and from rehearsals. The distance between their house and the high school where the rehearsals were held was 2.8 miles. He drove that distance 6 times during the day. How many miles did he travel that day?

a. Does this number sentence tell about the problem: $6 \times 2.8 = \square$?

b. Using what you know about rounding numbers, you can estimate the product. What is 2.8 rounded to the nearest whole number? What is 6×3? Is 18 an estimate of the product?

c. Copy this example, but leave out the part in color. Multiply as if you were multiplying 28 by 6. Do you get "168"? Is the answer 168? Why not? Is it 1.68? Is it 16.8? Put the decimal point where you think it belongs.

$$\begin{array}{r} 2.8 \\ \times\ 6 \\ \hline 168 \end{array}$$

d. Check the answer, using common fractions.

$$6 \times 2\frac{8}{10} = 6 \times \frac{28}{10} = \frac{6 \times 28}{10}, \text{ or } \frac{\triangle}{10}$$

Are $\frac{168}{10}$ and 16.8 names for the same number?

Estimate the answer for each example below. Then choose the correct product from those that are given.

2. $2 \times 48.8 = \square$	(976	9.76	97.6	.976)
3. $9 \times .7 = \triangle$	(6.3	.63	63	.063)
4. $7 \times 21.2 = \diamond$	(1.484	14.84	148.4	1484)
5. $8 \times .94 = \mathrm{N}$	(752	7.52	75.2	.752)

6. You know that a gallon of water weighs about 8.33 pounds. Then how much do 5 gallons of water weigh?

a. Begin by estimating the product. What is 8.33 rounded to the nearest whole number? What is 5×8? Then what is an estimate of the product?

b. Now copy this example, and multiply as if you were multiplying 833 by 5. Then put the decimal point where you think it belongs. Which of the following is the answer to the problem?

$$\begin{array}{r} 8.33 \\ \times\ \ 5 \\ \hline \end{array}$$

4,165 lb. 416.5 lb. 41.65 lb. 4.165 lb.

7. Which of the following is a correct replacement for the frame in the sentence at the right?

$2 \times .42 = \square$

a. 84 **b.** 8.4 **c.** .84

Check by using common fractions: $2 \times \frac{42}{100} = \frac{\triangle}{100}$.

8. Does the order in which we multiply change the product? Then does $.42 \times 2$ name the same number as $2 \times .42$?

9. Using any method you think is helpful, find the products. Express the products in decimal form.

a. $7 \times .5 = \triangle$ **b.** $.31 \times 4 = \square$
c. $3 \times .25 = \diamond$ **d.** $.1 \times 12 = \mathrm{N}$

10. Study Examples **9a–d** as listed above. In each example, is one of the numbers represented by a decimal and the other by a numeral for a whole number? Now study the numerals that represent the products you obtained. In each case, does the numeral for the product have the same number of places at the right of the point that the decimal in the example has?

11. Copy and multiply.

a	b	c	d	e	f
8	3.46	2.25	14	8.75	5.08
.6	7	9	1.01	3	5
4.8	24.22	20.25	14.14	26.25	25.40

1.a. Try estimating this product. Round 1.9 to the nearest whole number. Round 7.3. What is an estimate of the product?

$1.9 \times 7.3 = \square$

b. Copy the example, and multiply as if you were multiplying 73 by 19. Then put the decimal point where you think it belongs.

$$\begin{array}{r} 7.3 \\ \underline{1.9} \end{array}$$

c. Check your answer, using common fractions.

$$1\frac{9}{10} \times 7\frac{3}{10} = \frac{19}{10} \times \frac{73}{10} = \frac{\triangle}{100}$$

What is $\frac{1387}{100}$ expressed as a decimal?

2.a. Round .8 to the nearest whole number. Round 2.4. What is 1×2? What is an estimate of the product?

$.8 \times 2.4 = \triangle$

b. Copy the example, and multiply as if you were multiplying 24 by 8. Then put the decimal point where it belongs.

$$\begin{array}{r} 2.4 \\ \underline{.8} \end{array}$$

Check: $\frac{8}{10} \times 2\frac{4}{10} = \frac{8}{10} \times \frac{24}{10} = \frac{\triangle}{100}$

c. What is $\frac{192}{100}$ expressed as a decimal?

3. Look back at Exercise **1**. In **b**, you multiplied 73 by 19. In **c**, did you also multiply 73 by 19 when you found the product of the numerators of the common fractions?

In Exercise **2b**, did you multiply 24 by 8? In **c**, did you also multiply 24 by 8 when you found the product of the numerators of the common fractions?

4. Now examine the decimal places (that is, places at the right of the point) of the numerals in the examples.

a. How many decimal places are there in 7.3? in 1.9? How many decimal places are there in 13.87, the numeral for the product?

b. How many decimal places in .8? in 2.4? in 1.92?

5.a. Look at this example. Without multiplying, can you predict how many decimal places the numeral for the product has?

$.7 \times 3.1 = \triangle$

b. Now multiply. Express the product in decimal form.

6. What is the correct replacement for the frame in Sentence **a**?

a. $\frac{4}{10} \times 3\frac{21}{100} = \frac{4}{10} \times \frac{321}{100} = \frac{1284}{\square}$ **b.** $\begin{array}{r} 3.21 \\ \underline{\times\ .4} \end{array}$

Is the example in **b** another way of writing $\frac{4}{10} \times 3\frac{21}{100}$? In **a**, you saw that the product of 4 *tenths* and 321 *hundredths* was 1,284 *thousandths*. Now look at the decimal numerals. How many decimal places are there for tenths? for hundredths? How many decimal places do you think there are in the numeral for the product?

Copy the example in **b**, multiply, and put the decimal point where it belongs.

7. What is the correct replacement for the frame in Sentence **a**?

a. $\frac{99}{100} \times 1\frac{7}{10} = \frac{99}{100} \times \frac{17}{10} = \frac{1683}{\triangle}$ **b.** $\begin{array}{r} 1.7 \\ \underline{\times\ .99} \end{array}$

Is the example in **b** another way of writing $\frac{99}{100} \times 1\frac{7}{10}$? In **a**, did you find that the product of 99 *hundredths* and 17 *tenths* is 1,683 *thousandths*? Now look at the decimal numerals. How many decimal places are there in .99? in 1.7? How many decimal places do you think there are in the numeral for the product? Find the product and put the decimal point where it belongs.

8. Without multiplying, tell how many decimal places there are in the numeral for each product. Review Exercises **1–7** if you need help.

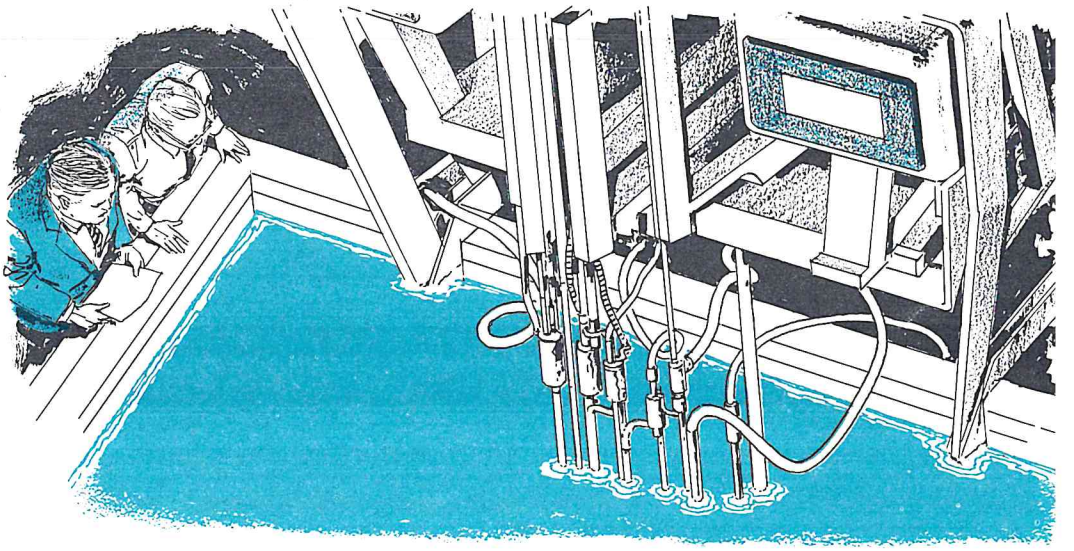
Check your answers by multiplying, using common fractions.

a. $.9 \times 4.8 = \square$ **b.** $.25 \times 7.6 = \triangle$

9. Find the products and express them in decimal form.

a. $8.9 \times 1.11 = \diamond$ **b.** $.7 \times .8 = \square$

Look at your work for Exercise **9**. For each example, compare the number of decimal places in the multiplier and the multiplicand with the number of decimal places in the product. Do you see a pattern? Describe it.



Using what you have discovered on pages 170–173, find the answers to these problems.

1. A control rod for one type of atomic reactor is 11.85 feet long. What is the total length of 6 such rods?

2. Each side of a square plot of land measures .8 mile. What is the perimeter of the plot?

3. What is the area of the plot of land in Exercise **2**?

4. A train travels at an average speed of 58.7 miles per hour. How many miles does it travel in 3 hours?

5. Another train travels at an average speed of 62.3 miles per hour. How many miles does it travel in 2.8 hours?

6. How much greater is the speed of the train described in Exercise **5** than that of the train described in Exercise **4**?

7. The width of a family room is 13.5 feet. The length is 19.5 feet. What is the floor area?

8. A certain part for a satellite cost $70.35. The government ordered 175 such parts. What was the total cost?

In each example below, choose the correct product from those that are given. Try to use the pattern you discovered to arrive at your answer without multiplying.

1. $.7 \times 16.4 = \diamond$ (114.8 11.48 1.148 .1148)

2. $1.2 \times .9 = \square$ (108 10.8 1.08 .108)

3. $.38 \times 10.4 = \triangle$ (3,952 395.2 39.52 3.952)

4. $1.6 \times .52 = \square$ (832 83.2 8.32 .832)

5. How many decimal places are there in the answer for this example: $.8 \times 4.7 = \text{N}$? for this example: $.35 \times 9.3 = \diamond$?

Find the product for each example.

6. How many decimal places are there in the answer for the example at the right? Explain why you think so.

$.93 \times 15.25 = \triangle$

What is the correct replacement for the frame in this sentence?

$$\frac{93}{100} \times 15\frac{25}{100} = \frac{93}{100} \times \frac{1525}{100} = \frac{141{,}825}{\square}$$

Is the product 141,825 *ten thousandths*? Then are there four decimal places in the decimal form of the product? Does the result fit the pattern you discovered earlier?

7. Without multiplying, tell how many decimal places there are in the answer for each of the following.

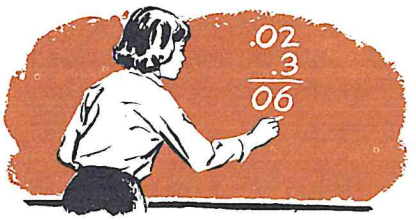
a.	**b.**	**c.**	**d.**	**e.**
27.25	.839	27.657	2.4	36.7259
1.37	1.9	.8	.99	.5

8. Find the products for Exercises **7a–e**.

9. Tell the number of decimal places in the answer for this example: $6 \times 2.0063 = \square$. Then multiply, and write the answer.

Can a numeral for a whole number (for instance, 6) be thought of as having no decimal places? Then is this statement true for multiplication with decimals?

▶In multiplying with decimals, the number of decimal places in the numeral for the product is equal to the sum of the number of decimal places in the multiplier and multiplicand.



1. To work the multiplication example shown at the left, Betty thought: "3 times 2 is 6; 3 times 0 is 0." Then, when she wanted to place the decimal point correctly, she was puzzled. She knew the numeral for the product should have three decimal places. But there are only two digits in "06." How would you write your answer?

a. Since .3 is less than 1, must the product of .3 and .02 be less than $1 \times .02$? Does this mean that the product must be less than .02? Then can the answer be .06?

b. Using common fractions, find the answer.

$$\frac{3}{10} \times \frac{2}{100} = \frac{6}{\square}$$

How do you write $\frac{6}{1000}$ as a decimal?

c. Is .006 the answer?

2. Here is another example. Multiply, just as if you were multiplying 4 by 2. Can the product for the example be 8? Can it be .8? What is the product?

.4
.2

3. To express your answer for Exercise **2**, did you write a 0 before the 8 and then place a decimal point before the 0 to make the figure mean hundredths?

What is the product for this example? Check your answer, using common fractions.

.9
.01

4. Multiply, and give the answer you think is correct. Then check each answer, using common fractions.

a	b	c	d	e
.03 .2	.03 .03	.002 .9	.01 .1	.8 .04
.41 .2	.065 .2	.19 .07	.15 .09	.018 .04

1. This chart tells the average number of miles that each of five different types of cars travels on one gallon of gasoline. On the average, how many miles does Car A travel on 10 gallons of gasoline? Find the correct replacement for the frame in this sentence.

Car A	16.4 miles per gallon
Car B	17.3 miles per gallon
Car C	17.8 miles per gallon
Car D	32.5 miles per gallon
Car E	36.7 miles per gallon

$$10 \times 16.4 = \square$$

2. How many miles does Car B travel on 10 gallons of gasoline? Car C? Car D?

3. Do you see a pattern you can use when you are multiplying by 10? Then, without using pencil and paper, tell how many miles Car E travels on 10 gallons of gasoline.

4. Many industries make use of "sheets" of metal. This chart tells the weight of a square foot of four different types of steel "sheets."

A	3.125 lb. per square foot
B	4.375 lb. per square foot
C	6.875 lb. per square foot
D	7.625 lb. per square foot

How many pounds do 100 square feet of Type A weigh?

$$100 \times 3.125 = \square$$

5. How many pounds do 100 square feet of Type B weigh? 100 square feet of Type C?

6. Do you see a pattern you can use in multiplying by 100? Then tell how many pounds 100 square feet of Type D weigh.

7. Are these sentences true, or are they false?

$10 \times 8.156 = 81.56$ $\qquad$ $100 \times 8.156 = 815.6$

What does $1{,}000 \times 8.156$ equal? $1{,}000 \times 3.294$? Do you agree with the following statement?

▶When multiplying a number expressed as a decimal
by 10, move the decimal point one place to the right;
by 100, move the decimal point two places to the right;
by 1,000, move the decimal point three places to the right.

1. Write numerals for the following, using digits.

a. Twenty-five thousand, six hundred four

b. Eight million

c. Four hundred ninety-two thousand, two hundred fifty-seven

d. Six thousand ten and eighty-seven hundredths

e. Three thousand one hundred forty-nine ten-thousandths

f. Twenty-six ten-thousandths

2. Copy and complete this chart to show what each digit in 816.73 represents.

8 1 6 . 7 3

- 3 is in hundredths' place and represents $3 \times \frac{1}{100}$
- 7 is in ? place and represents $7 \times ?$
- 6 is in ? place and represents $6 \times ?$
- 1 is in ? place and represents $1 \times ?$
- 8 is in ? place and represents $8 \times ?$

3. Make a similar chart for each of the following.

a. 7,509.634 **b.** 82,767.2349

4. Is $\frac{21}{5}$ greater than 4; that is, is it true that $\frac{21}{5} > 4$? Can you also compare these numbers using "<," the sign for "is less than"? Do it.

5. Using the sign ">," write a sentence comparing the numbers in each of these pairs.

a. $\frac{26}{9}$; 3 **b.** 5; 4.999 **c.** .009; .01 **d.** $\frac{5}{4}$; $\frac{9}{8}$

6. Using the sign "<," write a sentence comparing the numbers in each of these pairs.

a. 7; $\frac{41}{6}$ **b.** .003; $\frac{9}{100}$ **c.** 1.01; $\frac{11}{10}$ **d.** $\frac{97}{12}$; 8

7. Find a single numeral to replace N.

a. $(9 \times 1{,}000) + (7 \times 10) + (8 \times \frac{1}{10}) + (5 \times \frac{1}{100}) = N$

b. $(4 \times 100) + (3 \times \frac{1}{100}) + (6 \times \frac{1}{1000}) + (9 \times \frac{1}{10{,}000}) = N$

c. $(8 \times 100{,}000) + (7 \times 1{,}000) + (5 \times \frac{1}{10}) + (3 \times \frac{1}{10{,}000}) = N$

8. What does 10^2 equal? 10^3? 10^5? 10^7?

1. Copy each exercise and replace the question marks so that each fraction is equivalent to the fraction in the box.

a. $\boxed{\frac{5}{9}}$ $\frac{?}{18}$ $\frac{?}{27}$ $\frac{?}{36}$ $\frac{?}{63}$ $\frac{?}{90}$ **b.** $\boxed{\frac{7}{3}}$ $\frac{?}{6}$ $\frac{?}{9}$ $\frac{?}{12}$ $\frac{?}{30}$ $\frac{?}{75}$

Copy, and replace the frames.

	a	b	c	d
2.	$3\frac{2}{3} = \frac{\triangle}{3}$	$\frac{29}{6} = 4\frac{\square}{6}$	$9\frac{1}{4} = 8\frac{\triangle}{4}$	$6\frac{7}{6} = 7\frac{\triangle}{6}$
3.	$\frac{99}{8} = 12\frac{\triangle}{8}$	$\frac{14}{16} = \frac{7}{\square}$	$\frac{5}{12} = \frac{\square}{36}$	$12\frac{3}{2} = 13\frac{\triangle}{2}$
4.	$6 = \frac{\square}{3}$	$2\frac{5}{5} = \diamond$	$4 = 3\frac{\triangle}{6}$	$\frac{18}{8} = 2\frac{\triangle}{4}$

Find the missing numerals.

	a	b	c
5.	$2 - 1\frac{3}{8} = \square$	$1\frac{7}{10} + \frac{1}{5} = \triangle$	$2\frac{1}{4} + \square = 2\frac{7}{12}$
6.	$3\frac{2}{3} + 2\frac{4}{5} = \triangle$	$\diamond - 5\frac{3}{4} = 7\frac{1}{2}$	$6\frac{1}{12} - 5\frac{1}{8} = \square$

7. What is the reciprocal of $\frac{1}{2}$? of 3? of $\frac{4}{5}$? of $\frac{5}{4}$? of $2\frac{1}{3}$?

Copy, and find the missing numerals.

8.	$\frac{3}{5}$ of $20 = \mathrm{N}$	$\frac{1}{6} \times \frac{3}{5} = \square$	$6 \div \frac{4}{5} = \triangle$
9.	$40 \times \frac{3}{10} = \diamond$	$\frac{3}{4} \div \frac{1}{2} = \square$	$4\frac{1}{5} \times 2\frac{1}{3} = \triangle$
10.	$4 \div 1\frac{1}{2} = \mathrm{N}$	$1\frac{3}{4} \times \frac{4}{7} = \triangle$	$1\frac{3}{4} \div \frac{4}{7} = \square$
11.	$\frac{7}{8} \times 32 = \mathrm{N}$	$\frac{5}{8} \div \frac{10}{3} = \square$	$\triangle \times \frac{5}{3} = 0$

12.a. $(\frac{3}{4} \times \square) \times 12 = 12$ **b.** $\frac{2}{5} \times (\square \times 18) = 18$

13. Find the missing numeral in this sentence: $\frac{1}{4} \times \mathrm{N} = 1\frac{1}{8}$.

14. If $\frac{3}{5}$ of a certain number is $1\frac{1}{2}$, what is the number?

15. What fractional part of 36 is 21? Express your answer in simplest form.

16. Use the common denominator method to find the quotient for each of the following examples.

a	b	c	d
$\frac{9}{5} \div \frac{3}{5} = \mathrm{N}$	$\frac{1}{4} \div \frac{3}{4} = \square$	$\frac{7}{2} \div \frac{5}{2} = \triangle$	$\frac{4}{3} \div \frac{5}{6} = \diamond$

1.a. What does 10×8 equal? 10×8.3? 10×8.35?
b. What does 100×8 equal? 100×8.3? 100×8.35?
c. What does $1{,}000 \times 8$ equal? $1{,}000 \times 8.3$? $1{,}000 \times 8.35$?

2. Can we think of the number 8 as 8.0, 8.00, or 8.000? Can we think of 8.3 as 8.30 or 8.300? Does the pattern with multiplication by 10, 100, and 1,000 fit all the examples in Exercise **1**? For instance, to find the product for each example in **a**, did you move the decimal point one place to the right? Describe the pattern for the examples in **b**; in **c**.

3. A man measures a board and finds it to be $8\frac{3}{10}$ inches long. You know that he has measured it to the nearest tenth of an inch. Decimal fractions also tell how precise a measurement is. A measurement stated as "8.3 inches" is considered to be precise to the nearest ? of an inch.

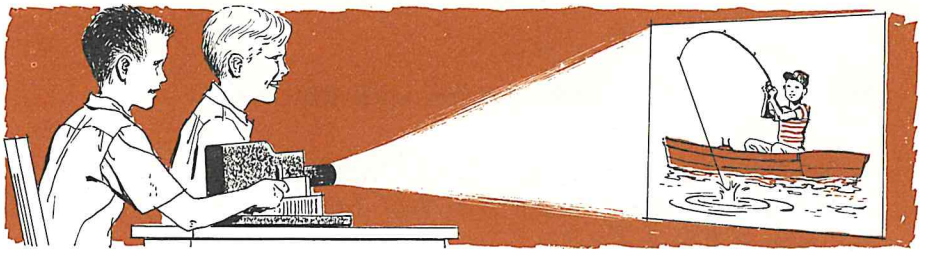
In Exercises **1** and **2**, you made use of the fact that 8.3 and 8.30 and 8.300 are all names for the same number. But if we are dealing with a measurement, "8.30 inches" *does not* mean the same thing as "8.3 inches." A measurement of "$8\frac{30}{100}$ inches" or "8.30 inches" is precise to the nearest ? inch. Is this more precise, or less precise, than a measurement stated as "8.3 inches"?

4.a. Fruits and vegetables are often measured by bulk—that is, by the space they occupy. One unit of **dry measure** is the bushel. One bushel is 4 pecks; each peck is 8 dry quarts; and each dry quart is 2 dry pints. How many quarts are there in a bushel? How many pints?
b. Expressed as a decimal, a peck is .25 bushel. How many bushels are 30 pecks?

A quart is .125 peck. How many pecks are 16 quarts?

5. If two numbers have a ratio of 3 to 1, is the first number 3 times as great as the second number? Which of these pairs of numbers have the ratio of 3 to 1?

a. 45 and 15 **b.** 7 and $2\frac{1}{4}$ **c.** $4\frac{1}{2}$ and $1\frac{1}{2}$



None of the problems on this page can be solved until some additional information is supplied. Study each problem and supply the information that will permit you to solve the problem. Be sure that your information is reasonable. Then solve the problem.

1. Sam paid \$3.64 for a roll of color film. After the film was exposed, he paid \$2.66 to have the color slides developed. How much did each color slide cost?

2. A bag of sugar costs \$.65. What is the cost of one pound?

3. Bill was told by his father to find the perimeter of a rectangular plot that he wanted to fence. The length was $22\frac{1}{2}$ ft.

4. Find the number of square inches in a pane of glass that is 2 feet 6 inches wide.

5. The sign in the store read, "Candy bars—6 for 31¢." Harold bought 6 candy bars. How much change did he receive?

6. Jim earned \$1.75 shoveling snow. How much did he have then to buy a birthday gift for his mother?

7. For a holiday dinner, Mrs. Burke bought a goose that weighed $14\frac{7}{8}$ pounds. How many people would the goose serve?

8. Mr. Herter drove at an average speed of 48.6 miles per hour. What distance did he drive in one day?

9. Jane received a box of pecans from her aunt in Georgia. She took one fourth of the pecans to school to share with friends. How many ounces of pecans did she take to school?

1. A board 7.6 feet long is to be cut into 4 parts of equal length. How long will each piece be? The division sentence for this problem is $7.6 \div 4 = \square$.

a. Try estimating the quotient. What is 7.6 rounded to the nearest whole number? What is $8 \div 4$? Then what is an estimate of the quotient?

b. Now work this division example as if you were dividing 76 by 4.

$$4\overline{)7.6}\quad ?\ ?$$

Which of the following do you think is the quotient: 19, 1.9, or .19? Place the decimal point in your answer.

c. Can division be checked by multiplication? Replace the frame with the numeral for the quotient; then see whether the sentence is true.

$$\square \times 4 = 7.6$$

In each of the following, estimate the quotient. Then, in **a**, divide as if you were dividing with whole numbers and place the decimal point where it belongs. Check, using the sentence in **b**.

	a	b
2. $12.4 \div 4 = \triangle$	$4\overline{)12.4}$	$\triangle \times 4 = 12.4$
3. $8.08 \div 8 = \diamond$	$8\overline{)8.08}$	$\diamond \times 8 = 8.08$
4. $3.5 \div 5 = \square$	$5\overline{)3.5}$	$\square \times 5 = 3.5$
5. $19.8 \div 2 = \diamond$	$2\overline{)19.8}$	$\diamond \times 2 = 19.8$
6. $3.45 \div 3 = \square$	$3\overline{)3.45}$	$\square \times 3 = 3.45$
7. $29.5 \div 5 = \triangle$	$5\overline{)29.5}$	$\triangle \times 5 = 29.5$

8. Look at your work for Exercises **2–7**. In each case, is the divisor represented by the numeral for a whole number? Now compare the dividend and the quotient. In each case, does the numeral for the quotient have the same number of decimal places as the numeral for the dividend?

9. The leader of the Crocodile Patrol had timed his patrol and found that, with rest periods, the patrol could hike about 2.3 miles per hour. How long will it take the patrol to hike a distance of 11.5 miles?

$$11.5 \div 2.3 = \triangle$$

a. Estimate the quotient. What is 11.5 rounded to the nearest whole number? 2.3 rounded to the nearest whole number? What is 12 ÷ 2? Then what is an estimate of the quotient?

b. Now work this division example as if you were dividing 115 by 23.

Is the quotient .5, or is it 5?

$2.3\overline{)11.5}$

c. Check by multiplication. Replace the frame in this sentence and see whether the sentence is true: $\triangle \times 2.3 = 11.5$.

In each of the following, estimate the quotient. Then, in **a**, divide as if you were dividing with whole numbers, and decide what the quotient is. Check, using **b**.

	a	b
10. $16.4 \div 4.1 = \square$	$4.1\overline{)16.4}$	$\square \times 4.1 = 16.4$
11. $4.5 \div .9 = \triangle$	$.9\overline{)4.5}$	$\triangle \times .9 = 4.5$
12. $28.8 \div .8 = \diamond$	$.8\overline{)28.8}$	$\diamond \times .8 = 28.8$
13. $25.2 \div 3.6 = \triangle$	$3.6\overline{)25.2}$	$\triangle \times 3.6 = 25.2$
14. $5.25 \div .75 = \diamond$	$.75\overline{)5.25}$	$\diamond \times .75 = 5.25$
15. $11.52 \div .96 = \square$	$.96\overline{)11.52}$	$\square \times .96 = 11.52$

1. Suppose you are asked to find the quotient for this example: $.9\overline{)18.}$ Can we think of .9 as $\frac{9}{10}$? Can 18 be expressed as 18.0 or 18 $\frac{0}{10}$ or $\frac{180}{10}$?

Then can we use common fractions and the common denominator method for dividing, as shown below?

$$18 \div \frac{9}{10} = \frac{180}{10} \div \frac{9}{10} = 180 \div 9, \text{ or } \triangle$$

Check your answer by multiplying: $\triangle \times .9 = 18.0$, or 18.

2. Using common fractions and the common denominator method, find the quotient for each example below.

a. $.5\overline{)15}$ **b.** $1.2\overline{)6}$ **c.** $.7\overline{)28}$ **d.** $2.5\overline{)125}$

3. You can also make use of the common denominator method without writing the example in common fraction form. For instance, suppose you are asked to find the quotient for the example $.4\overline{)8.}$

Since 8.0 is another name for 8, you can write the example as shown at the right. $.4\overline{)8.0}$

Now you divide as if you were dividing 80 by 4. What is $80 \div 4$?

Since you are dividing 80 tenths by 4 tenths, which of the following is the correct answer: .20, 2.0, or 20? Check.

4. In the example $.03\overline{)9}$, is the divisor expressed in hundredths? Then it is convenient to think of the dividend as hundredths. Can the number 9 be expressed as $\frac{900}{100}$ or 9.00?

Now copy the example as shown at the right. $.03\overline{)9.00}$

Think: You are dividing 900 hundredths by 3 hundredths. What is $900 \div 3$? Then what is the quotient?

5. Find the quotient for each of these examples.

a. $1.4\overline{)7}$ **b.** $.25\overline{)2}$ **c.** $.75\overline{)6}$ **d.** $1.25\overline{)20}$ **e.** $.9\overline{)270}$

6. Look back over your work for Exercise **5**. In any example, after you wrote zeros in the numeral for the dividend, did the numeral for the dividend ever have *fewer* decimal places than the numeral for the divisor?

You know that dividing is the inverse, or opposite, of multiplying. Let us use this fact to try to discover a pattern that will be helpful in division with decimals.

In each of the following exercises, study Sentence **a** and tell whether it is true or false. Then copy Sentence **b** and replace the frame with the correct numeral.

	a	b
1.	$3 \times .7 = 2.1$	$2.1 \div .7 = \diamond$
2.	$1.4 \times .6 = .84$	$.84 \div .6 = \square$
3.	$20 \times .8 = 16.0$	$16.0 \div .8 = \triangle$
4.	$1.02 \times 7 = 7.14$	$7.14 \div 7 = \square$
5.	$1.02 \times .7 = .714$	$.714 \div .7 = \diamond$

6. Look at each multiplication sentence. Does the number of decimal places in the product equal the sum of the number of decimal places in the multiplier and the multiplicand? Is this the pattern for multiplication you discovered earlier?

7.a. The sentences you wrote for **b** of Exercises **1–3** are shown at the right. Study each sentence. Is the number of decimal places in the quotient equal to the number of places in the dividend minus the number of places in the divisor?

$2.1 \div .7 = 3$
$.84 \div .6 = 1.4$
$16.0 \div .8 = 20$

b. Is this true in the other two division sentences you wrote?

8. What is $2 \times .4$? Then what is the quotient for Example **a**?

What is $.2 \times .4$? Then what is the quotient for Example **b**?

What is $.02 \times .4$? Then what is the quotient for Example **c**?

Do you agree that the following statement is true?

a. $.4\overline{).8}$

b. $.4\overline{).08}$

c. $.4\overline{).008}$

▶In a division example with decimals, the number of decimal places in the quotient is equal to the number of places in the dividend minus the number of places in the divisor.

Find the quotients. Check each answer by multiplication.

	a	b	c	d	e
1.	$6\overline{)12.6}$	$.6\overline{)12.6}$	$.6\overline{)1.26}$	$.6\overline{).126}$	$.06\overline{).126}$
2.	$3\overline{)4.89}$	$.3\overline{)48.9}$	$.3\overline{).489}$	$.03\overline{).489}$	$.003\overline{).489}$

3. Find the quotient for Example **a** at the right; for Example **b**.

a. $.5\overline{)1.0}$

b. $.5\overline{).5}$

c. $.5\overline{).2}$

In Example **a**, did you think, "What is 10 divided by 5?" and then write the quotient figure 2? In Example **b**, perhaps you thought, "What is 5 divided by 5?" and then wrote a 1. But now look at Example **c**. You cannot divide as if these were whole numbers unless you re-express the dividend. Is .20 another name for .2? Now find the quotient for the example as shown at the right.

$.5\overline{).20}$

4. Do .6 and .60 name the same number? Does writing another 0 at the end of the numeral .60 change its value? Writing zeros at the end of a decimal is called **annexing zeros.** Annexing zeros at the end of a decimal does not change its value.

Do 2 and 20 name the same number? But do 2 and 2.0 and 2.0000 all name the same number? You must think about the numbers you are expressing when you write zeros.

5. Which of the following pairs of numerals name the same number: 3.14 and 3.1400? 30 and 300? 30 and 30.00?

Find the quotients. Annex zeros when necessary. You may continue to annex zeros until the answer "comes out even."

	a	b	c	d	e
6.	$2.5\overline{).5}$	$.8\overline{)3.0}$	$.35\overline{).7}$	$.5\overline{).1}$	$.08\overline{).4}$
7.	$1.6\overline{).8}$	$7.5\overline{).6}$	$.012\overline{)2.4}$	$.05\overline{)7.4}$	$.026\overline{).39}$

Find the quotients. Check by multiplication.

	a	b	c	d	e
8.	$.5\overline{)3}$	$.35\overline{).105}$	$.65\overline{).13}$	$2.3\overline{).483}$	$.8\overline{)3.464}$
9.	$.45\overline{).18}$	$1.2\overline{).3}$	$.7\overline{).315}$	$.16\overline{).04}$	$15\overline{)4.65}$
10.	$1.6\overline{)1.2}$	$25\overline{)7.525}$	$.34\overline{).17}$	$5\overline{)4.05}$	$21\overline{)4.242}$

11. To make use of the pattern, must the numeral for the dividend have at least as many decimal places as the divisor? Then how will you express the dividend in this example: $.3\overline{)36}$? the dividend in these examples: $.12\overline{)72}$? $1.18\overline{)23.6}$? Work each of these examples.

Find the quotients. Check by multiplication.

12.	$.8\overline{)96}$	$.15\overline{)12}$	$1.6\overline{)352}$	$.008\overline{).04}$	$.05\overline{)2}$
13.	$.42\overline{)88.2}$	$.06\overline{)36}$	$.012\overline{)49.2}$	$2.5\overline{)750}$	$.157\overline{)314}$

14.a. To find the quotient for this example, Mary thought: "What is 63 ÷ 21?" Then she wrote the 3 as shown, and multiplied.

$$2.1\overline{).063}$$

$$\begin{array}{r} 3 \\ 2.1\overline{).063} \\ \underline{63} \end{array}$$

b. Is the answer 3? .3?

c. Do you think the answer must be some number of hundredths? That is, must the numeral have two decimal places? Which do you think is the answer: .30 or .03? Check by multiplying.

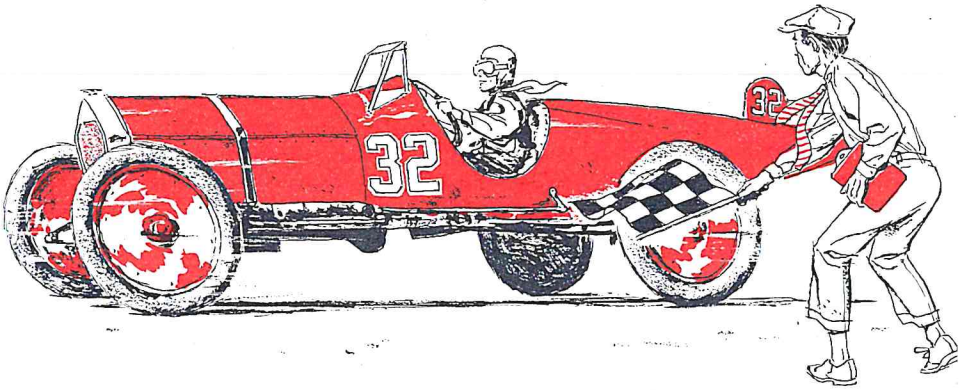
d. In **b**, you found that neither 3 nor .3 was the answer. Then, in **c**, you were asked if .30 was the answer. Since .30 is another name for .3, could you immediately ignore .30 as a possible answer?

15. Divide, and choose the correct quotient from the numbers named in parentheses.

a. $.2\overline{).004}$ (.2, .02, .20, 2)

b. $.004\overline{).2}$ (.5, 5, .05, 50)

c. $.7\overline{).049}$ (.70, .7, .07, .007)



1. The first Indianapolis Speedway race was run in 1911. The winning car traveled at an average speed of 74.59 miles per hour. The next year, the winning car traveled at an average speed of 78.72 miles per hour. How much greater was the speed of the winning car in 1912 than the speed of the winning car in 1911?

2. In 1961, fifty years after the first Indianapolis race, the winning car traveled at an average speed of 139.130 miles per hour. In 1962, the winning car traveled at an average speed of 140.292 miles per hour. What was the difference in the speed of these two cars?

3. If a car traveled at an average speed of 140.292 miles per hour, how many miles would it travel in 3 hours? How many miles would it travel in 10 hours?

4. Mr. Harris has an "antique" car that travels a distance of 36.5 miles on 5 gallons of gasoline. How many miles does the car travel per gallon of gasoline?

5. A certain brand of gasoline costs 33.9 cents per gallon. What will 15 gallons of this gasoline cost? Express your answer in dollars and cents. (Round to the nearest cent.)

6. A bus traveled a distance of 191.7 miles in 4.5 hours. What was its average speed?

1.a. Can you quickly express the common fraction $\frac{1}{10}$ as a decimal? Do it.
b. Express these common fractions as decimals: $\frac{7}{10}$; $\frac{8}{100}$; $\frac{25}{100}$; $\frac{375}{1000}$.
c. Can you easily write the **decimal equivalent** of any common fraction with a denominator 10, 100, 1,000, and so on? Tell what you do.

2. Now suppose you are asked to express $\frac{1}{2}$ as a decimal. Can you name a fraction with a denominator of 10 that is equivalent to $\frac{1}{2}$? Try it: $\frac{1}{2} = \frac{\square}{10}$.

What is $\frac{5}{10}$ expressed in decimal form? Then is this sentence true: $\frac{1}{2} = \frac{5}{10} = .5$?

3. Express $\frac{4}{5}$ as a decimal: $\frac{4}{5} = \frac{\triangle}{10} = \square$.

4. In Exercise **3**, when you wanted to express $\frac{4}{5}$ as a fraction with a denominator of 10, did you think: "What number times 5 equals 10?" Then what did you do? Was the procedure easy because 5 is a *factor* of 10?

5. Is 4 a factor of 10? That is, can 10 be divided evenly by 4? Is 4 a factor of 100? Then suppose you are asked to express $\frac{1}{4}$ as a decimal. Will you be able to find an equivalent fraction with a denominator of 100, and then use this to express $\frac{1}{4}$ as a decimal? Do it.

$$\frac{1}{4} = \frac{\triangle}{100} = \square$$

6. Express each of the following as a decimal.

a. $\frac{1}{5}$ **b.** $\frac{3}{4}$ **c.** $\frac{3}{20}$ **d.** $\frac{1}{50}$ **e.** $\frac{7}{25}$ **f.** $1\frac{7}{20}$

7. Is 8 a factor of 10? of 100? of 1,000? Check to make sure you have answered correctly. Then express $\frac{1}{8}$ as a decimal.

8. Express each of the following as a decimal.

a. $\frac{3}{8}$ **b.** $\frac{1}{40}$ **c.** $\frac{7}{8}$ **d.** $\frac{7}{40}$ **e.** $\frac{5}{4}$ **f.** $\frac{5}{2}$

9. There is another way to express a fraction as a decimal. You know that a fraction such as $\frac{7}{40}$ means $7 \div 40$. At the right, we have written $7 \div 40$ as a division example and have divided. Check each step to make sure it is correct.

$$\begin{array}{r} .175 \\ 40\overline{)7.000} \\ \underline{4\,0} \\ 3\,00 \\ \underline{2\,80} \\ 200 \\ \underline{200} \end{array}$$

Now look back at your answer for Exercise **8d**. Do the answers agree?

10. To express $\frac{3}{16}$ as a decimal, which of these examples can you use: $16\overline{)3}$ or $3\overline{)16}$? Copy the example and divide. Will you put a decimal point after the numeral for the dividend before you annex zeros? Why? Now divide. Annex zeros until the division comes out even.

11. Suppose you are asked to express $\frac{7}{15}$ as a decimal. Could you use the example shown at the right?

$$\begin{array}{r} .46 \\ 15\overline{)7.00} \\ \underline{6\,0} \\ 1\,00 \\ \underline{\;\;90} \\ 10 \end{array}$$

Study the work that has been done. Is it correct as far as it goes? Copy the example and continue the work until you have a decimal with four places.

Does the answer come out even? Will it come out even if you continue to divide, or will you get the digit 6 again and again?

12. In Exercise **11**, when you carried the work to four decimal places, was .4666 the numeral you obtained for the quotient? Suppose you wished to express the quotient correct to the nearest thousandth. Since the digit in ten-thousandths' place is 6, do you round downward, or upward? Then, to the nearest thousandth, the quotient is .467.

If you had been asked to express $\frac{7}{15}$ as a decimal correct to the nearest hundredth, would it have been enough to find the quotient to thousandths' place and then do the rounding?

13. Express each of the following as a decimal correct to the nearest thousandth.

a. $\frac{4}{15}$ **b.** $\frac{2}{9}$ **c.** $\frac{1}{7}$ **d.** $\frac{5}{12}$ **e.** $\frac{3}{7}$

1. Mr. Bonnard owns a piece of land with an area of 80 acres. He plans to divide it into 25 lots equal in area. What will be the area of each lot?

$$80 \div 25 = \square$$

Look at Example **a**. Instead of writing R5 above the example, we have divided the 5 by 25. Then the quotient is $3\frac{5}{25}$. Express $3\frac{5}{25}$ in simplest form.

Now study Example **b**. Here, instead of expressing $5 \div 25$ as a common fraction, we have written a decimal point and annexed a zero to the dividend. Copy and complete the work. What answer do you get?

Is $\frac{1}{5}$ equivalent to .2? Then are $3\frac{1}{5}$ and 3.2 names for the same number?

$3\frac{5}{25}$ or $3\frac{\triangle}{\diamond}$

a.
$$25\overline{)80}$$
$$\underline{75}$$
$$5$$

b.
$$25\overline{)80.0}$$
$$\underline{75}$$
$$5\,0$$
$$\underline{?\,?}$$

2. Check the two expressions for the quotient in **1**.

a. $3\frac{1}{5} \times 25 = \square$ **b.** $3.2 \times 25 = \square$

Do you get the dividend in each case?

Copy and divide. First, express the quotient as a mixed fraction. Second, work the example by using what you know about decimals. Annex zeros as needed.

	a	b	c	d	e
3.	$16\overline{)100}$	$40\overline{)190}$	$16\overline{)44}$	$15\overline{)18}$	$8\overline{)17}$
4.	$8\overline{)21}$	$25\overline{)36}$	$4\overline{)63}$	$25\overline{)85}$	$12\overline{)33}$
5.	$8\overline{)26}$	$36\overline{)189}$	$16\overline{)76}$	$12\overline{)27}$	$18\overline{)225}$

As you know, not all division examples come out even. Then the quotient can be rounded. In the examples below, find the quotient correct to the nearest thousandth.

6.	$3\overline{)1}$	$6\overline{)5}$	$3\overline{)2}$	$12\overline{)7}$	$9\overline{)13}$
7.	$12\overline{)3.65}$	$18\overline{)5.84}$	$9\overline{)7.08}$	$14\overline{)23.65}$	$15\overline{)97.7}$

1. Is this sentence true: $10 \times 3.625 = 36.25$? What is a correct replacement for the frame in this sentence: $36.25 \div 10 = \triangle$?

Check your answer by copying the division example and finding the quotient. $10\overline{)36.25}$

2. Copy, and find the quotients.

a. $10\overline{)84.97}$ **b.** $10\overline{)299.3}$ **c.** $10\overline{)127.38}$

3. In each of the examples in Exercises **1** and **2**, compare the answer with the numeral for the dividend. Do you see a pattern you can use to write the answer when a number expressed as a decimal is divided by 10? Describe the pattern.

a. What is $592.4 \div 10$? 10×59.24?
b. What is $8.67 \div 10$? $10 \times .867$?
c. Is the pattern for dividing a decimal by 10 the opposite of multiplying a decimal by 10?

4. To divide a number such as 736.4 by 100, you can use the method shown at the right. What is the correct replacement for the frame?

$$736.4 \div 100 = 736.4 \times \frac{1}{100}$$
$$= 736.4 \times .01$$
$$= \diamond$$

5. Using the method shown in Exercise **4**, find the quotients.

a. $823.99 \div 100$ **b.** $79.7 \div 100$ **c.** $347.25 \div 100$

6. In each of the examples in Exercises **4** and **5**, compare the answer with the numeral for the dividend. Do you see a pattern you can use to write the answer when a decimal is divided by 100? Is this pattern the opposite of the pattern you discovered when you multiplied a decimal by 100?

What is $624.99 \div 100$? 100×6.2499?

7. Can you predict the quotient for the example at the right? $829.3 \div 1{,}000 = \square$

If you are not sure of your answer, find the quotient by the method used in Exercise **4**.

8. Divide each of these numbers by 10; by 100; by 1,000.

a. 168.53 **b.** .915 **c.** 937.54

1. Study these examples. Will the common fraction that names the sum for **a** be equivalent to the decimal that names the sum for **b**? Why?

a. $\frac{2}{10} + \frac{7}{10} = \square$

b. $.2 + .7 = \square$

Name the sum in two ways—as a common fraction and as a decimal.

2. Sometimes in your work, you will find it convenient to use decimals instead of common fractions. Copy and complete the table below. If the decimal does not come out even, express your answer correct to the nearest thousandth.

$\frac{1}{2} = .?$	$\frac{1}{4} = .??$	$\frac{3}{4} = .??$	$\frac{1}{8} = .???$
$\frac{3}{8} = .???$	$\frac{5}{8} = .???$	$\frac{7}{8} = .???$	$\frac{1}{5} = .?$
$\frac{2}{5} = .?$	$\frac{3}{5} = .?$	$\frac{4}{5} = .?$	$\frac{5}{5} = ?.?$
$\frac{1}{3} = .???$	$\frac{2}{3} = .???$	$\frac{1}{6} = .???$	$\frac{5}{6} = .???$

3. Can you work the example at the right by renaming the mixed fractions as mixed decimals? Try it and see.

$$1\frac{1}{2} = 1.?$$
$$+\ 2\frac{4}{5} = 2.?$$

4. Work the following example in two ways—with common fractions and with decimals.

a. $3\frac{1}{4}$
$-2\frac{9}{20}$

b. $3.??$
$-2.??$

Are your results for **a** and **b** names for the same fractional number?

5. Find the sum or difference, using decimals. Check by using fractions.

a. $8\frac{9}{25} + 7\frac{17}{20} = \diamond$ **b.** $4\frac{1}{5} - 2\frac{9}{10} = \square$ **c.** $1\frac{49}{50} + 7\frac{3}{4} = \triangle$

6. Study these examples. Will the common fraction that names the product for **a** be equivalent to the decimal that names the product for **b**? Why? Multiply.

a. $\frac{1}{2} \times \frac{1}{4} = \square$

b. $.5 \times .25 = \square$

Are $\frac{1}{8}$ and .125 names for the same number?

You know that the decimal system is a base 10 system. Let us take another look at the way 10 is used in expressing numbers in the decimal system.

1. Is this sentence true: $100 = 10 \times 10$, or 10^2? What does $10 \times 10 \times 10$, or 10^3, equal? What does $10 \times 10 \times 10 \times 10$, or 10^4, equal?

2. You have used exponents to express 300 as 3×10^2. Similarly, 5,000 can be expressed as $5 \times 10^?$. How do you express 20,000 in this way?

3. What does $\frac{1}{10} \times \frac{1}{10}$ equal? Can we express $\frac{1}{10} \times \frac{1}{10}$, or $\frac{1}{100}$, as $\frac{1}{10^?}$?

What does $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$ equal? Can we express $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$, or $\frac{1}{1000}$, as $\frac{1}{10^?}$?

4. Are these sentences true: $.6 = 6 \times \frac{1}{10}$; $.05 = 5 \times \frac{1}{100}$, or $5 \times \frac{1}{10^2}$?

Copy, and make the correct replacements.

a. $.08 = 8 \times \frac{1}{100}$, or $8 \times \frac{1}{10^?}$

b. $.005 = 5 \times \frac{1}{\square}$, or $5 \times \frac{1}{10^?}$

c. $.0003 = 3 \times \frac{1}{\triangle}$, or $3 \times \frac{1}{10^?}$

5. Is the following sentence true: $3{,}462.579 = (3 \times 10^3) + (4 \times 10^2) + (6 \times 10) + 2 + (5 \times \frac{1}{10}) + (7 \times \frac{1}{10^2}) + (9 \times \frac{1}{10^3})$?

6. Express each of the following, using exponents.

a. 256.37 **b.** 8,972.468 **c.** 24,517.9384

7. Look back at Exercise **5** and the work you did for Exercise **6**. Do you see that tens' and tenths' places, hundreds' and hundredths' places, and so on, may be thought of as arranged around the ones' place?

EXTRA PRACTICE 299–302

If you make any mistakes, study the pages given at the right of the exercises.

Estimate the product first, then multiply. 170–173

	a	b	c
1.	$7 \times 3.09 = \diamond$	$1.87 \times 804 = \square$	$4.7 \times 2.79 = \triangle$
2.	$.81 \times 6.3 = \square$	$3.9 \times 5.2 = \triangle$	$2.43 \times 7.28 = \mathrm{N}$

Find the products. 170–173

3.

.363	614	.092
.9	.07	1.06

4. What does $10 \times .3125$ equal? 10×1.093? $100 \times .00052$? 100×16.85? $1{,}000 \times 16.85$? 177

Find the quotients. 182–187

	a	b	c	d
5.	$5\overline{)9.5}$	$.9\overline{)4.5}$	$.23\overline{)2.53}$	$.4\overline{).64}$
6.	$12\overline{)18.636}$	$.9\overline{)45}$	$1.2\overline{).0672}$	$.35\overline{)2.45}$

7. Express each fraction exactly as a decimal. 189–190

$\frac{7}{20}$ $\frac{8}{25}$ $\frac{3}{40}$ $\frac{7}{16}$

8. Express as decimals correct to the nearest thousandth. 190

$\frac{5}{12}$ $\frac{7}{9}$ $\frac{5}{8}$ $\frac{9}{16}$

9. Find the quotients correct to the nearest tenth. 190

$53\overline{)100}$ $3\overline{)100}$ $.26\overline{)4.8}$ $3.6\overline{)6.12}$

10. What does $16.09 \div 100$ equal? $5.67 \div 10$? $58.2 \div 1{,}000$? $.09 \div 10$? 192

11. Find the missing numerals. 172–173, 185–186

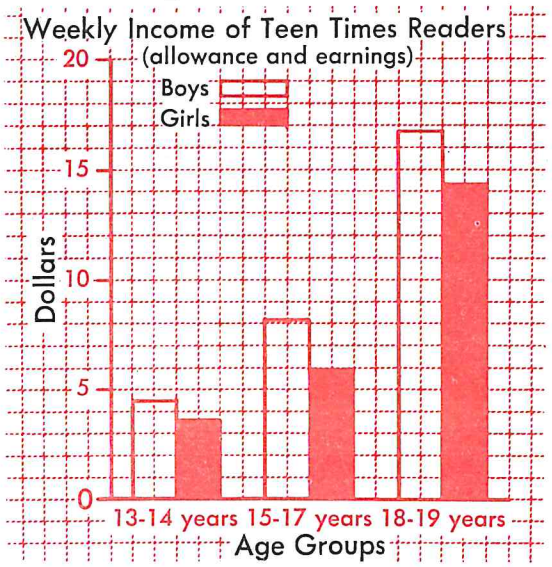
$3 \times \square = 3.6$ $\triangle \times .2 = .8$ $\diamond \times .2 = .08$ $4.5 \div .3 = \square$

$\mathrm{N} = 1.5 \times .06$ $\triangle \div 2.3 = 4$ $\diamond \times 4.2 = 168$ $.01 \times .01 = \square$

DISCOVERING MORE ABOUT NUMBERS: PAGES 329–331

7 Showing Data in a Bar Graph

The editors of the magazine *Teen Times* wanted to know the weekly income of their teen-age readers. The graph below shows the information they obtained by sending out questionnaires to all teen-age subscribers.



1. What information does the title of the graph give? What does the vertical scale show? How is it labeled?

2. This graph is called a **bar graph** and is helpful in making visual comparisons quickly.

In each age group, who has the greater income, the boys or the girls?

3. The graph shows that the difference between the boys' and the girls' income in the 13–14 year group is about 50 cents. About what is the difference in incomes for the 15–17 year group? the 18–19 year group?

4. About what is the difference in the income of the boys in the 13–14 year group and the boys in the 15–17 year group?

Weekly Income of *Teen Times* Readers (allowances and earnings)		
13–14 years:	Boys	$4.52
	Girls	$3.72
15–17 Years:	Boys	$8.14
	Girls	$5.98
18–19 years:	Boys	$16.80
	Girls	$14.25

5. At the left are the actual data used in preparing the graph. Can you make comparisons as quickly by glancing at the table as by glancing at the graph? Which provides more precise information?

6. Using the data in the table, give precise answers to the questions in Exercises **3** and **4**.

1. Alice wanted to make a vertical bar graph using the information given in this table.

City	Population	Telephones
New York	7,781,984	4,411,982
Chicago	3,550,404	1,894,012
Los Angeles	2,479,015	1,051,396
Philadelphia	2,002,512	1,083,041

Using square-ruled paper for her graph, Alice decided that she could use no more than 10 spaces along the scale for the tallest bar. If she let each space represent 100,000, would there be enough spaces for the tallest bar? If she let each space represent 1,000,000, would the tallest bar be less than 10 spaces? Which population figure is used in answering these questions—the largest figure or the smallest figure?

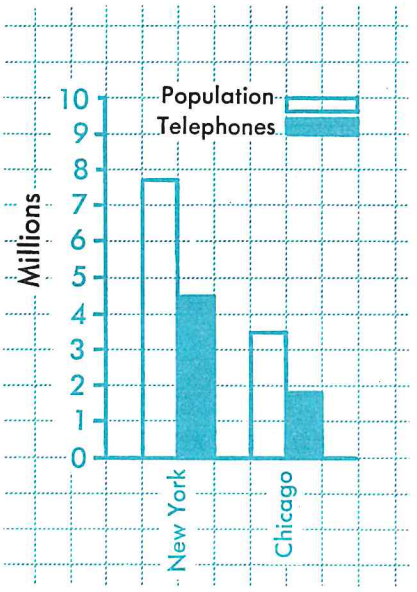
2. Alice let each space on the scale represent 1,000,000. To estimate where to place the tops of the bars, she decided to round the data.

a. What is 7,781,984 rounded to the nearest hundred thousand: 7,700,000 or 7,800,000?
b. Is 1,000,000 ten times 100,000? Then does 100,000 equal $\frac{1}{10}$ of 1,000,000? How many tenths of 1,000,000 is 800,000?
c. If 800,000 is $\frac{8}{10}$ of 1,000,000, can you represent 7,800,000 as 7.8 million?

3. What is 4,411,982 rounded to the nearest hundred thousand? Express 4,400,000 as millions and tenths of a million.

4. Copy and complete this table to show the data for the graph rounded to the nearest tenth of a million.

City	Population	Telephones
New York	7.8 million	4.4 million
Chicago	? million	? million
Los Angeles	2.5 million	? million
Philadelphia	? million	? million



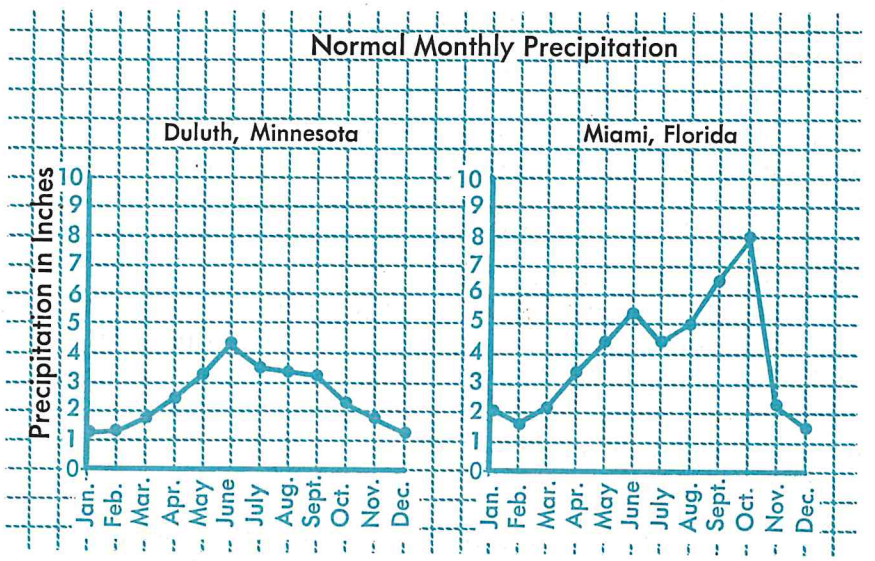
5. To make her bar graph, Alice first marked a scale along a vertical line on square-ruled paper. A line on which a scale for a graph is marked is called an **axis.** Beginning with zero, Alice marked the scale to show that 1 unit space stood for 1,000,000. How did she show that the numerals on the scale stood for millions?

6. Since the table showed the population of New York as 7.8 million, Alice estimated where 7.8 million would be represented on the vertical scale. She used this mark to indicate how tall the bar for population would be. Then, starting one unit to the right of the vertical scale, she drew the bar to represent the population. Next to this bar she drew a bar to represent the number of telephones and colored it blue. How did she label the pair of bars? How did she show which bar represented the population and which bar represented the number of telephones?

7. At the right of these completed bars, Alice began to outline the bars to represent the data for Chicago. She knew the bars representing the data for the two cities would need to be read easily, so she left a space between the two pairs of bars. What label should she use for the second pair of bars?

8. On square-ruled paper, draw a graph representing the data you completed in the table on page 197, Exercise **4**. Remember to leave a unit space between the pairs of bars. When the graph is finished, it will need a title. Think of a good title to write across the top.

1. Sometimes you wish to show information in ways for which a bar graph is not suitable. For example, if you are interested in showing changes in data, you should use a **broken-line graph** (sometimes called simply a **line graph**).



2. There are always two scales on a line graph. Often one of the scales represents time. Time is always marked on the horizontal axis. In the graphs shown above, what does one unit space represent on the vertical scale?

3. Do the two graphs above have a title? What does it tell you? Precipitation includes rain, sleet, snow, and so forth. How much precipitation is normal for Duluth in the month of January?

4. Which city receives the greatest amount of precipitation in one month? In which month does this occur? In which city does the least amount of precipitation occur in one month? In which months does it occur?

5. In which city does the greatest change in precipitation occur from one month to the next? During which two months does this change take place?

Normal Monthly Precipitation in Inches for Duluth and Miami												
	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Duluth	1.2	1.3	1.8	2.5	3.2	4.3	3.6	3.4	3.2	2.2	1.8	1.2
Miami	2.1	1.7	2.1	3.4	4.3	5.5	4.4	5.1	6.7	7.9	2.2	1.7

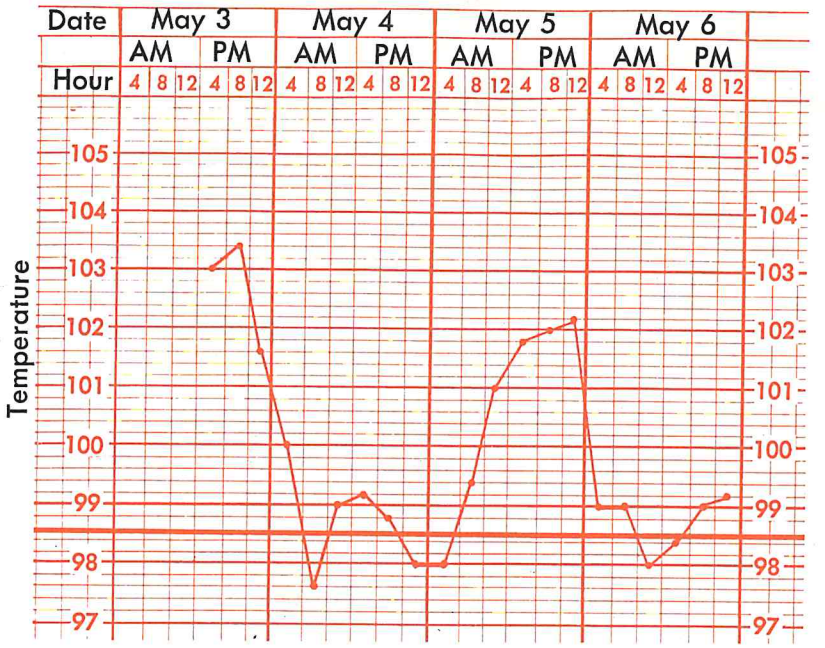
1. The table of data for the graphs on page 199 appears above. Using square-ruled paper, show the data as line graphs on the same pair of scales.

a. What will your vertical scale show? Look at the largest amount in the table of data and at the space you plan to use. Now decide what each unit space will represent.
b. Prepare a vertical scale and a horizontal scale that will fit the data. Label each.
c. Mark the points on the sheet of square-ruled paper according to the table of data. Use two pencils of different colors and make the graphs for the data of both cities on the same pair of scales.
d. Give the pair of graphs a title.

2. Using the pair of graphs you have prepared, answer the following:

a. What are the five wettest months in Duluth? in Miami? Are these the same months for both cities?
b. In Duluth, does the monthly precipitation increase, or decrease, from January to June? from June to December? In Miami, is there a constant increase in monthly precipitation from January to June? Is there a constant decrease from June to December?
c. Which month has the greatest normal precipitation in Duluth? in Miami? What is the difference in these amounts?

3. If each number of inches is rounded to the nearest whole number, what is the ratio of the greatest monthly precipitation for Miami to the greatest monthly precipitation for Duluth? Which gives a better comparison of precipitation in the two cities—the ratio or the difference?



1. Shown above is a special form of a line graph. It is a graph of a hospital patient's temperature and is referred to as a *temperature chart.* The numerals on the vertical scale represent degrees. In the space between degree marks there are five smaller spaces. Each small space represents a fractional part of a degree. What fractional part?

2. The red horizontal line represents the normal temperature of a human being. What temperature is represented by this line?

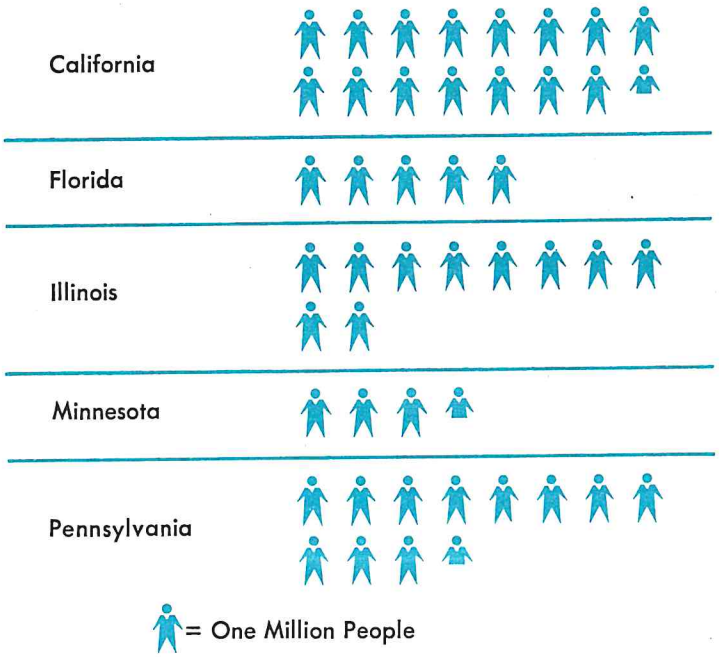
3. What was the patient's temperature when he was admitted to the hospital? What was it at 12 midnight on May 6?

4. What was the patient's maximum (highest) temperature? On what date did it occur? At what time?

5. What was the patient's minimum (lowest) temperature? On what date did it occur? How many hours elapsed between the patient's maximum temperature and his minimum temperature?

6. How much above normal was the patient's temperature at 8 A.M. on May 5?

1960 Population of Selected States



1. In another kind of graph that is used to make comparisons quickly, a small picture or symbol is used to represent a certain number of items. This type of graph is called a **pictograph.** Does the pictograph shown above have a title? What does it tell you?

2. Pictographs are usually used when the data include large numbers that are hard to understand and compare. In the pictograph above, how many people does one symbol represent? How do you know this?

3. How many symbols are shown for California? What do you think the last symbol in the group represents? What do you estimate the population of California to be?

4. Which state shown on the graph has the greatest population? How can you tell this? Which state has the least population? What do you estimate the difference in the populations of these two states to be?

5. At the right is the table of data that was used in making the pictograph on the opposite page. Does the graph show the data as precisely as the table? Why? To make a quick comparison of the populations of the states, would you use the table, or the graph?

1960 Population of Selected States	
California	15,717,204
Florida	4,951,560
Illinois	10,113,158
Minnesota	3,413,864
Pennsylvania	11,319,864

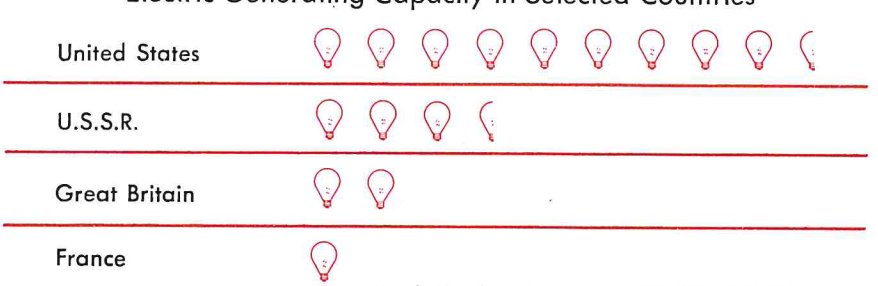
6. The data were rounded for drawing the graph. Were the data rounded to the nearest 10,000, nearest 100,000, or nearest 500,000? Could a graph be drawn to show the data rounded in either of the other ways? What difficulties might occur if this were attempted?

7. In drawing a pictograph, you must select a symbol to use in representing the data. What symbol was chosen to represent 1,000,000 people in the graph on the opposite page? What symbol might you use for the following sets of data?

a. The number of books in each of five different libraries
b. The number of houses constructed in each of four towns
c. Number of votes cast for each of three candidates
d. Beef production in each of five states
e. Petroleum production in each of four countries

8. Make a pictograph to represent the data in the following table. Remember, you must choose a symbol to use in representing the data, round the data if necessary, decide upon a unit value for each symbol, and give your graph a title.

East Side High School Student Enrollment	
Course	Number Enrolled
Academic	1,486
Commercial	932
Vocational	865



This pictograph shows how much electric power can be produced by all generating plants and dams in each of the countries listed. What unit of measure is used to show the amount of power each country can produce? If you do not know the meaning of "kilowatt," look it up in a dictionary or a science book.

1. What does the title of the pictograph tell you? Is the generating capacity given for all countries? for all European countries? What does each symbol represent? What does one half of a symbol represent? What would one fourth of a symbol represent?

2. In kilowatts, what is the generating capacity of Great Britain?

Do you think that the graph shows the generating capacity of each country with great precision?

3. What is the electric generating capacity of the United States? of the U.S.S.R.? The generating capacity of the United States is about how many times that of the U.S.S.R.? What is the ratio of the capacity of the United States to that of the U.S.S.R.?

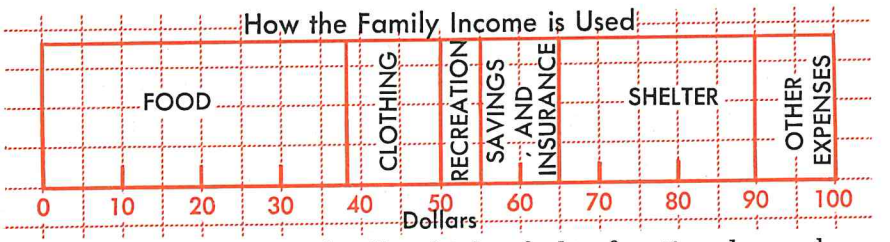
4. The electric generating capacity of Denmark is 1,870,000 kilowatts. What is this rounded to the nearest million kilowatts? What part of a symbol would be used to show the rounded number of kilowatts? Do you think it would be easy to show this part of a symbol in the pictograph? Would it be easy to read this part of a symbol? Do you see that a pictograph is not suitable for some data?

1. Robbie's family kept a record for one year of how they used the money his father earned. At the end of this year, they found how many hundred dollars had been spent and determined how each \$100 had been used. The results appear in this table. As you can see, for each \$100 earned, the family spent \$38 for food and \$12 for clothing. How much did they spend for recreation? for shelter?

Item	Amount Spent
Food	\$38
Clothing	12
Recreation	5
Savings & Insurance	10
Shelter	25
Other Expenses	10
TOTAL	\$100

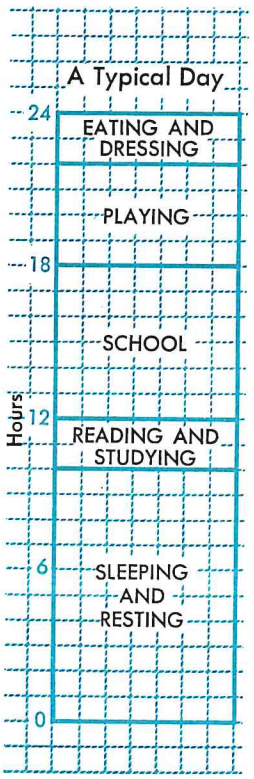
2. To find what part of the total money was used for an item, Robbie found what fractional part of \$100 was spent for that item. For example, $\frac{12}{100}$, or $\frac{3}{25}$, of the total amount was spent for clothing. What part of the total amount was spent for food? recreation? savings and insurance? shelter? other expenses?

3. One type of graph that shows quickly and easily the relation of the parts to the whole is called a **divided-bar graph.** In a divided-bar graph, we draw a single bar and then divide it into fractional parts. Robbie made a divided-bar graph like the one that appears below. How many scales does a divided-bar graph have? How many dollars are represented by each unit space on the scale?



4. Read the graph and tell which of the fractional numbers $\frac{1}{4}$, $\frac{1}{10}$, or $\frac{1}{20}$ best represents that part of each \$100 that was used for recreation; for shelter.

5. What decimal numeral best represents the part of each \$100 that was used for food? for clothing? for "other expenses"?



1. Bonnie kept a record for five school days of the time she spent on different activities. She used this information to draw a divided-bar graph for a typical day. A divided-bar graph can be horizontal or vertical. Bonnie drew a vertical graph like the one at the left. How many unit spaces did she show on the scale for her graph? Why?

2. Bonnie completed her graph for the data she had collected. On a typical day, how many hours does she spend eating and dressing? playing? reading and studying? in school? sleeping and resting?

3. The graph shows that Bonnie spent $\frac{4}{24}$, or $\frac{1}{6}$, of the day playing. Study the divided-bar graph and then tell what fractional part of the day she spent on each of the other activities.

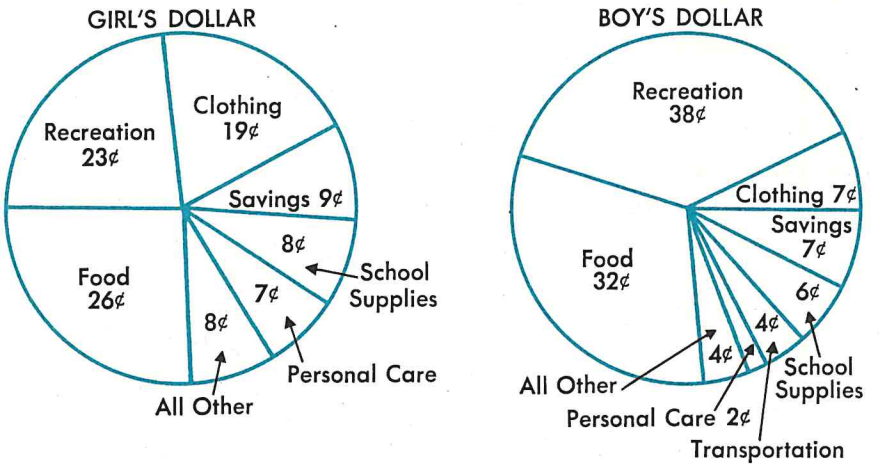
4. Bonnie discovered she spent twice as much time playing as reading and studying. What is the ratio of the time spent eating and dressing to the time spent sleeping and resting?

5. What is the ratio of the time Bonnie spent playing to the time she spent in school?

6. Did Bonnie spend as much time in school as she spent playing, eating, and dressing?

7. What is the total amount of time she spent in school and reading and studying? What fractional part of the day is this?

How the Teenage Dollar Is Spent



1. A divided-bar graph shows the relation of the parts to the whole. Another kind of graph that does this is the **circle graph.** Shown above are two circle graphs that illustrate how teenagers spend their money. Why are two circle graphs used? Bar graphs and line graphs have been used to compare sets of data. Can divided-bar graphs and circle graphs also be used to compare sets of data? What two sets of data are being compared in the circle graphs above?

2. Who spends a greater part of their money on food—boys or girls? a greater part on clothing? on personal care?

3. What item is shown on the circle graph for the boys that is not shown on the graph for the girls? Do you think this means the girls do not spend any money on transportation, or that they spend such a small part of each dollar that it cannot be shown on the graph?

4. What fractional part of the boys' dollar is spent on school supplies? What fractional part of the girls' dollar?

5. What is the ratio of the part the girls spend on food to the part the boys spend on food?

1. In the 8 grades of the Maywood School, there are 1,737 pupils. To find the average number of pupils per grade, will you divide?

Is the divisor a number less than 10? Then you can use **short division** to work this example. Copy the example and follow each step.

$$8\overline{)1{,}737}\quad 2$$

a. Think: "How many eights in 17 hundreds?" Are there as many as 200? as many as 300? Then write a 2 in hundreds' place. Does 2 hundreds × 8 equal 16 hundreds?
b. Instead of writing the "16," remember it and subtract. What is 17 hundreds − 16 hundreds? Now there is 137 left. Think of the 137 as 13 tens + 7. Are there as many as 10 eights in 13 tens? as many as 20? Then write a 1 in tens' place in the quotient. Does 1 ten × 8 equal 8 tens, or 80?
c. Do not write the "8". Remember it and subtract 8 tens from 13 tens. Is there 57 left? How many eights in 57? What is 7 × 8? Subtract 56 from 57. What is the remainder?
d. Do you get "217 R1" as your answer? Then what is the average number of pupils per grade?
e. Check your answer, using this sentence.

$$(217 \times 8) + 1 = N$$

2.a. In this division example, what will the first quotient figure be? Where will you write it?

$$9\overline{)4{,}608}$$

b. How many hundreds does 5 hundreds × 9 equal?
c. Copy the example and try to work it by short division. If you have any difficulty, you may do it the longer way. Then go back and think it through, using short division.

Divide, using short division.

	a	b	c	d	e
3.	$8\overline{)3{,}691}$	$5\overline{)4{,}979}$	$4\overline{)3{,}016}$	$9\overline{)8{,}640}$	$7\overline{)4{,}907}$
4.	$3\overline{)2{,}958}$	$5\overline{)4{,}325}$	$6\overline{)4{,}828}$	$2\overline{)9{,}098}$	$3\overline{)1{,}505}$
5.	$6\overline{)4{,}501}$	$9\overline{)7{,}857}$	$8\overline{)8{,}004}$	$3\overline{)1{,}400}$	$7\overline{)3{,}305}$

Find the missing numerals.

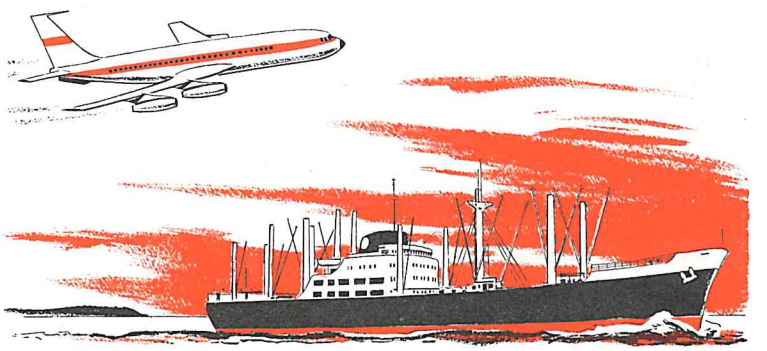
	a	b	c
1.	$5 \times \frac{5}{6} = \square$	$5 \div \frac{5}{6} = \triangle$	$\square \times \frac{1}{3} = 1$
2.	$\square - .6 = 1.1$	$1.8 + \triangle = 2.1$	$1\frac{1}{4} - \frac{7}{16} = \diamond$
3.	$4 \times 9.08 = \square$	$5.873 + 8.069 = \diamond$	$2.25 \div .45 = \triangle$
4.	$38.7 - 28.7 = \diamond$	$1.80 \div .036 = \triangle$	$3.1 \times 27.5 = \square$
5.	$3.65 + \square = 3\frac{65}{100}$	$\triangle \times .9 = 1.8$	$4.89 \div 3 = \diamond$
6.	$2.20 \div .4 = \square$	$1\frac{3}{5} + 2\frac{1}{6} = \diamond$	$12\frac{1}{4} - \square = 3\frac{1}{3}$

7. To measure distance, navigators of ships and airplanes use a unit of measure called the *international nautical mile*. To four decimal places, a nautical mile is 6,076.1155 feet. How much greater than a land, or *statute*, mile is a nautical mile?

8. Jim read in a book about ships that, to the nearest thousandth, there are 1.151 statute miles in a nautical mile. To the nearest tenth, how many statute miles are there in a nautical mile?

9. If 1 nautical mile is 1.151 statute miles, how far has an airplane traveled in statute miles when it has gone 10 nautical miles? 100 nautical miles? 1,000 nautical miles?

10. During a 24-hour period, a ship sailed 376.5 nautical miles. To the nearest tenth, how many statute miles is this?



1. The Harlow Company has a computer but does not have enough work to keep it in constant use. An agent has agreed to rent the computer to smaller businesses during the times that the Harlow Company does not use it.

The fee for computer time is \$100 per hour. The agent keeps \$20 of each \$100 that is received in computer rentals. The Harlow Company receives the remainder. What is the company's share of each \$100? Can we say that the ratio of the company's share to the hourly fee is 80 to 100?

2. What is the Harlow Company's share if \$200 is the total charge for computer time? if \$300 is the total charge? if \$600 is the total charge?

3. Copy and complete the table below so that it shows the company's share for each given total charge.

Number of dollars, company's share	80	160	?	?	?	?	?	?
Number of dollars, total charge	100	200	300	400	500	600	1,000	1,500

4. Think of the pairs of numbers, 80 and 100, 160 and 200, and so on. Does each pair have the same ratio?

5. Find the quotient for each example below.

$$80 \div 100 = \frac{\triangle}{100}, \text{ or } \frac{\Diamond}{5} \qquad 160 \div 200 = \frac{\square}{200}, \text{ or } \frac{\Diamond}{5}$$

6. Does the quotient, $\frac{4}{5}$, mean that in each pair of numbers the first number is $\frac{4}{5}$ of the second number? Will the company's share always be $\frac{4}{5}$ of the total charge?

7. At \$100 per hour, what is the charge for $1\frac{1}{2}$ hours of computer time? for $2\frac{1}{4}$ hours?

What is the company's share in each of these cases?

1. Read this numeral: 1,267,385,259. Copy and complete this chart to show what each digit in the numeral represents.

1,267,385,259

9	is in	ones'	place and represents	9
5	is in	?	place and represents	50
2	is in	?	place and represents	?
5	is in	?	place and represents	?
8	is in	?	place and represents	?
3	is in	?	place and represents	?
7	is in	?	place and represents	?
6	is in	?	place and represents	?
2	is in	?	place and represents	?
1	is in	?	place and represents	?

2. Find the number named by each of the following.

a. 10^5 **b.** 6^3 **c.** 5^4 **d.** 10^8 **e.** 10^9 **f.** 10^6

Use digits to express each of these numerals.

3. Three billion, seven hundred fifty million
4. Two billion, seventy-six million, four hundred thousand
5. Eight billion, nine hundred five

Find a single numeral to replace N.

6. $(5 \times 10^8) + (8 \times 10^5) + (7 \times 10) + 6 = N$
7. $(4 \times 10^9) + (7 \times 10^7) + (6 \times 10^3) = N$
8. $(2 \times 10^8) + (9 \times 10) + (9 \times \frac{1}{10}) + (3 \times \frac{1}{10^2}) = N$

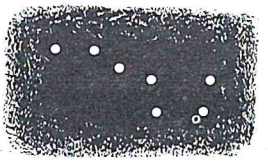
9. Suppose you are asked which represents the smaller number, $\frac{1}{4}$ or .28. Can you express $\frac{1}{4}$ as a decimal? Do it. Now tell which represents the smaller number.

10. Using the sign "<," write a sentence comparing the numbers in each of these pairs.

a. .47; $\frac{1}{2}$ **b.** 1.2; $\frac{11}{10}$ **c.** $\frac{180}{100}$; $1\frac{3}{4}$ **d.** $\frac{1}{25}$; .03

What is the distance around a rectangle or a square called? What is the amount of space inside a rectangle called?

Whenever you study figures and space, you are studying **geometry.** Let us look at some important ideas in geometry.



1. For a science project at school, Helen made "maps" to show how certain groups of stars look. At the left is her "map" of the Big Dipper. As you can see, Helen used dots to mark the locations of the stars. In geometry, we think of a location as a **point.** So we can say that each dot on Helen's "map" represents a location, or point, where a star appears. How many points are represented on the "map"?

2. Sometimes we use capital letters to name points. Below, we have marked two dots to represent points, and named them. One is named "point *A*." What is the other one named?

3. Can you think of a straight line that goes through *A* and *B*? Lay the edge of a ruler along the two dots if you need help in thinking of the line.

4. On a piece of paper, mark two points and label them X and Y. Can you think of a line that goes through these points?

Now draw a picture of the line you have thought of. Does your picture look like this?

5. In Exercises **3** and **4**, you have been thinking of lines and have drawn a picture of one line that you thought of. A line is an idea; the picture we make is a **model,** or interpretation, of a line. When you look at the edge of a ruler, do you think of a line? Then we say that the edge of a ruler is also a model of a line. Since we think of a point as a location, a point is also an idea.

What do we use as a model, or interpretation, for the idea of a point?

6. When you draw a picture of a line on a piece of paper, you have to stop at the edges of the paper. But can you think of the line as going on and on?

▶In geometry, we think of a line as going on and on in both directions.

7. To show that a line goes on and on in both directions, we draw arrowheads, as in the picture at the right. Two points on the line have been named. Using these points, we can name the line as "line *CD*." In writing, we can indicate "line *CD*" as $\overleftrightarrow{CD}$. We read this as "line *CD*."

How would you name the line you drew for Exercise **4**? Now show another way you can write "line *XY*."

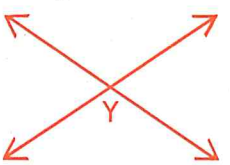
8.a. On a piece of paper, mark two points, *A* and *B*. Draw a line through *A* and *B*. Mark arrowheads on the drawing to show that the line goes on and on in both directions.

b. Mark dots for other points on the line. Label them *C*, *D*, *E*, *F*, and so on. Mark points until you use all the letters of the alphabet. Could you mark still other points?

When you make a mark for a point, are you making a kind of "rough picture" of a point? You know that we think of a point as a location, and you also know that a line goes on and on in both directions. Is there any limit to the number of points on a line?

▶In geometry, we may think of a line as a set of points.

9. Mark a point on your paper. Represent two lines that go through the point. Can you think of another line that goes through the point? two other lines? five other lines? Draw a few of these lines.



10. Mark two different points on your paper. Now draw a line that goes through these points. Does your drawing represent the only line that goes through these two points? We say that exactly one line goes through two points.

1. At the left is a picture of a line through two points, P and Q. Can you think of points at the right of Q? at the left of P? points between P and Q?

We call the part of the line from P to Q a **line segment** or a **segment.** We can name it "segment PQ" and write it as $\overline{PQ}$. A line segment consists of two points and the part of the line that is between these points. The two points at the ends are called **end points.** What are the end points of $\overline{PQ}$?

2. You know that a line does not have a beginning or an end. Then does a line have end points?

3. Does this picture show both a line, $\overleftrightarrow{AB}$, and a segment, $\overline{AB}$? Describe $\overline{AB}$.

4. One point, R, is marked on this picture of a line. The point R separates the line into two sets of points: the set of points at the right of R and the set of points at the left of R.

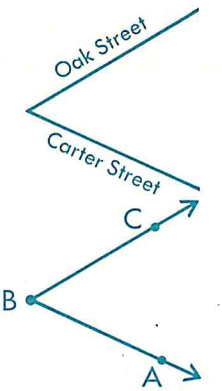
5. This picture shows a part of a line through R. Is this a picture of a line segment? Why not? As you can see, this part of a line consists of a point, R, and all the points at one side of R. We call such a figure a **ray.** A ray has one end point. What does the arrowhead indicate?

6. This picture shows a point, R, on a line. Two other points on the line have been given names. This helps to name a ray. We can name the ray consisting of R and all the points on the Y side of R as "ray RY," written $\overrightarrow{RY}$. Notice that the end point is named first.

Name the ray consisting of R and all the points on the X side of R. Make a drawing of $\overrightarrow{RX}$.

7. When you speak of a ray of sunlight, you are using "ray" in the geometric way. The sun is the end point. If you think of a searchlight beam as a ray, what is the end point?

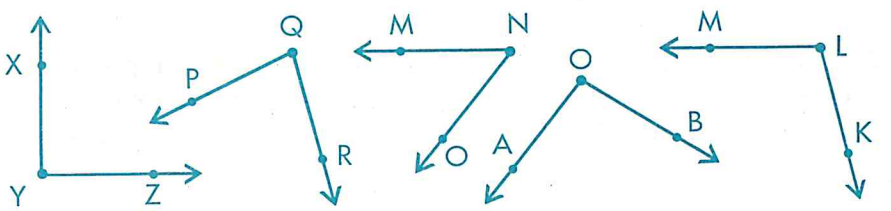
You have probably heard the word "angle" used many times in your life. For instance, people speak of the "angle" formed by two streets that meet.



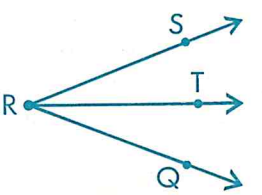
1. In geometry, we say an **angle** is a set of points consisting of two rays with a common end point. The common end point is called the **vertex** of the angle, and the rays are the **sides.**

The angle represented in this diagram is formed by the rays *BA* and *BC*. What is the common end point? We can name this angle "angle *ABC*" or "angle *CBA*." Notice that we always use the vertex letter in the middle when we name an angle. Sometimes, when there is no possible confusion about an angle, we can use just the letter at the vertex as the name of the angle. For instance, we can describe this angle as "angle *B*."

2. Write three different names for each of the angles shown below.

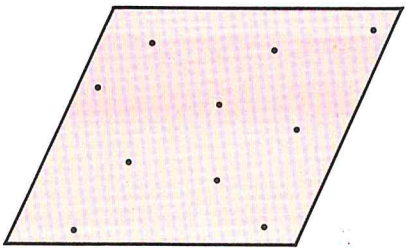


3. At the right is a diagram showing three rays with a common end point. Can you see that there are three angles that you can name? One of these angles is angle *QRS*. Name the other two, using three letters to name each of these angles.



If someone spoke of angle *R* in this diagram, would you have any way of knowing which of the three angles was meant? In such a case, it is important to use three letters to name the angle you mean.

You know that a sheet of glass or a sheet of paper suggests a flat surface. Now imagine a flat surface that extends outward in all directions without any limit. In geometry, this idea is called a **plane**. We can use sheets of paper to help us think of planes.



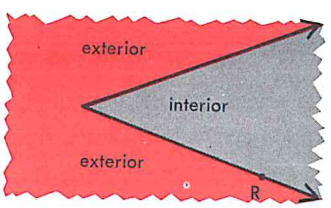
1. On a sheet of paper, mark several points. Can you mark many points on the paper? You know a plane extends in all directions without any limit. Do you see that the number of points in a plane is unlimited?

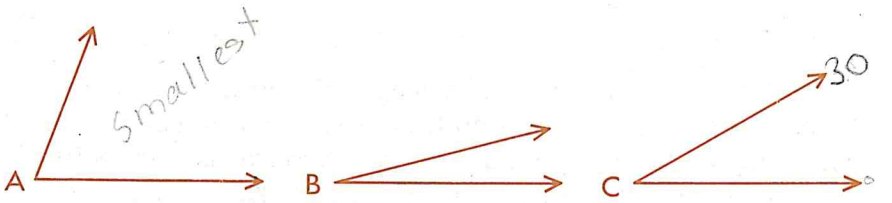
2. Choose any two of the points you marked, and label them A and B. Remember that your sheet of paper represents a part of a plane that extends without limit.

Draw a line that passes through points A and B. Mark a point C on that line; a point D; a point E. Is point C in the plane of the paper? point D? point E? Do you think all the points on the line are in the plane of the paper?

3. You know that there are an unlimited number of points in a plane. Do you think there are an unlimited number of lines in a plane? an unlimited number of rays? an unlimited number of angles?

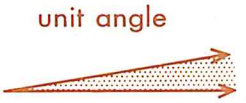
4. The picture below shows an angle and the plane in which it lies. Does an angle *separate* a plane into two sets of points? Notice that one part of the plane is labeled the *interior* of the angle and the other part the *exterior* of the angle. Is a point such as R in the interior of the angle? the exterior? We say it is *on* the angle. Is the angle itself a set of points? Then can we think of the plane in terms of three sets of points?

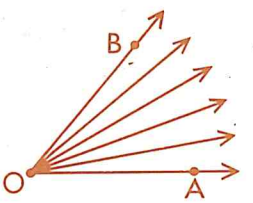




1. Shown above is a picture of three angles. Which would you say is the smallest angle? the largest angle? If you need help in deciding, think of these as representing pairs of scissors. Which angle is only slightly open? Which angle is open the widest?

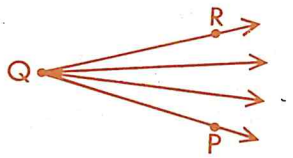
2. To measure an angle, we have to use a unit that is of the same kind. Suppose we choose a unit angle as shown below. Then, to find the measure of any other angle, such as angle *AOB*, we must find out how many of these unit angles are needed to "fill up" the interior of angle *AOB*. In the picture, we have subdivided the interior of angle *AOB* into angles that are unit angles. How many unit angles were marked off from ray *OA* to ray *OB*? How many unit angles is the measurement of angle *AOB*?

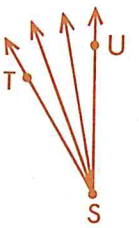


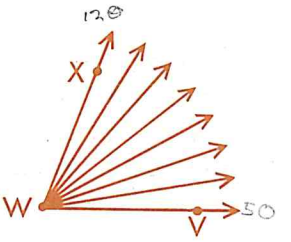


We say the *measure* of angle *AOB* is 5, and its *measurement* is 5 unit angles.

3. Shown below are three more angles whose interiors are subdivided into the unit angles chosen in Exercise **2**. In unit angles, give the measurement of angle *PQR*; of angle *UST*; of angle *VWX*.





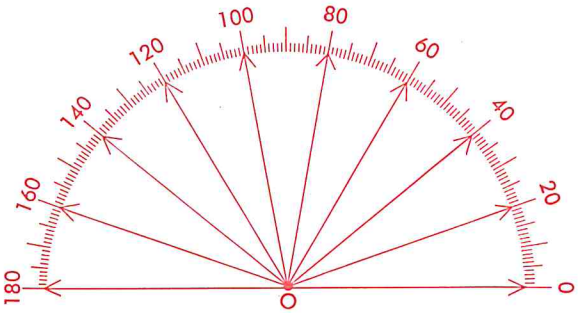


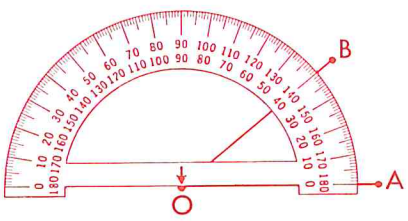
1. What do we mean by a *standard unit* of measure?

2. To find the measure of any angle, we need a standard unit of measure and a scale. Think of a line and a point O on it. Through O we can draw rays so that these rays, along with the rays on the original line, form 180 angles, all alike.

These 180 angles and their interiors "fill up" one side of the line. Now think of the rays numbered in order from 0 to 180, forming a scale. In the picture, only some of the rays are shown. The others are suggested by the little marks. Are the ray that is numbered 0 and the ray that is numbered 180 on the same line?

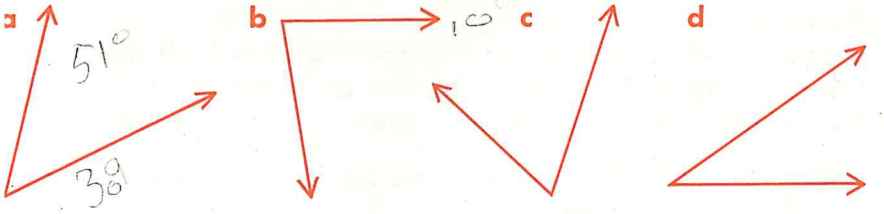
As a standard unit of measure, we use one of the 180 small angles. Its measurement is called **one degree,** written **1°**. We may also say that its measure is 1.



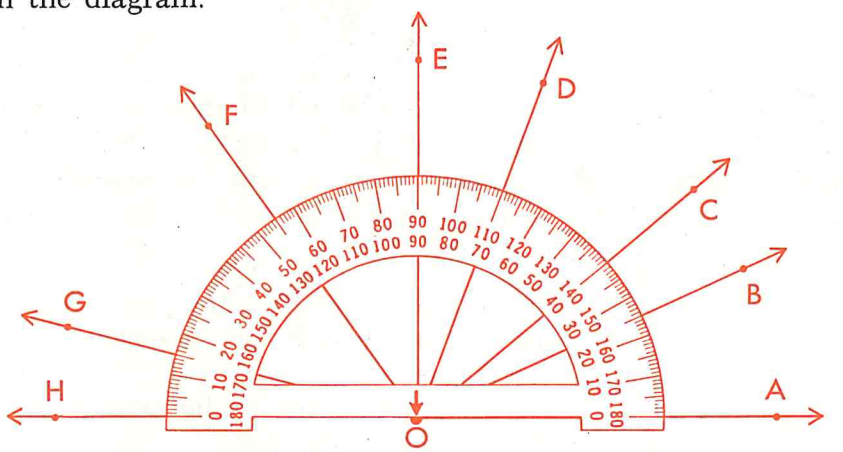


3. You have probably seen a special instrument that is used in measuring angles—the *protractor*. In constructing the scale in Exercise **2**, we were actually illustrating the idea of a protractor. The unit of measure on the protractor is the degree. To find the measure of an angle, place the arrowhead of the protractor at the vertex, as shown, and the 0 line along one ray of the angle. Where does the other ray of the angle cross the scale? What numeral appears on the scale there? What is the measurement of the angle?

1. Use a protractor to find the measurement of each of the angles shown below.



2. Below, you see a protractor that has been placed on a set of rays. All of the rays have the same end point, *O*, and the arrowhead of the protractor is at *O*. Name five different angles in the diagram.



3. Each of the angles named below is shown in the diagram. What is the *measurement* of each?

a. angle *AOB* **b.** angle *AOC* **c.** angle *AOE* **d.** angle *AOG*

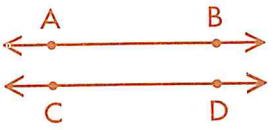
4. Suppose you are asked to find the *measure* of angle *DOE*. At what numeral does ray *OD* cross the scale? At what numeral does ray *OE* cross the scale? Then do you think the measure of angle *DOE* is 20?

5. What is the measure of each of these angles?

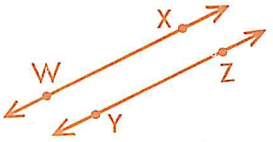
a. angle *EOF* **b.** angle *FOG* **c.** angle *AOG* **d.** angle *AOH*

1. Use a sheet of paper as a model of a plane and draw two lines that cross, or **intersect.** In how many points do they intersect?

You know that a line goes on indefinitely in both directions. But no matter how far these lines go on in the plane, will they intersect in more than the one point that you have shown?



2. Now try to imagine two lines that will not intersect at all. If you need help in imagining this, think of railroad tracks that go on and on in straight lines. Two lines in the same plane that do not intersect are called **parallel lines.**

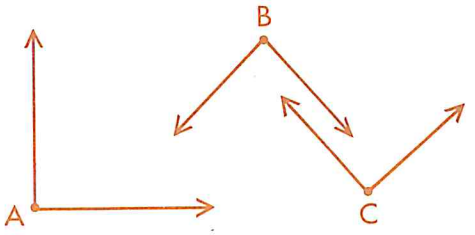


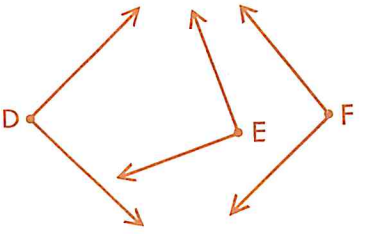
3. In this picture, two parallel lines, WX and YZ, are represented. It is sometimes convenient to speak of line segments as being parallel. By agreement, we say that line segment WX is parallel to line segment YZ because they lie on parallel lines. Could we say, in Exercise **2**, that segment AB is parallel to segment CD?

4. On pages 217–219, you studied a standard unit of measure and learned how to find the measurements of angles. To the nearest degree, what is the measurement of the angle shown here?

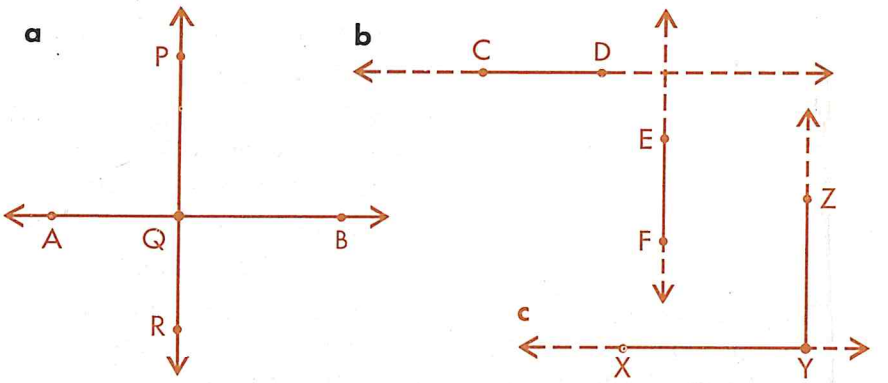
▶An angle of 90° is called a **right angle.**

5. Which of the angles shown below are right angles? Which are not?





6. As you know, an angle is formed by two rays with a common end point. However, we can also speak of the angles formed by intersecting lines or line segments. For example, in **a** below, lines PR and AB intersect in point Q, and we can think of the rays QP and QB as forming an angle. Can you name three other angles that are shown in **a**?

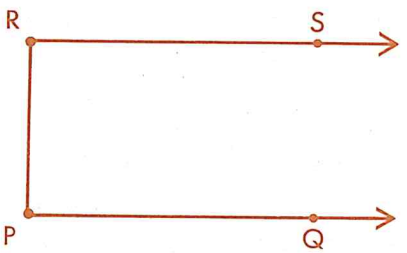


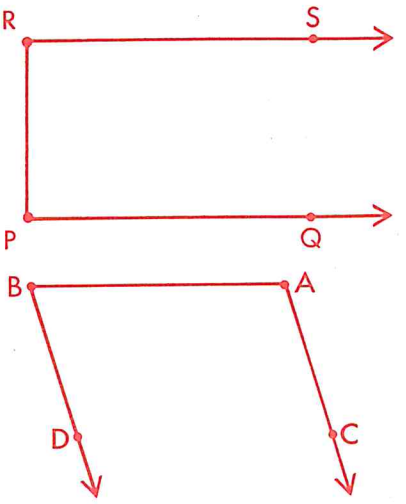
In each diagram, find the measurement, correct to the nearest degree, of one of the angles formed.

In each case, when you chose an angle and measured it, did you find the measurement of the angle to be 90°? In **b**, do segments EF and CD intersect? Do the lines of which they are parts intersect?

▶Two intersecting lines are said to be **perpendicular** if the angles formed at the point of intersection are right angles. We say that rays, or line segments, are perpendicular if the lines on which they lie are perpendicular.

7. In the diagram at the right, what is the measurement of angle PRS? of angle RPQ? Then is $\overrightarrow{PQ}$ perpendicular to $\overline{PR}$? Is $\overrightarrow{RS}$ perpendicular to $\overline{PR}$? Notice that we say a ray is perpendicular to a segment. Is it true that $\overleftrightarrow{PR}$ is perpendicular to $\overleftrightarrow{PQ}$? to $\overleftrightarrow{RS}$?

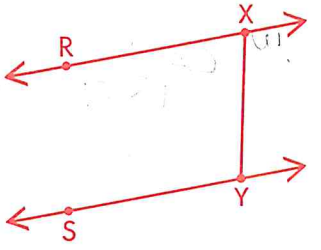


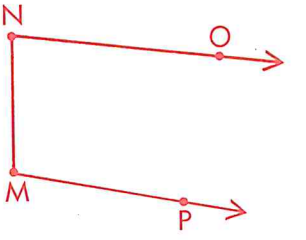


1. At the left is the diagram used in Exercise **7**, page 221. You have found that each of the rays is perpendicular to $\overline{PR}$. What is the sum of the measures of the angles PRS and RPQ? Are $\overrightarrow{RS}$ and $\overrightarrow{PQ}$ in the same plane? Does $\overrightarrow{RS}$ seem to be parallel to $\overrightarrow{PQ}$?

2. In the second diagram, what is the measure of angle BAC? of angle ABD? What is the sum of their measures? Do $\overrightarrow{BD}$ and $\overrightarrow{AC}$ seem to be parallel?

3. Think of $\overrightarrow{BD}$ and $\overrightarrow{AC}$ as parts of railroad tracks, and $\overline{AB}$ as a railroad tie. Do both of the angles that you measured lie on the same side of the tie and between the tracks?



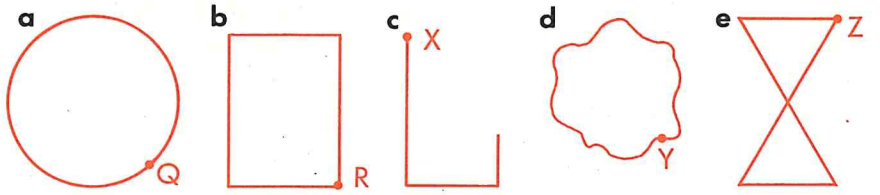


4. In the diagram at the left, above, are angles XYS and YXR in positions similar to the positions of the angles measured in Exercise **2**? Find the measures of angles XYS and YXR. What is the sum of their measures? Do $\overrightarrow{XR}$ and $\overrightarrow{YS}$ seem to be parallel?

▶For two angles in a position similar to the position described in Exercise **3**, let us make the following agreement. If the sum of their measures is 180, then two of the rays lie on parallel lines. If the sum of their measures is not 180, then the rays are not on parallel lines.

5. In the diagram at the right, above, is $\overrightarrow{NO}$ parallel to $\overrightarrow{MP}$? Use the above agreement to help you decide.

1. In geometry, any set of points in a plane is called a **plane figure.** Below are pictures of five sets of points in a plane—that is, five plane figures.

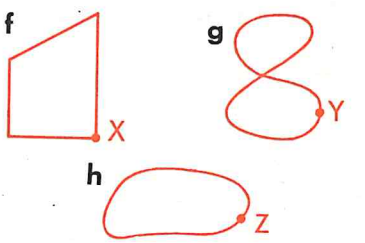


Study Figure **a**. In drawing this figure, could you begin at point Q, draw the figure, and end at Q again—*without* lifting your pencil from the paper and without retracing any part of the figure? We call such a figure a **closed figure.**

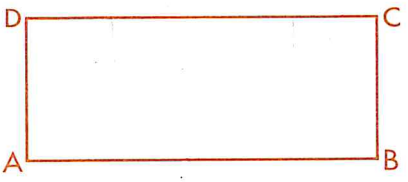
Study Figure **b**. Could you begin at *R*, draw the figure, and end at *R* again—without lifting the pencil or retracing any part of the figure? Is Figure **b** a closed figure?

Can Figure **c** be drawn in the same way (beginning and ending at X)? Then Figure **c** *is not* a closed figure. Is Figure **d** a closed figure? Figure **e**?

2. When a closed figure does not intersect itself, we call it a **simple closed figure.** Which of these drawings represent simple closed figures? Which does not represent a simple closed figure?



3. Does each of the drawings below represent a simple closed figure? Can we think of each figure as consisting of line segments joined together, end to end? A simple closed figure that consists only of line segments is called a **polygon.** So each of the drawings represents a polygon. Any polygon with four sides is a **quadrilateral.** Which of the figures on this page are quadrilaterals?



1. In earlier grades and in this book, you have worked with rectangles. Now, however, let us study the rectangle from the point of view of geometry. Do you think there are some things that are true of all rectangles? In geometry, we call these things **properties** of a rectangle.

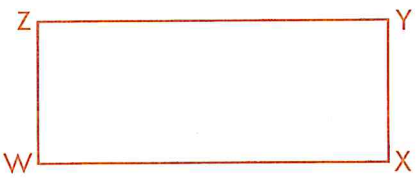
a. The rectangle shown above consists of line segments AB, BC, CD, and DA. These line segments are called the sides of the rectangle. How many sides does a rectangle have? Is every rectangle a quadrilateral?

We can name the rectangle above as "rectangle $ABCD$." Sides AB and CD are called opposite sides. Is there another pair of opposite sides? Name them.

b. Measure angles A, B, C, and D. Is each a right angle?

c. Since angles A and D are right angles, what is the sum of their measures? Then are $\overline{DC}$ and $\overline{AB}$ parallel?

d. What is the sum of the measures of angles A and B? Then are $\overline{DA}$ and $\overline{CB}$ parallel?



2. Shown here is another rectangle. Check to see if all its angles are right angles. Are the opposite sides parallel? How do you know?

▶ Is a rectangle a quadrilateral whose opposite sides are parallel and whose angles are right angles?

3. In rectangle $ABCD$, find the lengths of the opposite sides $\overline{AB}$ and $\overline{DC}$. Are the lengths equal? Now find the lengths of the other pair of opposite sides. Are these lengths equal?

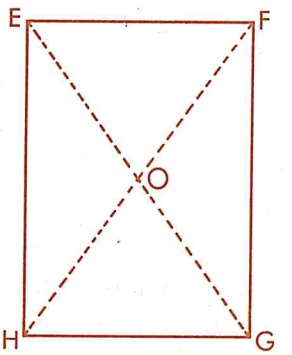
4. Repeat the procedure of Exercise **3** with the pairs of opposite sides in rectangle $WXYZ$.

▶ Is it true that the lengths of opposite sides of a rectangle are equal?

5. Can the vertex of an angle be described as the point at which the two sides of the angle meet? Look at rectangle $WXYZ$ in Exercise **2**. Is point Z the vertex of one angle of the rectangle? Name the vertex of another of the angles of this rectangle.

The four points, W, X, Y, and Z, are called **vertices** (pl. of *vertex*) of the rectangle. What are the four vertices of the rectangle in Exercise **1**?

6. Use a thin sheet of paper and carefully trace the drawing of rectangle $EFGH$. Now draw two line segments, EG and FH, as suggested by the dashes. Each of the line segments EG and FH is called a **diagonal** of the rectangle. A diagonal is a segment that joins two vertices but *is not* a side of the figure.



Now find the lengths of the diagonals. Are the lengths equal?

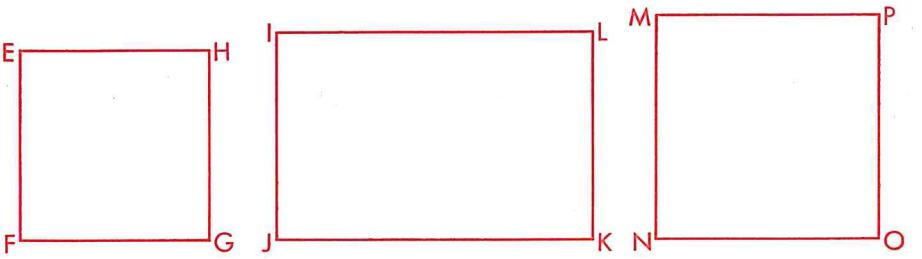
7. Trace rectangle $ABCD$ (Exercise **1**). Then draw the diagonals and measure them. Are they equal in length? Do you think that the diagonals of rectangle $WXYZ$ (Exercise **2**) are equal in length? Measure them and see. Look at your work for Exercises **5–7** and then tell whether you agree with the following statement.

▶ The diagonals of a rectangle are equal in length.

8. You know that a rectangle is a figure that lies in a plane. Is it a simple closed figure? Does it have an interior? an exterior? In Exercise **6**, the point where $\overline{EG}$ and $\overline{FH}$ intersect has been named O. How would you describe where point O lies? Now look at point H. Would you say it is in the interior of the rectangle? Is it *on* the rectangle?

9. Draw a rectangle and mark two points that lie in the part of the plane exterior to the rectangle. Mark two points that lie in the interior of the rectangle. Mark two points that lie on the rectangle.

1. a. Shown below are three polygons. Measure the angles of each. Are they all right angles? Are the opposite sides of each figure parallel? Then is each figure a rectangle?
b. Measure the sides of each figure. Is each a square? Which is not a square?



▶ Do you agree that we can describe a square as a rectangle all of whose sides have the same length?

2. In a rectangle, is each pair of intersecting sides perpendicular? (For instance, in rectangle *IJKL*, above, is $\overline{IL}$ perpendicular to $\overline{IJ}$?) Give a reason for your answer.

Since a square is a rectangle, is this true for a square?

3. You know that a rectangle has diagonals that are equal in length. Since a square is a rectangle, are the lengths of its diagonals equal?

Trace the squares *EFGH* and *MNOP*, above. Draw the diagonals and measure them. Are they equal in length for each square?

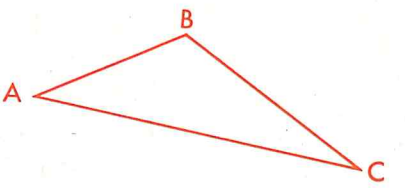
4. a. Now look at the diagonals $\overline{FH}$ and $\overline{EG}$. Do they seem to be perpendicular? Use your protractor to measure an angle formed by these intersecting segments. What do you find?

Measure one of the angles formed by the intersecting diagonals $\overline{MO}$ and $\overline{NP}$. Are these diagonals perpendicular?

b. Trace rectangle *IJKL* and draw the diagonals. Are they perpendicular? Draw another rectangle that is not a square and see whether its diagonals are perpendicular.

Does a square have all of the properties of a rectangle and other properties that a rectangle *does not* have?

1. Shown at the right is triangle *ABC*. How many sides does a triangle have? How many vertices? Is a triangle a simple closed figure? a polygon? Explain.



Does a triangle have an interior and an exterior?

2. Draw a triangle in which one angle is a right angle. Such a triangle is called a **right triangle.**

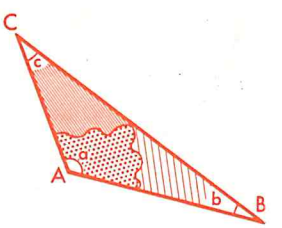
Now try to draw a triangle with two right angles. What do you find?

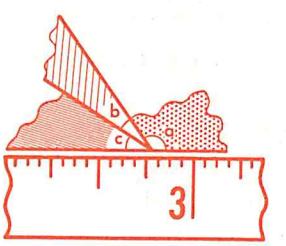
Can a triangle have one angle with a degree measure that is greater than 90? Draw a diagram.

3. Measure to the nearest degree each of the angles in triangle *ABC*. Find the sum of these measures. Compare the results with those of your classmates.

4. Draw a triangle and measure to the nearest degree each of its angles. What is the sum of the measures?

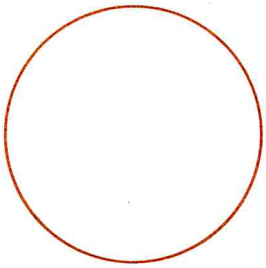
5. In Exercises **3** and **4**, you used your protractor to find the sum of the degree measures of the angles of a triangle. Now try this experiment: Draw a triangle on your paper. Shade the interior as in the diagram. Cut out the triangular region; then cut along the wavy markings so that you will have a part of the interior of each angle. Place the three parts at a mark of your ruler, as shown.





Do the pieces of paper that represent the measures of the angles form an angle whose measure is 180? Does this indicate that the sum of the degree measures of the angles of a triangle is 180?

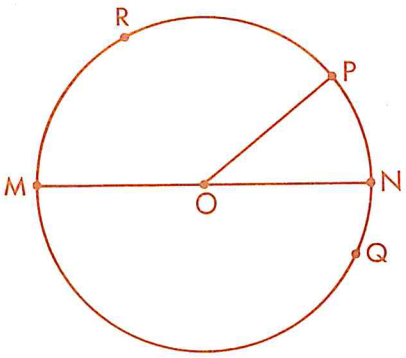
▶ Do you agree that for any triangle the sum of the degree measures of the angles is 180?



1. Like a rectangle, a square, or a triangle, a circle is a plane figure. Is a circle a simple closed figure? How do you know? Is it a polygon? Why not?

A circle is a special kind of simple closed figure. All of its points are the same distance from a fixed point in the same plane. The fixed point is called the **center.**

Does a circle have an interior? Is part of the plane exterior to the circle? In the diagram above, the center of the circle is not marked. Would you mark a point for the center *inside* the circle, *on* the circle, or *outside* the circle?



2. Shown here is a circle with center point O and two line segments that have a relationship to the circle.

a. One end point of OP is the center of the circle. The other is on the circle. Such a segment is called a **radius.**

Imagine a line segment $\overline{OQ}$. Is $\overline{OQ}$ a radius? Measure the distance from O to P; from O to Q. Are $\overline{OQ}$ and $\overline{OP}$ equal in length?

Imagine another segment, $\overline{OR}$. Find the distance from O to R. Is the length of $\overline{OR}$ equal to the length of $\overline{OP}$?

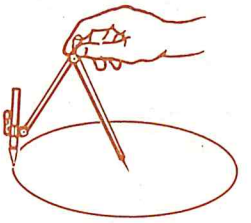
When we speak of all points on a circle as being "the same distance" from a fixed point, is the "distance" the length of a radius?

b. Think of other points on the circle. Can you imagine an unlimited number of points on a circle? Then does a circle have an unlimited number of **radii** (pl. of *radius*)?

3. $\overline{MN}$ is a **diameter** of the circle above. Any line segment that has its end points on the circle and passes through the center is a diameter. Can you imagine other diameters?

4. Is the length of a diameter twice the length of a radius?

5. We use a **compass** to draw a circle. One leg of the compass holds a pencil; the other leg has a sharp steel point. Push the steel point gently into the paper. Hold the paper with one hand; with the other hand, swing the pencil leg around.



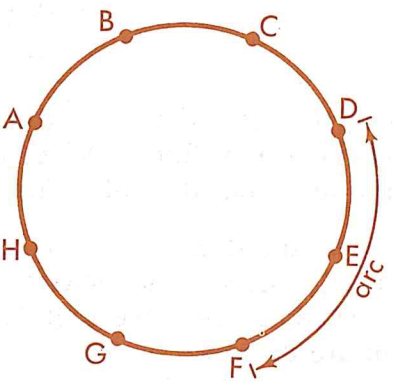
6. Open the compass to a distance of 1 inch. Draw a circle having a radius of 1 inch. Draw five diameters for this circle. Without measuring, tell what the length of each diameter is.

Can you imagine an unlimited number of diameters for a circle?

7. Draw a circle having a radius of:

a. $1\frac{1}{2}$ in. **b.** 2 in. **c.** $\frac{3}{4}$ in. **d.** 3 in.

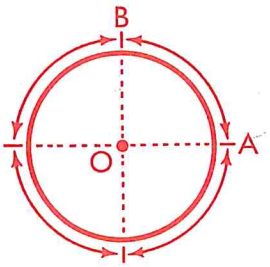
8. Now let us take another look at the circle; that is, the simple closed figure. This picture shows a circle with various points on it named. Using such points, we can talk about parts of the circle. A part of a circle is an **arc.** We name an arc by its end points and a point contained in the arc. For example, the arc indicated in the diagram is arc *DEF*.



What is the name of the arc that has *A* and *C* as its end points and contains point *B*; the arc that has *A* and *G* as its end points and contains point *H*?

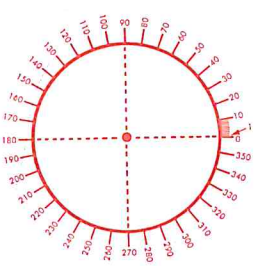
9. Look at the diagram for Exercise **2**. Diameter *MN* separates the circle into two special arcs. Each is a **semicircle.** One can be named arc *MRN*. Name the other semicircle.

1. Draw a circle and then draw a diameter for the circle. Fold the drawing along the diameter. Do the two semicircles have the same length?



2. Draw a circle and a diameter for the circle. Then draw a diameter perpendicular to the first diameter. Is the circle divided into four arcs of equal length?

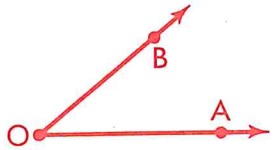
3. In the diagram, does angle AOB have the center of the circle as its vertex? Such an angle is called a **central angle.** Use the letters X, Y, and Z to name one of the central angles in your own diagram.



4. Now imagine a circle that is divided into 360 equal arcs. Each semicircle is divided into how many equal parts? In this drawing, the numerals show the marks for each ten arcs, beginning at 0. To measure arcs, we use a unit called an **arc degree.** An arc degree is $\frac{1}{360}$ of a circle; that is, the arc from 0 to 1 is a 1-degree arc. The arc from 0 to 10 is a 10-degree arc. Is the arc from 70 to 80 also a 10-degree arc? Name some other 10-degree arcs. What is the measure of the arc from 0 to 30? from 0 to 90?

5. In the diagram above, do you see a central angle that is a right angle? What is its measure in angle degrees?

Do the sides of the angle "cut off" an arc? What is its measure in arc degrees?

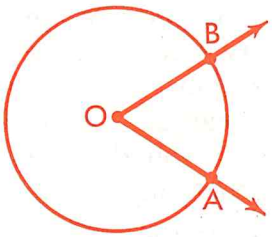


6. On a thin sheet of paper, draw an angle AOB of 40°. Then lay the drawing over the diagram in Exercise **4** so that O is at the center of the circle, and $\overrightarrow{OA}$ lies over the right-hand side of the diameter. What is the measure, in arc degrees, of the arc "cut off" between the sides of the angle?

7. Repeat the procedure of Exercise **6**, using an angle of 60°. What is the measurement, in arc degrees, of the arc cut off between the sides of the angle?

Since a central angle measured in angle degrees and its arc measure in arc degrees have the same number, we use the same symbol, (°), for an arc degree as for an angle degree.

8. Is angle *AOB* a central angle? The measurement, in angle degrees, of angle *AOB* is 65°. Then what is the measurement of arc *AB* in arc degrees? If the measurement of angle *AOB* were 85°, what would be the measurement of arc *AB*?

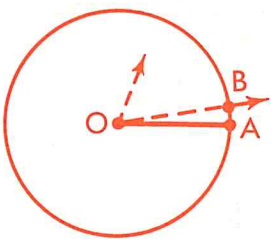


If the measurement, in arc degrees, of arc *AB* were 70°, would the measurement of angle *AOB* also be 70°? If the measurement of arc *AB* were 115°, what would be the measurement of angle *AOB*?

9. What is the measurement, in arc degrees, of one half of a circle? one fourth of a circle? one third? one sixth?

10. What is the measurement of the central angle corresponding to each of the arcs mentioned in Exercise **9**?

11. Draw a circle and mark radius *OA*. In the diagram, the measurement of arc *AB* is 10°. What is the measurement of angle *BOA*?



If you want to continue counterclockwise and draw an arc *BC* of 60°, can you do it by drawing an angle *BOC*? Explain. On your diagram, continue counterclockwise around the circle from *B* to mark and label arcs with the following measurements in arc degrees.

a. arc *BC*: 60° **b.** arc *CD*: 80° **c.** arc *DE*: 30°
d. arc *EF*: 25° **e.** arc *FG*: 90° **f.** arc *GH*: 65°

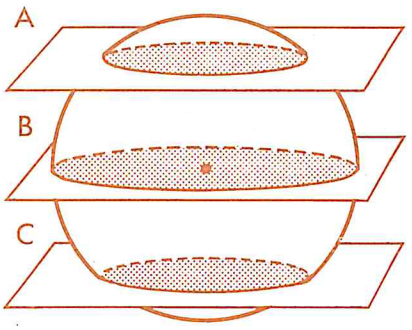
Do you find *H* and *A* to be the same point? What is the arc-degree measure of a circle?

1. The geometric shape suggested by a tennis ball or basketball is called a **sphere.** Do all of the points of a sphere lie in the same plane? We call a sphere a "space figure" or a geometric solid.

A sphere is a figure such that all of its points are the same distance from a given point, the **center.**

2. One half of a sphere is called a **hemisphere.** A good model of a hemisphere can be found on an ice cream scoop. Can you describe several uses of the word "hemisphere" in geography?

3. Is the surface of the earth approximately a sphere? Does the *axis* of the earth pass through the North Pole and the South Pole? Would you say that the center of the earth is a point *on* the axis, or *not on* the axis?



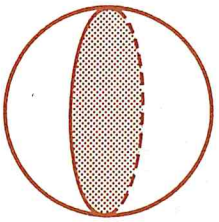
4. Look at the diagram. Imagine the horizontal plane, marked *A*, intersecting the sphere, somewhat the way a knife might cut through an orange. Do the plane and the sphere intersect in a circle?

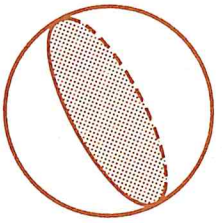
Imagine a plane, *B*, in a lower position where the center of the sphere is on the plane. Do the plane and the sphere intersect in a circle? But in what way is this circle different from the first circle? Now look at plane *C*. Do the plane and the sphere again intersect in a circle?

Which of the three circles is the largest?

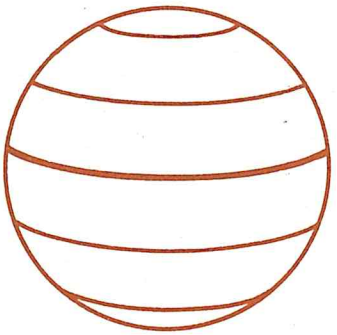
In geometry, when a plane passes through the center of a sphere, the circle that is formed on the sphere is called a **great circle.** Any other circle on a sphere is called a **small circle.**

5. The drawings below represent spheres. Each drawing shows the results when a plane is passed through the center of a sphere. Is the circle formed on the sphere? Does each plane divide the sphere in half?

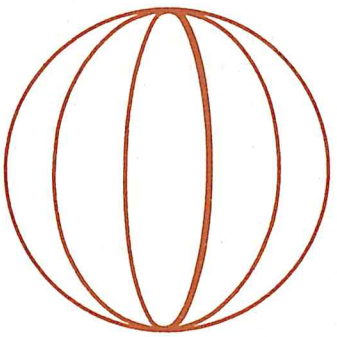




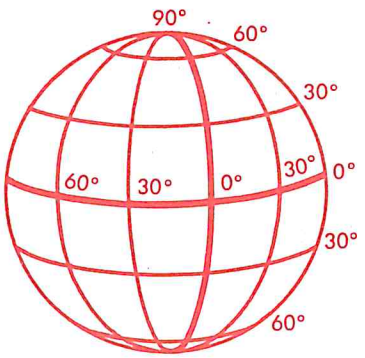
6. Are there many great circles on a sphere? Do you think there are many small circles on a sphere?



7. From earlier work in arithmetic and other subjects, you know that we locate places on earth by thinking of the earth as covered with two sets of circles. This drawing shows one set of circles. What is the name given to the great circle that is shown in this picture? All of the other circles are small circles. They are called **parallels of latitude.**



8. Each of the planes that form the second set of curves passes through the North Pole and the South Pole. Does each plane then pass through the center of the sphere? Then is each of these circles a great circle? Each half of such a circle is called a **meridian.** The one that passes through Greenwich, England is called the **prime meridian.**



You know that we speak of latitude and longitude in locating points on the earth. This picture shows both meridians and parallels of latitude.

1. How many arc degrees are there on the equator? Degrees measured east from the prime meridian are called **east longitude.** Degrees measured west from the prime meridian are called **west longitude.** What is the longitude of the prime meridian? What is the greatest east longitude measurement you can have? the greatest west longitude?

2. The equator is given a latitude of 0°. Each meridian is thought of as two arcs: one arc north of the equator and one arc south of the equator. Since a meridian is a semicircle, is each of these arcs one fourth of a circle, or 90 arc degrees? Where is 90° south latitude? 90° north latitude?

3. If the distance from the equator to the North Pole may be represented by 90 arc degrees, what distance is represented by 45 arc degrees?

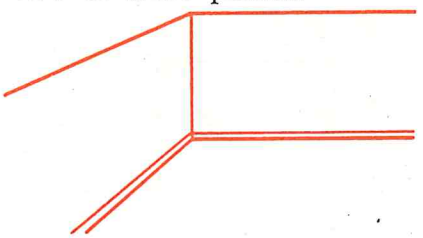
4. On a world map, find the parallel of latitude marked 20 that is north of the equator. Does any place that is located on this parallel have a latitude of 20° north? Now suppose you are told that a place is at 20° north latitude, 40° west longitude. Does this locate the place for you? We write this location as 20°N., 40°W.

5. Try this on your own. On a map, find the cities located approximately at each of the following places.

a. 30° N., 90° W. **b.** 40° N., 120° W. **c.** 20° N., 90° E.
d. 60° N., 20° E. **e.** 0° latitude 50° W. **f.** 20° N., 100° W.

In earlier work, you used models to help you think about the intersection of spheres and planes. Let us now use models to think about the intersection of two or more planes.

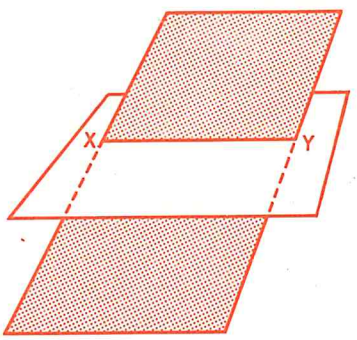
1. Think of one wall of your classroom as a model of a plane. Think of a neighboring wall as a model of another plane. Would you say that the two planes intersect in a line?



Remember that a model of a plane shows only part of a plane. The plane goes on and on indefinitely. Then is the figure formed by the intersection of the two planes a line segment, or is it a line?

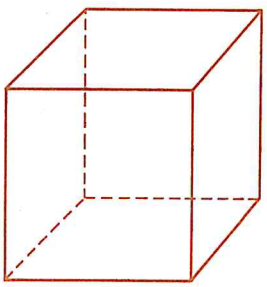
2. Think of the ceiling of your classroom as a plane. Think of one wall as another plane. What is the figure formed by the intersection of these two planes?

3. This drawing shows a model of two planes that intersect. The line in which they intersect is called XY. Do you see that line XY lies in both planes? We can say that each of the planes *passes through* line XY. To make a model for intersecting planes, you can use two index cards. On one card, draw line segment XY. You will use this segment to represent the line that it is a part of; that is, $\overleftrightarrow{XY}$. Make a slit along the line XY and insert your other card through the slit. You now have a model of two planes passing through a line.



Lift out the second card; then pass it through the slit so that it slants in a different way. Does it represent a different plane passing through $\overleftrightarrow{XY}$? By slanting this card in different ways, can you represent many different planes passing through the line? Are there an unlimited number of planes that may pass through a line?

As you know, any set of points in a plane is called a plane figure. A space figure, or solid figure, is also a set of points. However, not all of the points of a solid figure lie in the same plane.



1. This picture shows a cardboard carton. Each of its sides and the top and bottom are the same size.

Think of each side as a model of a square and its interior. Then does each side represent a part of a plane? Is the carton a model of a special kind of space figure? What is the name of this figure?

2. In geometry, the "top" and "bottom" of such a figure are often called its **bases.**

Study the diagram. Count the sides and the bases. How many are there altogether? Each is called a **face** of the cube.

3. Look at a pair of faces that intersect. Since the faces are parts of planes, what is the figure formed where two faces intersect—a line or a line segment?

4. Count the number of line segments in which two faces intersect. How many are there? Each such segment of the cube is called an **edge** of the cube.

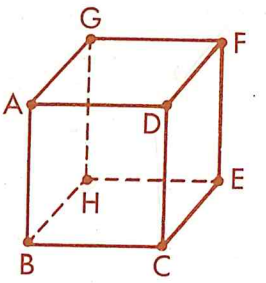
5. Find a point where two edges intersect. Is there a third edge that intersects the other two in the same point? Can we think of this point as a corner point of the cube? We call such a point a **vertex** of the cube. How many vertices does a cube have?

6. In geometry, "solid" has a special meaning. A cube is a geometric solid. But the space inside a cube is not a part of the cube. It is only the "skin" that forms a geometric solid. For example, a cube is formed only by parts of six planes.

Since each face of a cube consists of a square and its interior, we say that each face is a **square region.** How many square regions form a cube?

7. In this diagram, eight points on the cube have been labeled so that we can speak about various edges and angles.

What do we call any one of these points? more than one of these points?



8. Is angle ABC a right angle? How do you know? Are there other right angles in a cube? Name as many as you can.

9. How many right angles can you find in any one face? Then how many right angles are there in a cube? If you did not name this many angles in your answer for Exercise **8**, find and name those you missed.

10. Is it true that any two edges of a cube that intersect are perpendicular?

11. Do the edges $\overline{AB}$ and $\overline{CD}$ intersect? Is $\overline{AB}$ parallel to CD? How do you know? Name at least three other pairs of edges that are parallel to each other.

12. Suppose the length of $\overline{BC}$ is 3 feet. What is the area of face $ABCD$?

Are all the edges of a cube the same length? Then what is the area of each of the other faces? What is the sum of the areas of all of the faces? This is called the **total area** of the cube.

13. If the length of $\overline{BC}$ had been 4 feet, what would have been the area of face $ABCD$? Since you know the area of one face, can you use the sentence below to find the total area? Explain.

$$6 \times 16 = \square$$

14. Each of the following measurements represents the length of an edge of a cube. What is the total area of the cube in each case?

a. 10 ft. **b.** $1\frac{1}{2}$ ft. **c.** $2\frac{1}{2}$ in. **d.** 7 yd. **e.** 16 ft.

EXTRA PRACTICE 303–306

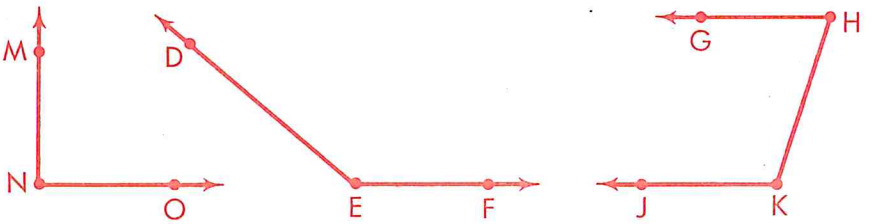
If you make any mistakes, study the pages given at the right of the exercises.

1. How many lines can be drawn through one point? through two different points? 213

2. Represent angle *RST* on your paper. What letter is at the vertex? Name the two rays that form the sides. 215

3. Shade the interior of angle *RST*. Mark a point Q that is in the exterior. 216

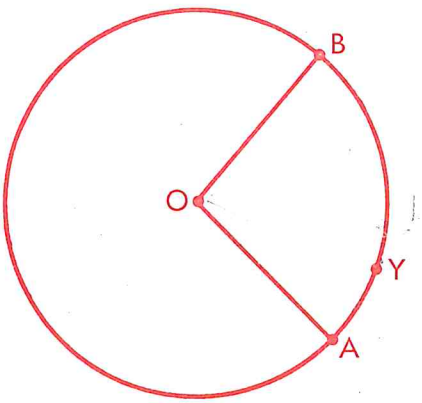
4. Find the measurement of each of these angles. 218



5. Which angle is a right angle? Name the two rays in that diagram that are said to be perpendicular. 220–221

6. In how many points may two different lines intersect? Do two lines in a plane always intersect? 220

7. Using the diagram, name a pair of parallel lines. 222



8. What is the degree measure of central angle *BOA*? What is the arc-degree measure of arc *BYA*? 230–231

9. Find the arc-degree measures of the arcs *BY* and *YA* that form arc *BYA*. 230–231

10. How many faces does a cube have? What is the total area of a cube if each edge is 8 feet long? 236–237

DISCOVERING MORE ABOUT NUMBERS: PAGES 332–334

Per Cent

8

The picture shows a farm on which strawberries are raised. Read the sign that appears in the picture. "25%" is read "twenty-five per cent" and means 25 parts per 100 parts. Anyone who wishes to help pick strawberries on this farm is not paid in money but is allowed to keep 25 baskets of berries per 100 baskets he picks.

1. The boys and girls in the picture plan to have a booth at their school fair where they will sell the strawberries they are allowed to keep.

a. If they pick 100 baskets, how many do they keep?
b. If they pick 200 baskets, how many do they keep?
c. On the first day, they picked 500 baskets of berries. How many did they keep?

2. On the second day, the owner of the farm told the children that they could keep 30% of the berries they picked. How many baskets could they keep per 100 picked? On this day they picked 200 baskets. How many did they keep?

3. The Midville Boy Scouts agreed to deliver handbills to 60% of the houses in town. This means they will deliver handbills to 60 houses per 100 houses in Midville. Suppose there are 1,000 houses in town. To how many houses will the Boy Scouts deliver handbills?

1. If 25% means 25 parts per 100 parts and 60% means 60 parts per 100 parts, what does 90% mean? 20%? 5%? 1%? 100%?

2. Does a score of 85% mean 85 points out of 100 points? What does a score of 90% mean? 95%? 100%?

3. Another way to look at a score of 85% is $\frac{85}{100}$ of the total number of points that could be made. If you look at the other scores in this way, what does 90% mean? 95%? 100%?

4. Does each of your answers to Exercise **3** have 100 as the denominator? Express each as a decimal numeral.

5. a. This store is advertising games and toys at "20% off." This means that for a game that originally cost \$1 you pay 20 cents less. At this rate, how much less would you pay for a game that originally cost \$5? **b.** Does 20 cents per dollar mean the same as 20 cents per 100 cents? Does it mean the same as $\frac{20}{100}$ of a dollar? What does $\frac{20}{100} \times 5$ equal? Do you get the answer that you did in **a**? Can you use the decimal .20 instead of $\frac{20}{100}$? Try it and see: $.20 \times 5 = \square$.

6. At the same sale, John bought a chemistry set. The original price was \$10. At 20% off the original price, how much did he save by buying the set on sale?

$\frac{20}{100} \times 10 = \triangle$

Could you find the answer by using a decimal? Do it.

7. You have seen that we can think of 20% as $\frac{20}{100}$ or .20. How would you express 27% as a common fraction and as a decimal?

8. Express each of the following as a common fraction with a denominator of 100; then as a decimal.

a. 39%	**b.** 47%	**c.** 59%	**d.** 73%	**e.** 91%
f. 45%	**g.** 70%	**h.** 25%	**i.** 63%	**j.** 10%

9. a. Miss Adams is principal of an elementary school. One day she said that 10% of the pupils were absent. If the school has 800 pupils in all, how many were absent?

Since 10 pupils per 100 pupils were absent, do we know that $\frac{10}{100}$ of all the pupils were absent? Then, to find the number of pupils that were absent, you can use this sentence: $\frac{10}{100} \times 800 = \diamond$.

b. Is $\frac{10}{100}$ equivalent to $\frac{1}{10}$? What does $\frac{1}{10}$ of 800 equal? Do you get the same answer as in **a**?

$$\frac{1}{10} \times 800 = \diamond$$

10. a. One snowy day Miss Adams said that 25% of the pupils were absent. How many pupils were absent?

b. Is $\frac{25}{100}$ equivalent to $\frac{1}{4}$? Then can you use the sentence at the right to work the example in another way? Try it.

$$\frac{25}{100} \times 800 = \triangle$$

$$\frac{1}{4} \times 800 = \triangle$$

11. Do you think we can express 25% as .25 or $\frac{25}{100}$ or $\frac{1}{4}$? What are three ways to express 10%?

12. Suppose you are told that 50% of the pupils in Miss Adam's school belong to clubs. Does this mean that $\frac{50}{100}$ or .50 of the pupils belong to clubs? What is $\frac{50}{100}$ expressed in simplest form?

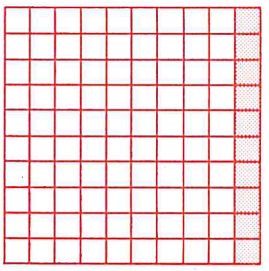
Then, to find the number of pupils that belong to clubs, can you use this sentence: $\frac{1}{2} \times 800 = \square$? Do it.

13. Suppose the school had only 650 pupils. What is 50% of this number?

Is 50% of 650 the same as $\frac{50}{100} \times 650$? as $\frac{1}{2} \times 650$?

14. If a store is having a "50% off" sale, how much would a person save on a book priced at \$4? \$6? \$5? The saving would always be what fractional part of the original price? Give your answer as a common fraction in simplest form.

▶ Do you see that we can express per cents as common fractions and as decimal fractions?



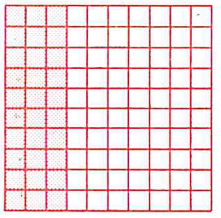
1. The square region shown at the left is divided into 100 small square regions. Each of the small regions is what fractional part of the large square region? Express your answer both as a common fraction and as a decimal.

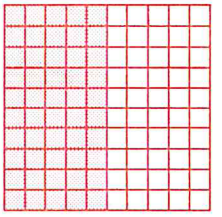
What per cent of the large square region is a small square region?

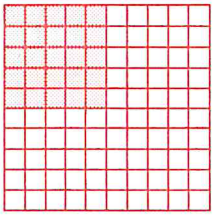
a. How many small square regions are shaded? How many hundredths of the large square region are shaded? Then what per cent of the large square region is shaded?

b. How many hundredths of the large square region are *not* shaded? Then what per cent of the large region is not shaded?

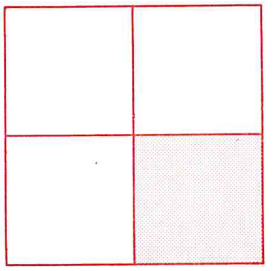
2. Tell what per cent of each of the following diagrams is shaded and what per cent is not shaded.

a 

b 

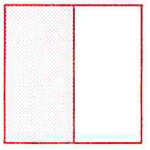
c 

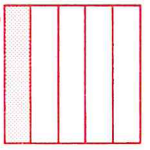
3. In each diagram above, does the per cent for the shaded part plus the per cent for the unshaded part equal 100%?

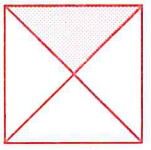


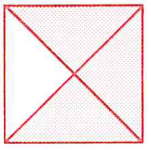
4. What fractional part of this diagram is shaded? Can you express $\frac{1}{4}$ as an equivalent fraction with a denominator of 100? Do it: $\frac{1}{4} = \frac{\square}{100}$. What per cent of the diagram is shaded?

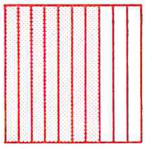
5. For each diagram below, tell the per cent that is shaded.

a 

b 

c 

d 

e 

1. Jim took a test and said that he answered correctly 4 of each 5 questions. Does this statement tell you *how many* questions Jim answered correctly? But can we say that the ratio of the number of questions he answered correctly to the number of questions on the test was "4 to 5"?

2. How many questions did Jim answer correctly if there were 10 questions on the test? 20 questions on the test? 50 questions on the test? 100 questions on the test? Copy and complete the table below.

Number of questions answered correctly	4	?	?	?	?
Number of questions on the test	5	10	20	50	100

3. Find the quotient for each example. Express your answer as a common fraction in simplest form.

a. $4 \div 5 = \square$ **b.** $8 \div 10 = \square$ **c.** $80 \div 100 = \square$

Do you get the same answer in each case? Does the quotient, $\frac{4}{5}$, mean that the number of questions answered correctly is always $\frac{4}{5}$ of the total number of questions?

4. Think of the pairs of numbers 4 and 5, 8 and 10, 16 and 20, and so on. You know that each pair has the same ratio. Then does "80 to 100" express the same idea as "40 to 50" or "4 to 5"?

5. When we speak of "80%," do we mean "80 per 100"; that is, "80 to 100"? Do you see that we can interpret a per cent as a ratio?

6. As you have seen, we can also think of 80% as $\frac{80}{100}$. What decimal numeral can be used to express 80%?

7. Express 65% as a ratio. Express 12% as a ratio. What fraction with a denominator of 100 do you use to express 65%? 12%? Express each as a decimal fraction.

8. Express each of the following ratios as a per cent.

a. 3 to 5 **b.** 7 to 10 **c.** 9 to 20 **d.** 40 to 50

1. Of the 100 words on a test, Bob spelled 86 words correctly. His score was $\frac{\square}{100}$, or $\square\%$.

2. On a 50-word spelling test, Jane had 45 words correct. Did she have $\frac{45}{50}$ of the test correct? Since $\frac{45}{50}$ is equivalent to $\frac{\triangle}{100}$, Jane's score was $\triangle\%$.

3. On a geography test, Harold gave the correct answers to 19 of the 20 questions. What was his score expressed as a per cent? Use the sentence below to find the answer.

$$\frac{19}{20} = \frac{\diamond}{100}, \text{ or } \diamond\ \%$$

4. Helen has \$20. Each dollar represents $\frac{1}{20}$ of her money. What fraction with a denominator of 100 is equivalent to $\frac{1}{20}$?

$$\frac{1}{20} = \frac{\square}{100}$$

Then each dollar represents what per cent of the total amount of money that Helen has?

5. Helen spent \$15 of her \$20. Did she spend $\frac{15}{20}$ of her money? Since $\frac{15}{20} = \frac{\triangle}{100}$, what per cent of her money did Helen spend?

6. During vacation, Larry earned \$50. He saved \$37, or $\frac{37}{50}$ of what he earned. Since $\frac{37}{50} = \frac{\diamond}{100}$, what per cent of his earnings did Larry save?

7. How many dollars did Larry spend? What per cent of his earnings did Larry spend?

8. Express as per cents.

a. $\frac{26}{50}$ **b.** $\frac{4}{25}$ **c.** $\frac{5}{10}$ **d.** $\frac{2}{5}$

9. You know that 20% can be expressed as $\frac{20}{100}$ or .20 or $\frac{1}{5}$. How would you express 15% as a fraction with a denominator of 100? as a decimal? as a fraction in simplest form?

10. Express each of the following as a decimal fraction and as a common fraction in simplest form.

a. 35% **b.** 18% **c.** 12% **d.** 26% **e.** 75%

Express each of the following hundredths as a per cent.

	a	b	c	d
1.	$\frac{99}{100}$	$\frac{50}{100}$	$\frac{30}{100}$	$\frac{17}{100}$
2.	$\frac{1}{100}$	$\frac{3}{100}$	.65	.98
3.	.02	.10	.05	.04

For each of the fractions below, write an equivalent fraction with a denominator of 100; then write the equivalent decimal numeral; then express it as a per cent.

4.	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{9}{20}$
5.	$\frac{3}{50}$	$\frac{3}{10}$	$\frac{1}{4}$	$\frac{2}{5}$
6.	$\frac{17}{25}$	$\frac{1}{25}$	$\frac{3}{4}$	$\frac{49}{50}$

Express each of the following per cents as a decimal.

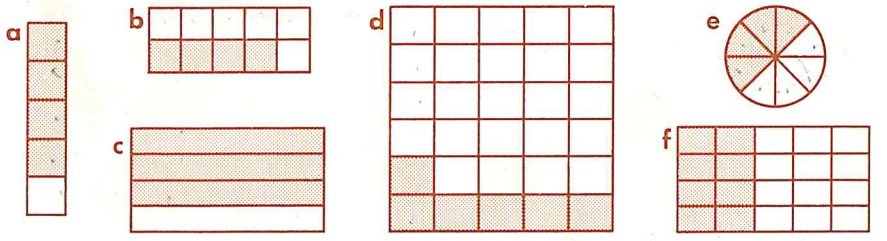
7.	19%	12%	33%	6%
8.	39%	17%	73%	9%

Express each per cent as a common fraction in simplest form.

9.	95%	4%	50%	45%
10.	40%	2%	20%	80%

11. Express each of the per cents in Exercises **9** and **10** as a decimal.

12. For each of the diagrams below, tell what part is shaded. Use a fraction in simplest form; then a common fraction with a denominator of 100; then a decimal; then a per cent.



1. To express $\frac{3}{20}$ as a per cent, you begin by finding an equivalent fraction with a denominator of 100. Do you think: "What number times 20 equals 100?" Then what do you do to find the missing numerator? Is the procedure easy because 20 is a factor of 100?

$\frac{3}{20} = \frac{\square}{100}$, or $\frac{3}{20} = \frac{\triangle}{100}$

Can you easily express each of these fractions as an equivalent fraction with 100 as a denominator: $\frac{1}{4}$, $\frac{11}{50}$, $\frac{7}{25}$? In each case, is the denominator a factor of 100?

2. Now suppose you want to express $\frac{7}{8}$ as a per cent. Is 8 a factor of 100? (Can 100 be divided evenly by 8?) Then let us find another way to express $\frac{7}{8}$ as a per cent.

Can you quickly express any decimal as a per cent? For instance, can you name the per cent that is equivalent to .39? .56? .85? Do it.

Then, if you find the decimal that is equivalent to $\frac{7}{8}$, you will be able to express $\frac{7}{8}$ in per cent form. You know that $\frac{7}{8}$ can be expressed as a decimal by dividing its numerator by its denominator. In the division example at the right, the work has been carried out to two decimal places. Then the remainder has been expressed as a fraction. The "$\frac{1}{2}$" means $\frac{1}{2}$ of a hundredth. Then is .87$\frac{1}{2}$ equivalent to 87$\frac{1}{2}$%? So $\frac{7}{8}$ is equivalent to 87$\frac{1}{2}$%.

$$\begin{array}{r} .87\frac{4}{8}, \text{ or } .87\frac{1}{2} \\ 8\overline{)7.00} \\ 64 \\ \hline 60 \\ 56 \\ \hline 4 \end{array}$$

3. a. To express $\frac{1}{3}$ as an equivalent decimal, which of these division examples will you use: $3\overline{)1.00}$ or $1\overline{)3.00}$?

Write the example on your paper and carry your work to two decimal places. Express the remainder as a fraction.

b. Copy, and fill in the missing numerals: $\frac{1}{3} = \triangle$ hundredths, or $\triangle$%.

4. Express each as an equivalent decimal, then as a per cent.

a. $\frac{2}{3}$ **b.** $\frac{1}{6}$ **c.** $\frac{5}{8}$ **d.** $\frac{5}{6}$ **e.** $\frac{7}{12}$ **f.** $\frac{1}{8}$ **g.** $\frac{3}{16}$ **h.** $\frac{3}{8}$

Per cents are used not only in arithmetic classes. They are used in other subjects you study in school. Outside of school, too, you will find that a knowledge of per cents is necessary.

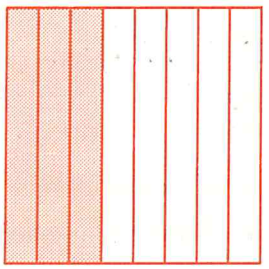
The following table lists certain per cents and the decimal expression and the equivalent fraction for each. These per cents are so often used that much time is saved by knowing them. Study the table.

Fraction	Decimal	Per cent	Fraction	Decimal	Per cent
$\frac{1}{4}$	.25	25%	$\frac{1}{10}$	.10	10%
$\frac{1}{2}$	.50	50%	$\frac{1}{5}$	.20	20%
$\frac{3}{4}$	.75	75%	$\frac{2}{5}$	.40	40%
$\frac{100}{100}$	1.00	100%	$\frac{3}{5}$	.60	60%
$\frac{1}{100}$	.01	1%	$\frac{4}{5}$	.80	80%
$\frac{1}{25}$	.04	4%	$\frac{1}{20}$	.05	5%

1. Copy and complete the table below.

	Fraction	Decimal	Per cent		Fraction	Decimal	Per cent
a.	$\frac{1}{8}$	?	?	**b.**	$\frac{1}{3}$	?	?
c.	$\frac{3}{8}$	?	?	**d.**	$\frac{2}{3}$	?	?
e.	$\frac{5}{8}$	?	?	**f.**	$\frac{1}{6}$	?	?
g.	$\frac{7}{8}$	?	?	**h.**	$\frac{5}{6}$	?	?
i.	$\frac{1}{16}$	?	?	**j.**	$\frac{1}{12}$	?	?

2. The use of diagrams may help you discover another way of finding a per cent that is equivalent to a given common fraction, such as $\frac{3}{8}$. Think of a diagram divided into eighths. If all of the diagram were shaded—that is, if $\frac{8}{8}$ were shaded—what per cent would be shaded?



Since $\frac{3}{8}$ of the diagram shown is shaded, the per cent that is shaded is $\frac{3}{8}$ of 100%. Copy the sentence and find the missing numeral. Does your answer check the results for **1c**?

$\frac{3}{8} \times 100 = \square$

3. Use the method of Exercise **2** to check the per cents you found for **a**, **b**, **f**, **i**, and **j**, of Exercise **1**.

1. The Milady Hat Shop had a spring sale. Included in the sale were 72 hats of a certain style. During the sale, 75% of these hats were sold. How many of the hats were sold?

To find the number of hats that were sold, do you have to find 75% of 72? What is the decimal expression for 75%? So 75% of 72 can be found by finding .75 of 72. Can it also be found by finding $\frac{75}{100}$ of 72? $\frac{3}{4}$ of 72? Why?

Find the missing numeral in each of the sentences below.

$.75 \times 72 = \square$ $\quad \frac{75}{100} \times 72 = \square$ $\quad \frac{3}{4} \times 72 = \square$

Did you get 54 as the missing numeral in each of the above sentences? Sometimes it is easiest to work with a fraction which has 100 as a denominator, sometimes it is easiest with a fraction in lowest terms, and sometimes the decimal numeral is easiest. Use your judgment.

2. To find 25% of 220, can you use each of the sentences below? Explain; then choose the sentence you wish and find the missing numeral.

$.25 \times 220 = \square$ $\quad \frac{25}{100} \times 220 = \square$ $\quad \frac{1}{4} \times 220 = \square$

3. To find 40% of 400, can you use this sentence: $.40 \times 400 = \triangle$? Write two other sentences you could use. Then choose one sentence and use it to find what 40% of 400 is.

4. Find what each of the following equals.

a. 75% of 56
b. 5% of 20
c. 15% of \$4.40
d. 24% of 50
e. 20% of \$15
f. 75% of 80 miles
g. 8% of \$100
h. 45% of \$10
i. 15% of 30 feet
j. 80% of 100 yards
k. 4% of 500 gallons
l. 35% of 20 pounds
m. 25% of \$120
n. 10% of 15 inches

1. The sign in this bank tells us that the bank pays 4% **interest** on money that has been in a savings account for a year. This means that the bank adds 4 cents for every 100 cents in a savings account for a year. Will the bank pay 4 dollars interest per 100 dollars in the account? Will the bank pay interest equal to $\frac{4}{100}$, or .04, of the number of dollars in the account?

Jack had \$50 in his savings account for one year. How much interest did the bank pay Jack?

2. Joan has \$250 in her savings account. How much interest will her money earn in one year at 4%?

3. Banks charge interest on borrowed money. When a bank charges 6% interest on borrowed money, it means that the bank charges 6 dollars interest per 100 dollars borrowed for one year. In other words, the bank charges $\frac{6}{100}$, or .06, of the amount borrowed.

To help pay for a new car, Mr. Clark borrowed \$1,000 for a year from a bank that charged 6% interest. At the end of the year, he paid the bank \$1,000 plus the interest. How much did he pay?

First find the interest. Then, to find the amount Mr. Clark paid, add the interest to the amount borrowed. Do these two sentences tell about the problem?

$.06 \times \$1{,}000 = \square$ $\$1{,}000 + \square = \triangle$

Find the missing numerals.

4. Mr. White borrowed \$253 for one year at 6% interest. How much interest did he pay? How much did he pay the bank at the end of the year? Write two sentences as in Exercise **3** and then find the missing numerals.

In working with a per cent such as $33\frac{1}{3}\%$ or $12\frac{1}{2}\%$ or $6\frac{1}{4}\%$, it is convenient to use the common fraction form.

1. You have seen that $33\frac{1}{3}\%$ is equivalent to $\frac{1}{3}$. But suppose you did not remember this relationship and you wanted to find the common fraction that is equivalent to $33\frac{1}{3}\%$.

Just as 50% can be expressed as $\frac{50}{100}$, $33\frac{1}{3}\%$ can be expressed as $\frac{33\frac{1}{3}}{100}$. The fraction $\frac{33\frac{1}{3}}{100}$ is called a **complex fraction,** since one of its terms is a fractional number (in this case, the numerator $33\frac{1}{3}$). But any fraction indicates division, and $\frac{33\frac{1}{3}}{100}$ can be thought of as $33\frac{1}{3} \div 100$. Now divide, and express the answer in simplest form.

$\frac{33\frac{1}{3}}{100}$

$$33\tfrac{1}{3} \div 100 = \frac{100}{3} \div 100 = \frac{100}{3} \times \frac{1}{100}, \text{ or } \frac{\triangle}{\square}$$

Have you found that $33\frac{1}{3}\%$ is equivalent to $\frac{1}{3}$?

2. To find the fraction (in simplest form) that is equivalent to $12\frac{1}{2}\%$, you can begin by expressing $12\frac{1}{2}\%$ as a complex fraction. Can $\frac{12\frac{1}{2}}{100}$ be thought of as $12\frac{1}{2} \div 100$? Now divide, and express the answer in simplest form. $12\frac{1}{2} \div 100 = \frac{25}{2} \div 100 = \frac{25}{2} \times \frac{1}{100}$, or $\frac{\triangle}{\square}$.

$\frac{12\frac{1}{2}}{100}$

What is the fraction (in simplest form) that is equivalent to $12\frac{1}{2}\%$?

3. What is $12\frac{1}{2}\%$ of 80? $33\frac{1}{3}\%$ of 75?

4. Express each of the following per cents as a complex fraction.

a. $6\frac{1}{4}\%$ **b.** $87\frac{1}{2}\%$ **c.** $14\frac{2}{7}\%$ **d.** $66\frac{2}{3}\%$ **e.** $37\frac{1}{2}\%$ **f.** $16\frac{2}{3}\%$

5. Find the common fraction (in simplest form) that is equivalent to each of the per cents in Exercise **4**.

6. Find each of the following: $87\frac{1}{2}\%$ of 40; $16\frac{2}{3}\%$ of 186; $66\frac{2}{3}\%$ of 66.

7. How much interest would \$400 earn in one year at $4\frac{1}{2}\%$? at $5\frac{1}{2}\%$?

1. One season Bert pitched in 20 games and won 17 of them. What fractional part of the number of games that he pitched did Bert win? What per cent of the games did he win?

$$\frac{17}{20} = \frac{\square}{100}, \text{ or } \square\%$$

2. Jack pitched in 25 games and won 21 of them. What fractional part of the number of games that he pitched did Jack win? What per cent of the games did he win?

3. "Pitching averages" are based on the fractional part, or per cent, of the number of games pitched that are won. Is it sometimes difficult to tell which of two fractional numbers is larger (for instance, $\frac{17}{20}$ and $\frac{21}{25}$)? Is it easy to compare per cents? Who won the larger per cent of his games—Bert or Jack? Then who had the better pitching record?

4. Don won 5 games of the 8 that he pitched. Ted won 7 of the 10 games that he pitched. Who had the better pitching average? Tell the per cent that each won.

In each of the following, begin your work by finding what fractional part one number is of the other number.

	a	b
5.	12 is what per cent of 48?	65 is what per cent of 325?
6.	18 is what per cent of 360?	75 is what per cent of 225?
7.	What per cent of 104 is 13?	What per cent of 300 is 100?
8.	16 is what per cent of 192?	What per cent of 16 is 6?
9.	What per cent of 6 is 1?	What per cent of 42 is 28?

10. The Midway Grocery had a sale. Mrs. Wylie bought a can of peas for 20 cents. Usually, she has to pay 26 cents for a can of peas. What fractional part of the usual price did Mrs. Wylie pay?

a. Suppose you are asked what per cent of the usual price she paid. Can $\frac{20}{26}$ be easily expressed as an equivalent fraction with a denominator of 100? Why not?

b. Can you divide to find the answer? The division example is shown at the right. Is the quotient closer to .76 or to .77? We say that to the nearest per cent the answer is 77%.

$$\begin{array}{r} .76\tfrac{24}{26} \text{ or } .76\tfrac{12}{13} \\ 26\overline{)20.00} \\ 182 \\ \hline 180 \\ 156 \\ \hline 24 \end{array}$$

11. What per cent of 68 is 37?

$$\frac{37}{68} = \triangle\%$$

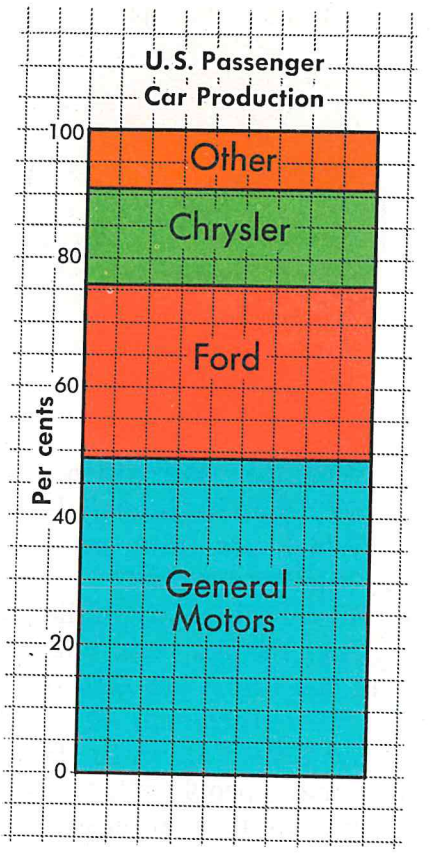
Can we use the division example at the right to find the answer? Express your answer to the nearest whole per cent.

$$\begin{array}{r} .54 \\ 68\overline{)37.0} \\ 340 \\ \hline 300 \\ 272 \\ \hline 28 \end{array}$$

Find answers for the following.

	a	b
12.	35 is what per cent of 75?	28 is what per cent of 29?
13.	30 is what per cent of 30?	What per cent of 32 is 16?
14.	What per cent of 13 is 5?	17 is what per cent of 78?

1. The divided-bar graph at the right shows, for a recent year, passenger-car production in the United States by leading manufacturers. Notice that per cents are used to show the share of each manufacturer. On pages 205 and 206, you used fractions for the same purpose. Notice that the scale indicates the per cents. What does each small space on the scale represent?



a. What per cent of the cars registered were manufactured by General Motors? by Ford? by Chrysler? by others?

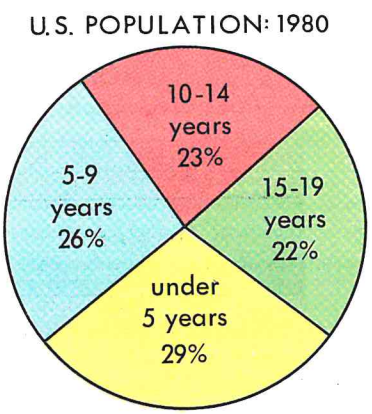
b. Are you able to tell from the graph how many cars were sold by each manufacturer? Can you tell the per cent of cars sold by each manufacturer? Do these per cents give an idea of how successful each manufacturer's models were that year?

2. The table gives data similar to those shown in the graph above, but for the preceding year. Use the same scale as in the above graph and construct a divided-bar graph, using these data.

Compare the two graphs. In which year was the per cent greater for each of the manufacturers?

U. S. Passenger Car Production (Preceding Year)	
General Motors	46%
Ford	31%
Chrysler	13%
Other	10%

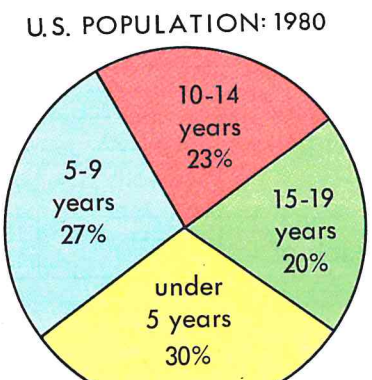
3. In the year for which the data are given in Exercise **2**, about 5,600,000 new cars were manufactured. About how many were produced by each of the manufacturers?



1. It is important for future planning to have some idea of what the population of the United States may become. Recently the government estimated the population for 1980. The estimate indicates there will be about 109,000,000 individuals under 20 years of age. This circle graph shows the per cent of the 109,000,000 that will be in each of four age groups.

a. What per cent will be under 5 years? from 5 through 9 years? from 10 through 14 years? from 15 through 19 years?
b. Which age group will have the largest number of people? the smallest number?
c. About how many millions will there be in each age group? To find the answers, will you use the number 109,000,000? Will you find a per cent of this number?

2. The government also estimated that the total population of the United States in 1980 will be about 260,000,000. About how many people will be 20 years old or more? What per cent of the total population will be in the under-20 age group?



3. By a different estimate, the total population in 1980 will be 246,000,000, with 96,000,000 individuals under 20 years of age. The graph shows the per cent of the 96,000,000 that will be in each age group.

a. About how many will there be in each of the four groups?
b. What per cent of the total population will be in the under-20 age group? in the 20-and-over age group?

1. A men's club wants to present a school with some new library books that cost \$60. The men hope to obtain this amount by selling balloons at a school fair. They will make a profit of 30% on the balloons they sell. What must the total sales be if they are to have a profit of \$60?

If there is a profit of 30% on the total sales, will the profit be $\frac{30}{100}$, or $\frac{3}{10}$, of the total sales? Then if we let N represent the total sales they must have, can we write this number sentence?

$$\frac{3}{10} \times N = 60$$

From your work with reciprocals, you know how to find a replacement for N. What is the reciprocal of $\frac{3}{10}$? Then is the following a true sentence?

$$\frac{3}{10} \times (\frac{10}{3} \times 60) = 60$$

What does $\frac{10}{3} \times 60$ equal? Then is 200 the replacement for N? In dollars, what must the total sales of the balloons be?

Is your answer correct? Check, using this sentence: 30% of $200 = \square$; that is, $\frac{3}{10} \times 200 = \square$.

2. The attendance at a baseball game was about 45,000. The announcer said that the stadium was 80% filled. About how many people can the stadium hold?

If we let N represent the number of people the stadium can hold, do we know that 80% of N is 45,000? What is 80% expressed as a common fraction in simplest form? Write a number sentence and use reciprocals to find the replacement for N.

Find the missing numerals.

3. a. 25% of N is 100 **b.** $16\frac{2}{3}$% of N is 6

4. a. 40% of N is 760 **b.** 350 is 8% of N

5. a. 25 is 1% of N **b.** 100% of N is 75

1. This year there are 640 boys and girls in Cherry Hill School. The school board made a study and predicts that next year there will be 720. What is the amount of increase? What is the per cent of increase?

a. The original number was 640. The predicted number is 720. Does the sentence $720 - 640 = N$ tell the amount of increase? What is the amount of increase?

b. The original number was 640. The amount of increase is 80. The amount of increase is what fractional part of the original number? Now find the simplest form of this fractional number and then the equivalent per cent.

$$\frac{80}{640} = \frac{\triangle}{\square}, \text{ or } \underline{?}\%$$

This per cent is the "per cent of increase."

2. The senior class this year has 120 members and next year will have only 105. What is the amount of decrease? What is the per cent of decrease?

a. Amount of decrease: $120 - 105 = N$

b. Per cent of decrease: N is what per cent of 120?

▶ A per cent of increase (or decrease) is always found by answering the question "What per cent of the *original number* is the amount of increase (or decrease)?"

3. A manufacturer of model trains increased his daily output from 1,500 to 1,545. What was the per cent of increase?

4. Jane's father said the warm winter had saved him money. The cost of heating their home a year ago was \$275, and this year it was \$242. What was the per cent of decrease?

5. A new car decreases in value each year. A car that costs \$3,424, when new, decreases in value 50% during two years. By how much does it decrease in value? What is it worth then?

6. At the end of three years a car that originally cost \$3,424 has a value of \$856. What is the amount of decrease? What is the per cent of decrease?

7. The store mentioned on page 240 advertised toys for sale at "20% off." Another way to say "20% off" is to say the store sold the toys at a **discount** of 20%. This means the buyer pays 20% less than the regular selling price.

a. A boy bought a table game at the sale for which the regular price was \$3.50. What is the amount he saved, or the amount of the discount?

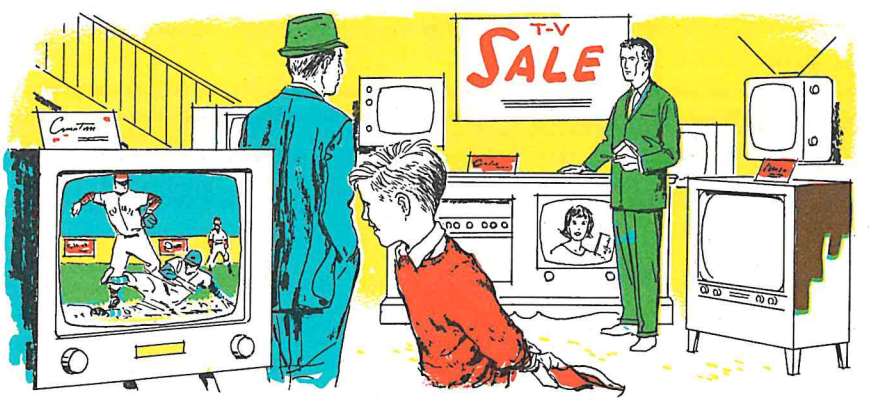
b. How much did the boy have to pay for the game?

8. A camera priced at \$49.50 is sold at a discount of $33\frac{1}{3}\%$. What is the amount of the discount?

9. Do you see that the *amount* of a discount can be thought of as an amount of decrease? Can we think of the *per cent* of discount—for instance, the $33\frac{1}{3}\%$ in Exercise **8**—as a per cent of decrease?

10. A man had \$200 in a savings bank. At the end of the year, the bank sent him a statement showing that he had \$208 in his savings account. What was the per cent of increase? What is the usual name for the money added to a savings account by a bank?

11. Mr. Ames is offering a 20% discount on a television receiver which is marked to sell at \$355. What is the amount of decrease in price? What will be the sale price of the television receiver?



1. A store owner ordered $245 worth of sports equipment and outdoor games for summer stock. He is allowed a discount of 2% if he pays the bill within 10 days. How much may be subtracted from the amount of the bill if he pays it within the 10-day period? What will he pay for the goods?

2. Sally and her mother bought new furniture for her room. The regular price was $375, but they paid only $281.25. How much did they save? What per cent did they save?

3. What per cent of $2,000 is $1,200?

4. A steel bar weighs 81 pounds, and an aluminum bar of the same size weighs 27 pounds. The weight of aluminum is what per cent of the weight of steel?

5. Mr. Lester had 324 young chickens that he was raising for the market. He sold 75% of them to the buyer for a supermarket. How many did he sell? What per cent did he have left?

6. One day there were 46 pupils absent from the Everett School. The principal said that this was 8% of the total number of pupils in the school. How many pupils does the school have in all?

7. At the college bookstore, Peter's father receives a 15% discount. In preparation for their summer vacation, he bought $10.15 worth of books. To the nearest cent, how much did he save?

8. This year, because of a severe electrical storm, the number of people attending the spring music concert was only 80% of the attendance last year. This year 276 attended. How many attended the concert last year?

9. John used to charge $1.00 an hour for mowing lawns. Now he charges $1.25 an hour. What is the per cent of increase?

10. A tennis racket is marked down from $12.50 to $10. What is the per cent of decrease?

1. What is 10×100? 100×100? 100×165?

2. Study Sentence **a** at the right. Can we rename 200 as 2×100? Then can we rewrite the original sentence as Sentence **b**? When you want to find the product of three or more numbers, does the way in which we pair the numbers, or the order in which we multiply, change the result? Then can we find the product of 200 and 300 by using Sentence **c**?

a. $200 \times 300 = \square$
b. $(2 \times 100) \times (3 \times 100) = \square$
c. $(2 \times 3) \times (100 \times 100) = \square$

What is the correct replacement for the frame?

3. Study this set of sentences. Tell how Sentence **b** was obtained from Sentence **a**; sentence **c** from Sentence **b**.

a. $400 \times 212 = \Diamond$
b. $4 \times 100 \times 212 = \Diamond$
c. $(4 \times 212) \times 100 = \Diamond$

What is 4×212? 848×100? Then what is the product of 400 and 212?

4. Find the product of 600 and 832 by using a set of sentences such as those shown in Exercise **3**.

$600 \times 832 = \Delta$

5. If you are asked to find the product of 300 and 781, can you do the work mentally? If not, do it any way you wish.

Find the missing numeral in each of the following.

	a	**b**	**c**
6.	$300 \times 300 = \Delta$	$200 \times 500 = \square$	$600 \times 900 = \Diamond$
7.	$100 \times 375 = \square$	$100 \times 868 = \Diamond$	$100 \times 909 = \Delta$
8.	$200 \times 123 = \Delta$	$300 \times 311 = \square$	$400 \times 204 = \Diamond$
9.	$600 \times 623 = \square$	$700 \times 824 = \Delta$	$900 \times 182 = \Diamond$
10.	$800 \times 893 = \Diamond$	$500 \times 414 = \square$	$400 \times 975 = \Delta$

1. Study this sentence. Will the sentence be true if N is replaced by 100? 200? 300? 400? 500?

$N \times 312 < 128{,}915$

What is the largest number of hundreds that makes the sentence true? Then what is the first quotient figure for this example: $312\overline{)124{,}800}$? Where will you place this figure? Now divide 128,915 by 312. Check your answer.

2. To estimate the quotient for this example, can you think of 185 as about 2 hundreds? Can you think of 62,135 as 600 hundreds + 2,135? What is 600 ÷ 2? Then what is the first quotient figure you will write? Where will you place this figure? Copy the example, and work it. Check your answer.

$185\overline{)62{,}135}$

3. To estimate the quotient for this example, can you think of 693 as about 7 hundreds? Can you think of 15,825 as 150 hundreds + 825? What is 150 hundreds ÷ 7 hundreds? Is the quotient about 200, or about 20? Where will you write the first quotient figure? Copy the example and work it. Check.

$693\overline{)15{,}825}$

Copy and divide. Check.

	a	b	c	d
4.	$63\overline{)\$52{,}794}$	$58\overline{)29{,}485}$	$85\overline{)8{,}557}$	$99\overline{)30{,}477}$
5.	$700\overline{)42{,}855}$	$729\overline{)\$29{,}889}$	$634\overline{)1{,}450}$	$515\overline{)\$18{,}540}$
6.	$221\overline{)17{,}985}$	$436\overline{)40{,}694}$	$112\overline{)5{,}948}$	$810\overline{)46{,}420}$
7.	$370\overline{)9{,}814}$	$368\overline{)29{,}890}$	$182\overline{)11{,}700}$	$571\overline{)30{,}763}$
8.	$523\overline{)16{,}941}$	$343\overline{)24{,}353}$	$804\overline{)21{,}219}$	$244\overline{)20{,}322}$
9.	$920\overline{)32{,}200}$	$721\overline{)40{,}696}$	$136\overline{)41{,}705}$	$324\overline{)67{,}278}$
10.	$240\overline{)97{,}334}$	$423\overline{)351{,}218}$	$830\overline{)332{,}075}$	$521\overline{)364{,}905}$

If you make any mistakes, study the pages given at the right of the exercises.

EXTRA PRACTIC 307–308

Express each per cent as a decimal and then as the equivalent fraction in lowest terms. 242–244

	a	b	c	d	e
1.	50%	25%	5%	8%	2%
2.	48%	4%	10%	65%	16%

Express each of the decimals as a per cent. 245

3.	.95	.75	.44	.09	.02

Express each as a fraction. 245

4.	$33\frac{1}{3}\%$	75%	80%	25%	$66\frac{2}{3}\%$

Express each as a per cent. 245

5.	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{5}$

6. Find the missing numeral in each. 247–248

a. 40% of 150 is ?
b. 25% of 75 is ?
c. 10% of \$4,200 is ?
d. 6% of \$50 is ?

7. Find the missing numeral in each. 250

a. An increase of \$5 from \$40 is an increase of ?%.
b. A decrease from \$20 to \$5 is a decrease of ?%.
c. A decrease from \$60 to \$50 is a decrease of ?%.
d. An increase from \$50 to \$60 is an increase of ?%.

8. Give each answer to the nearest whole per cent. 255

a. What per cent of 20 is 12?
b. What per cent of 200 is 150?
c. 10 is what per cent of 87?
d. 26 is what per cent of 77?

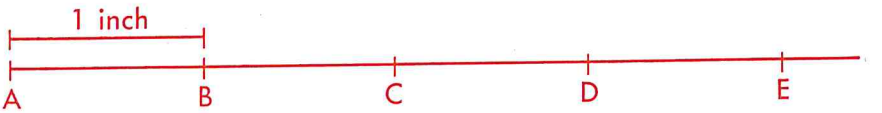
9. Find the missing numeral in each. 256

a. 50 is $12\frac{1}{2}\%$ of N
b. 120 is 75% of N
c. 2% of N is 20
d. 40% of N is 32

DISCOVERING MORE ABOUT NUMBERS: PAGES 335–336

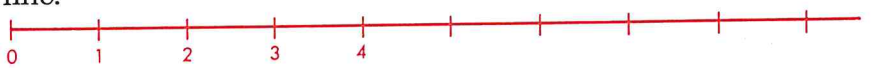
9 Thinking about the Number Line

1. You know that to make a scale for measuring we select a standard unit of measure—for instance, 1 inch—and we draw a line segment. Then, beginning at the left end of the segment, we mark off unit distances, one after another. In the drawing below, we have labeled the point at the left end of the segment with the letter *A*. On a scale, would 0 appear instead of *A*? What numeral would appear instead of *B*? *C*? *D*? *E*?



2. Could the segment be extended in order to make a scale that could measure a distance of 6 inches? 12 inches?

3. Shown below is a model of what we have called a number line.



a. To make this model, we use a procedure similar to the one used in making a scale for measuring. We choose a convenient distance as the unit distance, or *unit space*. Beginning with the point at the left end of a line segment, we mark off one unit space after another, as shown, and label the points with the numerals 0, 1, 2, 3, and so on. Does the numeral 3 stand for the *number* of unit spaces from 0 to this point? Can we associate a number with each of the points that we have marked on the model of the number line?

b. Could we divide each of the unit spaces into fractional parts—for instance, into halves? What number would you associate with the point that is halfway between 1 and 2?

c. Draw a number line and divide each unit space into fourths of a unit space. Then label each of the points you have marked on your line.

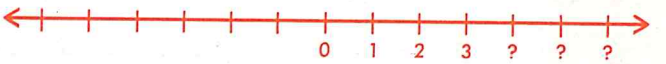
1. Does the number-line drawing we have been using represent a *line*, or does it actually represent a *ray*?

a. Is there an end point? Does the diagram indicate a part of a line that goes on indefinitely in one direction? Are these the characteristics of a ray?

b. What number would you associate with the point that is 1,000 spaces to the right of 0? Do you think that the spaces could be laid off on the ray as far as you please?

2. You know that a line goes on and on indefinitely in both directions. Let us see whether we can associate numbers with the points of a line rather than with a ray.

a. On your paper represent a line. Choose any point on it and label it 0. Mark off unit spaces to the left and right of the zero point. Your diagram should look like the one below.



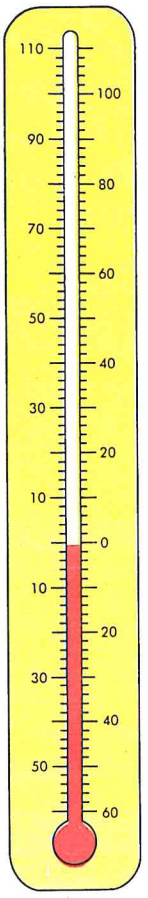
b. What numbers can you associate with the points you have marked to the right of 0? Label these points.

c. If you wanted to label a point for $\frac{1}{2}$, would this point be between 0 and 1? Would the points for each of these numbers be between 0 and 1: $\frac{3}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{99}{100}$? Use any proper fraction you wish to name a fractional number. Is the point for this number between 0 and 1?

d. Think of the fractional numbers named by these improper fractions and mixed fractions: $\frac{3}{2}$, $2\frac{1}{4}$, $1\frac{1}{10}$, $\frac{8}{5}$, $\frac{51}{10}$. Is the point for each of these to the right of 1?

e. Would the point for any fractional number you know be to the right of 0?

If you used all the numbers you know, would you still have marked only points to the right of 0? On the following pages, we will see whether we can invent numbers that we can associate with the points to the left of 0.



1. The illustration at the left represents an outdoor thermometer. Notice that temperatures from 60° below 0° to 110° above 0° are shown on the scale.

How many equal spaces are there between 0° and 10° on the scale? Then how many degrees are represented by each of these smaller spaces?

2. As you probably know, a temperature of 10° below 0° is usually represented as "−10°." Use this method to represent a temperature of 20° below 0°; a temperature of 13° below 0°; a temperature of 8° below 0°.

3. When a temperature is given as "27°" does it mean 27° below 0°, or 27° above 0°? What does a temperature of 38° mean? a temperature of −38°?

4. Think of the drawing below as a thermometer scale in a horizontal position. The marks for temperatures above 0° are to the right of the point marked 0°. The marks for temperatures below 0° are to the left of the point marked 0°. For convenience, a mark has been made for each degree, and each of these points has been labeled.

What is the label for the point that is one unit space to the left of 0°? the point that is two unit spaces to the left of 0°? For each point at the left of the point marked 0°, does the label indicate the number of unit spaces between that point and the point marked 0°? Does each of these labels indicate the point is *to the left* of the point marked 0°?

5. How many unit spaces from 0° is 4°? How many unit spaces from 0° is −4°? Are 4° and −4° the same distance from 0°? Are they in opposite directions from 0°?

Are 6° and −6° the same distance from 0°? Are they in opposite directions from 0°? If you marked points for 30° and −30°, would the points be the same distance from 0°, but in opposite directions from 0°?

We can call a pair such as −4° and 4° **opposites.** What is the opposite of 6°? the opposite of −30°? of −12°?

6. The opposite of $\frac{1}{2}°$ is $-\frac{1}{2}°$. Where would you mark a point for $-\frac{1}{2}°$? Where would you mark a point for $-2\frac{1}{2}°$? What is the opposite of $-2\frac{1}{2}°$?

7. The way in which we have been indicating temperatures above and below 0° suggests a way in which we can write numerals for our new numbers. For instance, the new number that is the opposite of 1 is −1. How would you express the new number that is the opposite of 3? the opposite of 7?

In your work for page 263, you drew a number line and labeled points to the right of 0. Now, fill in the numerals for the points to the left of 0.

What would be a good name for the new numbers? Could we call them "minus numbers"? Perhaps we could, but usually these numbers are called **negative numbers.**

8. We have been using the word "opposite." In your daily life, are there many things you can think of as opposites? For instance, what would be the opposite of a profit of $5? What would be the opposite of a loss of 3 pounds in weight?

9. Suppose you represented a loss of $5 as −5. How would you represent a loss of $8? Now imagine that a newsdealer has a loss of $5, and then another loss of $8. Would his total loss be the sum of his two losses? Using a □ to represent his total loss, we can write the sentence below. Notice that parentheses are used to avoid confusion of the plus sign with the numeral that follows it.

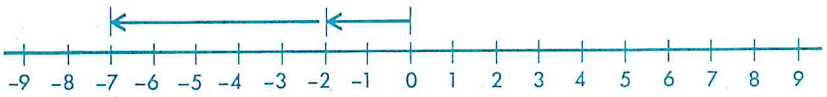
$$-5 + (-8) = \square$$

What is the correct replacement for the frame?

1. You know that you can use the number line to illustrate addition. For instance, to show the addition of 2 and 4 on the number line, you begin at 0 and draw an arrow that shows a move of two spaces to the right. Then, from 2, you draw an arrow showing a move of four spaces to the right. At what numeral does the second move end?

2.a. Draw a number line and represent the addition of 3 and 6.
b. Did you begin at 0 and draw an arrow that represented a move of three spaces *to the right*, and then another arrow that represented a move of six spaces *to the right*?

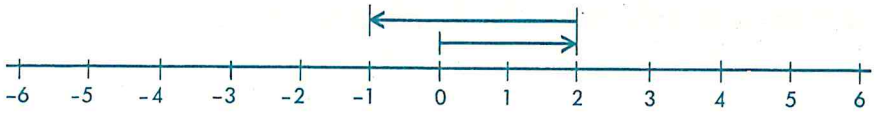
3. Just as we draw arrows to the right to represent a sentence such as $2 + 5 = 7$, we draw arrows to the left to represent addition involving negative numbers. For instance, to represent the addition sentence $-2 + (-5) = \square$, we begin at 0 and draw an arrow to show a move of two spaces *to the left*. Then, from -2, we draw an arrow to show a move of five spaces *to the left*.



At what numeral does the second move end? Then what is the correct replacement for the frame?

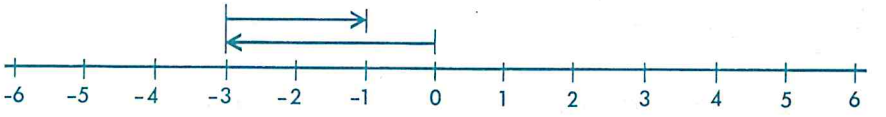
4. Now use a number line to represent the addition of -3 and -6. Begin at 0 and represent a move of three spaces *to the left*; then, from -3, represent a move of six spaces *to the left*. At what numeral does the second move end? Then what is the correct replacement for the frame in the following sentence: $-3 + (-6) = \diamond$?

5. Find the correct replacement for the frame: $2 + (-3) = \square$

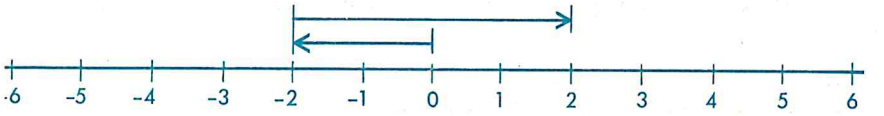


Starting at 0 you draw an arrow to show a move two spaces *to the right;* then, from 2, you represent a move of three spaces *to the left.* At what numeral does the second move end? Then what is the correct replacement for the frame?

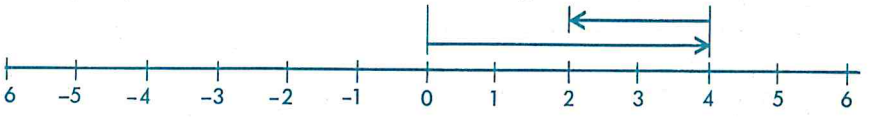
6. To represent the sentence $-3 + 2 = \square$, start at 0 and draw an arrow to represent a move of three spaces to the left. Then, from the point for -3, you represent a move of two spaces. In which direction? At what numeral does the second move end? What is the correct replacement for the frame?



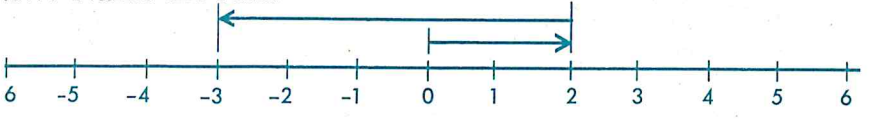
7. Does the drawing below represent this addition sentence: $-2 + 4 = \triangle$? What is the correct replacement for the frame?



8. Does the drawing below represent this addition sentence: $4 + (-2) = \triangle$? What is the correct replacement for the frame?



9. What addition sentence is represented in the drawing below? Name the sum.



Using drawings of a number line as on page 267, represent each of these pairs of sentences and name the sum in each case.

	a	b
1.	$4 + (-1) = \diamond$	$-1 + 4 = \diamond$
2.	$-3 + (-2) = \triangle$	$-2 + (-3) = \triangle$
3.	$-4 + 2 = \square$	$2 + (-4) = \square$
4.	$5 + (-2) = \triangle$	$-2 + 5 = \triangle$

5. Study the results of the pairs of examples in Exercises **1–4**. Do you agree that the order in which we add does not change the sum?

Using drawings of a number line, find the sums.

6.	$3 + (-3) = \square$	$-3 + 3 = \square$
7.	$-5 + 5 = \triangle$	$5 + (-5) = \triangle$
8.	$\frac{1}{2} + (-\frac{1}{2}) = \mathrm{N}$	$-\frac{1}{2} + \frac{1}{2} = \mathrm{N}$

9. Study the pairs of sentences in Exercises **6–8**. In each case were we finding the sum of a number and its opposite? Is the sum 0 in each case?

10. Make a drawing of a number line and represent this sentence: $7 + (-3) = \triangle$.

11. In your work this year, you have also represented subtraction sentences on the number line. Make a drawing and represent this sentence $7 - 3 = \triangle$.

Compare this drawing with the one you made for Exercise **10**. Do the two drawings appear the same?

12. Make a drawing to show this sentence: $5 + (-2) = \square$. Now make a drawing to represent this sentence: $5 - 2 = \square$. Do the two drawings appear the same?

13. Could you write a subtraction sentence that would give the same result as the addition sentence $7 + (-1) = \mathrm{N}$? Try it and see.

1. Mary kept a record of her gain or loss in weight, month by month. In April she lost three pounds and wrote a "−3" in her record. In May she gained one and a half pounds and wrote "$1\frac{1}{2}$" in her record. What does the "−2" mean?

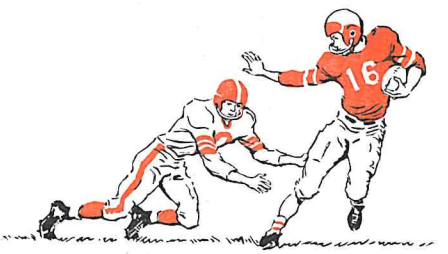
April −3
May $1\frac{1}{2}$
June −2

2. Suppose you are asked to find the over-all change in her weight during April and May. Does this number sentence tell about the problem: $-3 + 1\frac{1}{2} = \triangle$? What is the correct replacement for the frame? (If you wish, you can draw a number line and represent the addition by moves.)

When you have written a numeral that replaces the frame, tell what this numeral means in the problem situation.

3. Using any method you wish, find a single numeral that represents the total change in Mary's weight during the three-month period. Then tell what the numeral means in the situation.

4. On one play, Bayville High's football team lost eight yards. On the next play, it gained three yards. If the loss is represented by the numeral "−8," what numeral would represent the gain of three yards? a loss of one and a half yards?



5. Does Sentence **a** below tell about the over-all result of the first two plays? What does Sentence **b** tell about?

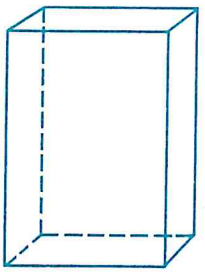
a. $-8 + 3 = \diamond$ **b.** $(-8 + 3) + (-1\frac{1}{2}) = \square$

Find the correct replacement for the $\diamond$ and tell what the numeral means. Then find the correct replacement for the $\square$ and tell what this numeral means.

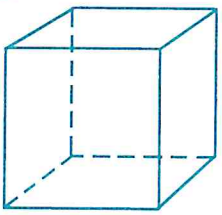
6. Betty kept a record of the yards that her brother gained or lost when he carried the ball. The record is shown at the right. Using any method you wish, find a single numeral to represent the sum of these gains and losses. After you find the numeral tell what it means in the situation.

−5
7
9
−3
$-1\frac{1}{2}$
2

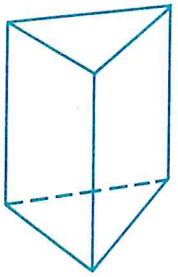
A



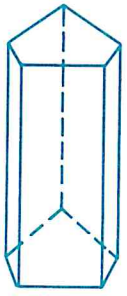
B



C



D



Let us take another look at some geometric shapes.

1. All of the drawings on this page represent **prisms,** but each one is a different type of prism. Do all of the points of a prism lie in the same plane? Is a prism a space figure?

What is the shape of the base of prism *A*? A prism takes its name from the shape of its base, so prism *A* is called a **rectangular prism.**

How many faces does prism *A* have? Are two of the faces also bases? Often we use the phrase **lateral faces** to describe the faces that are not bases. How many lateral faces does prism *A* have?

A breakfast cereal box is a good model of a rectangular prism. Name some other models of rectangular prisms.

2. Can a cube be described as a rectangular prism? Why? How is a cube different from a rectangular prism such as prism *A*?

3. What is the shape of the bases of prism *C*? Then what would you call prism *C*? How many bases does prism *C* have? How many lateral faces?

4. You may not know the name for the shape of the base of prism *D*. Do the edges of the base have the shape of a polygon of five sides? A polygon of five sides is called a **pentagon,** from the Greek word *pente* meaning "five." Prism *D* is called a **pentagonal prism.**

5. How many bases does prism *D* have? How many lateral faces?

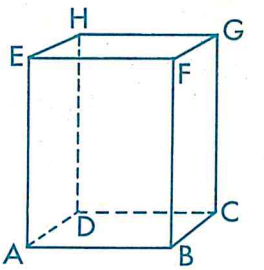
6. Study the prisms. Are the two bases of prism *A* the same size and shape? Are the two bases of prism *C* the same size and shape? prism *B*? prism *D*?

7. Look at prism *A*. Think of each lateral face as a rectangle and its interior. Then can we say that the lateral faces are rectangular regions? Are the bases also rectangular regions?

8. Think of each base of prism *C* as a triangle and its interior. What would you call a base of this prism?

9. In each of the four prisms on page 270—and in the prism below—the lateral faces are rectangular regions. We can say the boundaries of the lateral faces are rectangles.

For instance, in this prism, rectangle *ABFE* is the boundary of a lateral face.



a. Are $\overline{AB}$ and $\overline{FE}$ opposite sides of the rectangle? Are they also edges of the bases? Is $\overline{AB}$ parallel to $\overline{FE}$? Why?

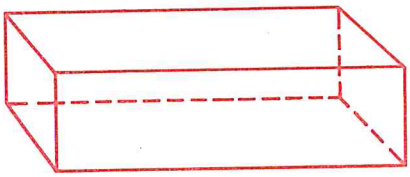
b. Is *BCGF* a rectangle? Is $\overline{BC}$ parallel to $\overline{FG}$? Why?

▶When a pair of intersecting segments in one plane is parallel to a pair of intersecting segments in another plane, the planes are said to be parallel. Do *ABCD* and *EFGH* lie in parallel planes? We say that the bases of this prism are parallel.

10. Are the bases of prism *A* parallel? prism *B*? prism *C*? prism *D*? Explain.

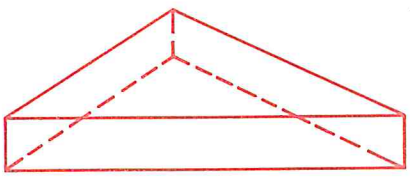
11. Do all of the prisms shown on page 270 have the same number of bases? Do all of the prisms have the same number of lateral faces?

▶For each of the prisms, is it true that the bases are parallel? Are the bases the same size and shape?

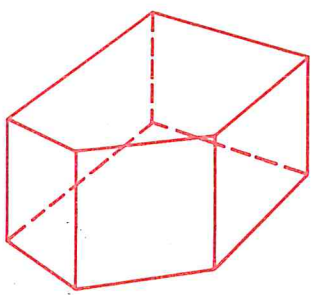


1. What is the name of the space figure shown at the left? How many vertices does it have? How many edges? At a vertex, how many edges intersect? Do two faces intersect in an edge? A face and a base intersect in an ?. How many edges form the boundary of a lateral face?

2. What would your answers have been to the questions in Exercise **1** if the prism shown had been a cube?



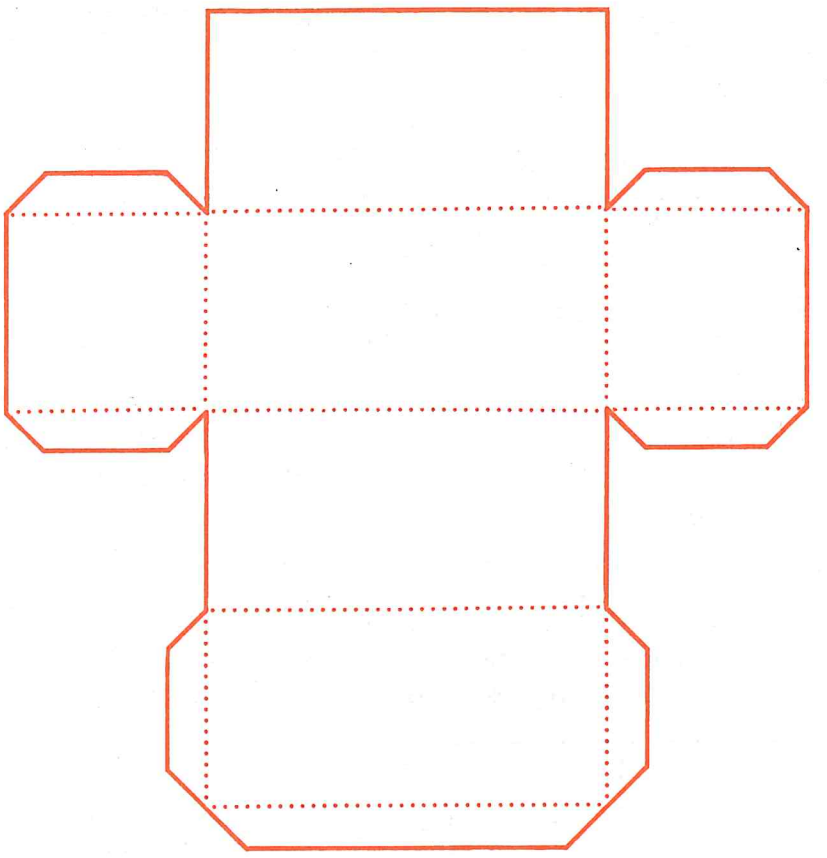
3. In the prism shown at the left, what is the shape of the bases? the lateral faces? A face and a base intersect in an ?. Two faces intersect in an ?. How many vertices does the triangular prism have? How many edges? How many edges intersect in a vertex? How many form the boundary of a lateral face?



4. This drawing represents a pentagonal prism. How many vertices does it have? How many edges? At a vertex, how many edges intersect? Two faces intersect in an ?. A face and a base intersect in an ?. How many edges form the boundary of a face?

5. You have been studying some members of the family of prisms. Do you notice any patterns—that is, anything that is common to all of these prisms? Tell what you know about the bases; about the faces.

Do the members of the family have the same number of vertices? the same number of edges? Do the same number of edges intersect in a vertex? Do two faces always intersect in an edge? Can you say the same thing about a face and a base?

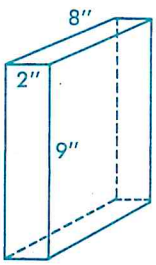


. Shown above is a form that can be followed in constructing
ı model of a rectangular prism.

Copy the form on heavy paper. (You can make your diagram
arger if you wish.) Cut carefully along the solid line segments
n the diagram. Fold along all of the dashed segments and
›aste the flaps.

:. Make a model of a rectangular prism in which the longer
dges are 4 inches and the shorter edges are $2\frac{1}{2}$ inches. Be sure
ɔ include flaps in your diagram. Which parts of your diagram
epresent the bases of the prism? How do you know?

. Try making a model of a cube that has an edge of 2 inches.



1. The drawing shows a breakfast cereal box that has the shape of a rectangular prism.

a. Only three edges have their lengths indicated. With this information, can you find the lengths of all the other edges? Explain.

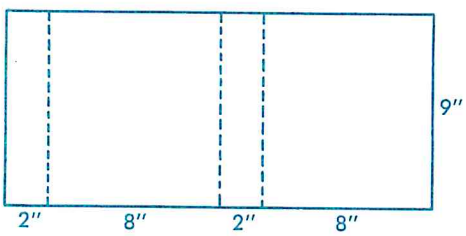
b. Is each lateral face a rectangular region? Is there a pair of faces that have the same size and shape? Is there another pair of faces that have the same size and shape? Explain.

c. What is the area of each lateral face of the box?

d. What is the sum of the areas of the four lateral faces? The sum of the areas of the lateral faces of a prism is called the **lateral area** of the prism.

2. What is the perimeter of a base of the prism?

3. Now imagine the top and bottom of the cereal box removed and a cut made along one of the vertical edges. Then we can fold the box, or the representation of the lateral faces, out flat. At the left is a diagram of the result.



a. When the lateral faces are folded out flat, do they form a rectangular region? Why?

b. What is the height of the rectangular region? What is the length?

c. Is the length of this region equal to the measurement you got in Exercise **2**? Then is the length of this rectangular region the same as the perimeter of the base?

d. Find the area of the rectangular region, using your answers for **b**. Do you get the same result that you did in **1d**?

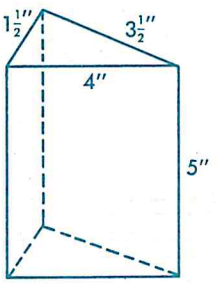
Then is the following sentence true?

▶ The lateral area equals the perimeter of the base times the height of the prism.

Find the lateral area of each of the rectangular prisms whose dimensions are as follows.

	a			b		
	length	width	height	length	width	height
1.	$10\frac{1}{4}$ in.	$2\frac{3}{4}$ in.	$10\frac{1}{4}$ in.	7 in.	3 in.	15 in.
2.	8.5 ft.	2.8 ft.	3.5 ft.	$12\frac{1}{2}$ ft.	$11\frac{1}{2}$ ft.	$8\frac{1}{2}$ ft.
3.	$9\frac{3}{4}$ in.	$2\frac{1}{4}$ in.	$11\frac{3}{4}$ in.	24 ft.	$14\frac{1}{2}$ ft.	8 ft.

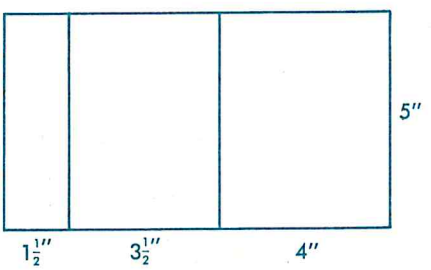
4. Shown at the right is a model of a triangular prism with the lengths of certain edges indicated.



5. Is each lateral face a rectangular region? Can you find the area of each lateral face? What is the sum of the areas of the lateral faces? Would you call this sum the lateral area of the triangular prism?

6. Suppose we removed the bases and made a cut along one edge of the prism, then folded the lateral faces out flat.

The diagram at the right shows the result. Is it a rectangular region? Why? What is the area? Is your answer the same as for **a**?

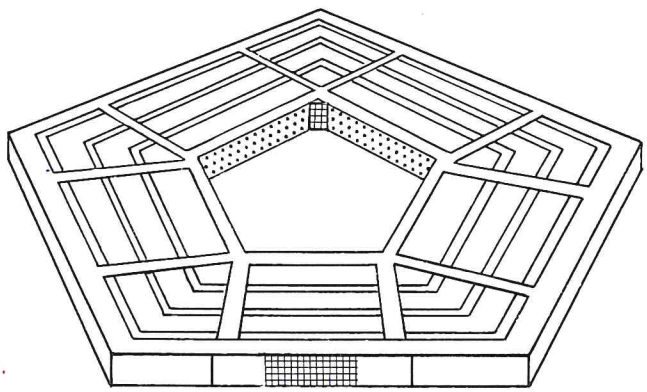


7. What is the perimeter of the base of the prism? Is this the same as the length of the rectangular region? Then is the lateral area of the triangular prism equal to the perimeter of the base times the height? Is the lateral area of a rectangular prism also equal to the perimeter of the base times the height? Do you think this would be true of any prism?

8. Find the lateral area of a triangular prism if the perimeter of its base is $17\frac{3}{4}$ inches and its height is $1\frac{1}{2}$ inches.

6. Find the lateral area of each of the triangular prisms whose dimensions are given below.

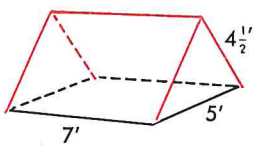
	Length of sides of base	Height
a.	.3 in., .4 in., .5 in.	.8 in.
b.	$3\frac{1}{8}$ yd., $1\frac{3}{4}$ yd., $2\frac{5}{8}$ yd.	$5\frac{1}{4}$ yd.
c.	231 ft., 125 ft., 205 ft.	22 ft.



7. Each side of the base of a pentagonal prism is 7 inches in length. If the height of the prism is 19 inches, what is its lateral area?

8. Each side of the Pentagon near Washington, D.C., is 921 feet long. What is the perimeter of the base?

The building has 5 stories. If 14 feet are allowed for each story, how high is the Pentagon? Then what is the lateral area of the Pentagon?



9. The pup tent shown in the drawing has a canvas floor as well as canvas side walls. The dimensions of the floor are 5 feet by 7 feet. The dimensions of each of the side walls are $4\frac{1}{2}$ feet by 7 feet.

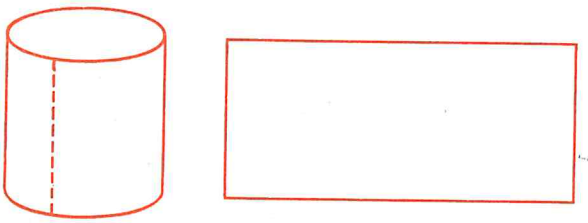
a. Suppose that the floor and the side walls of the tent were made of the same type of canvas. How many square feet of canvas (with no allowance for sewing and so forth) would be needed to make the floor and the side walls?

b. Usually, however, the floor canvas of a tent is different from that used in the side walls. How many square feet of canvas would be needed for the side walls? How many square feet of the other canvas would be needed for the floor?

1. The juice can shown at the right is a model of the geometric shape called the **cylinder.** As in the case of a prism, it is the "skin" and not the contents that is the cylinder.

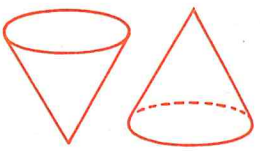
a. Does the model have only flat regions?
b. What is the shape of the bases? Are both of them the same size and shape? We call them circular regions.

2. Imagine the label removed from the can without tearing or cutting. If you made bases, could you form a model of a cylinder, using the label?

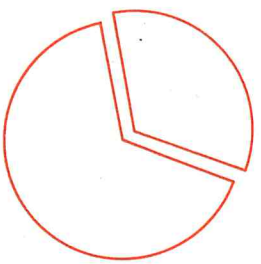


a. Now cut the label as shown by the dashes. When you lay the label flat, what shape does it have? What do we call the region? Does this suggest a way for you to make a model of a cylinder, using sheets of paper?

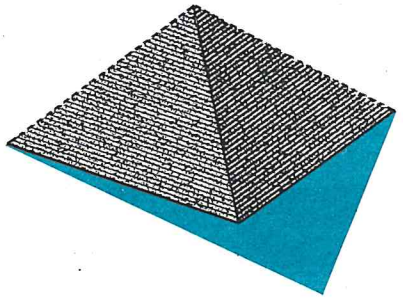
3. The paper drinking cup in the picture is a model of a space figure called a **cone.** How is a cone like a cylinder? How is it different? The circular region of the cone is usually called its base.



4. You can make a model of a cone with a circular base. Begin by drawing a circle on your paper. Then cut out the circular region and remove the interior of a central angle that has a measurement of about 120°. Then trace a shape that you can cut out for the base.

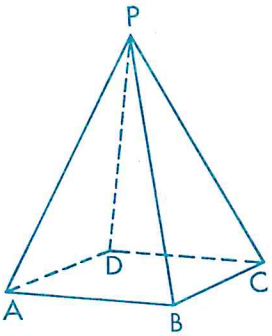


5. List other objects that are models of cylinders; that are models of cones.



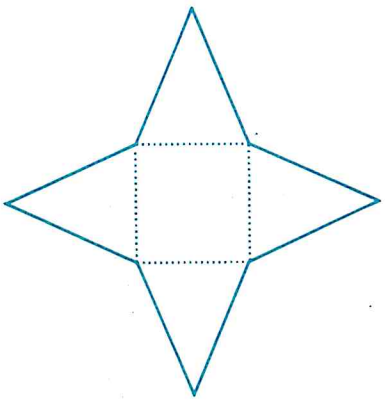
1. When the Egyptian Pharaohs built their tombs, the geometric shape they used as a model was the **pyramid.**

a. How is a pyramid like a cone? How is it different?
b. The bases of the Egyptian pyramids are square regions. Could the base of a pyramid be a rectangular region?
c. How many faces does the pyramid in the picture have? What is the name of the boundary of a face? Each face is a triangular region.



2. The drawing at the left represents a rectangular pyramid.

a. What is the base called?
b. Are there any faces that are *not* triangular regions?
c. How many vertices does this pyramid have? How many edges?
d. Do the same number of edges intersect at each vertex? How many edges intersect in A? in B? in C? D? P?



3. You can make a model of a square pyramid by using the diagram at the left.

Draw a square with sides that are each 3 inches long. Then, on each side of the square, draw a triangle (for a face) whose other two sides are each $3\frac{1}{2}$ inches long.

Cut out your model, fold along the dotted segments and fasten the edges with tape.

If you make any mistakes, study the pages given at the right the exercises. 262

. Draw a model number line and, using the following umerals, show how numbers are assigned to points on a line: -3, 3, −2, 2, −1, 1, 0. 264–265

. What is the opposite of a temperature of 6°? The opposite f $-12\frac{1}{2}$°?

. What is the opposite of −3? the opposite of −300? the pposite of 13? 265

. Draw a model number line and show the following addition ntences on it. 266–267

. $-1 + (-3) = \diamond$
$-\frac{1}{2} + (-1) = \square$
b. $-2 + 5 = \triangle$
d. $-2\frac{1}{2} + 2 = N$

. Find the correct replacement for the frame or N in each f the following sentences. 268

. $-3 + 3 = N$
$1 + (-4) = \square$
. $6 + (-9) = N$
b. $5 + (-5) = \square$
d. $-6 + 8 = \square$
f. $-11 + 7 = \square$

. How many sides does a base of a triangular prism have? a entagonal prism? a rectangular prism? 270–271

. How many edges does a cube have? How many vertices? s the area of each face the same? How many faces are iere? 272

. Find the total area (the area of all the faces) of a cube the length of one edge is 5 inches. 274–276

. What is the lateral area of a pentagonal prism if its height s 12 inches and each side of a base is 3 inches long. 274–276

0. A pyramid has a base whose boundary is a square. Give wo names you could use to describe this pyramid.
This pyramid has how many faces (disregarding the base)? Vhat is the shape of each of these faces? 278

DISCOVERING MORE ABOUT NUMBERS: PAGES 337–338

EXTRA PRACTICE 310

1. Copy, and replace each frame with a decimal. 149–1

a. $\frac{45}{100} = \square$ **b.** $\frac{35}{1000} = \triangle$ **c.** $\frac{5}{1000} = \diamond$ **d.** $6\frac{708}{10,000} = \square$

2. Tell what the figure 8 means in each of these numerals. 1

a. 8,469.601 **b.** 469.608 **c.** 46,960.8 **d.** .4690

3. Round each to the nearest hundredth. 162–16

a. 8.765 **b.** 469.608 **c.** 1.074 **d.** 7,849.01

4. Express each as a per cent. 240–24

a. $\frac{1}{3}$ **b.** .35 **c.** $\frac{5}{8}$ **d.** .2

5. Write the fraction in lowest terms that corresponds to each of the following. 24

a. 25% **b.** 50% **c.** $12\frac{1}{2}\%$ **d.** 95%

6. You know that 605.67 equals $(6 \times 100) + (5 \times 1) + (6 \times \frac{1}{10}) + (7 \times \frac{1}{100})$. Write a similar expression for each of the following. 101, 16

a. 76.7698 **b.** 7,496.1

7. Copy, and replace each frame. 148, 155–15

a. $1.5 = \frac{\square}{10}$ **b.** $.105 = \frac{\triangle}{1000}$ **c.** $8.735 = \frac{873}{\square}$

d. $78.05 = \frac{7805}{\square}$ **e.** $8.7456 = \frac{\triangle}{10,000}$ **f.** $11 = \frac{110}{\triangle}$

8. Copy, and replace each frame. 265–26

a. $-1 + (-10) = \square$ **b.** $-8 + 7 = \triangle$ **c.** $-3 + \triangle = 0$

d. $15 + (-15) = \square$ **e.** $-\frac{1}{2} + 1 = \triangle$ **f.** $6 + (-5) = \diamond$

9. Tell which sentences are true and which are not true. 16

a. $2^3 < 5^2$ **b.** $3^3 < 5^2$ **c.** $4^2 = 2^4$

d. $10^6 = 100,000$ **e.** $5^3 > 10^2$ **f.** $6^3 > 200$

10. Represent angle ABC on your paper. Use the letters D and E to name two points in the interior of angle ABC. Use the letters F and G to name two points in the exterior of angle ABC. 21

1. Copy, and replace the frame with a decimal. 158–170, 182

a. $5.1 + 6.2 = \square$ **b.** $7.58 - \square = 4.64$ **c.** $\square + 7.6 = 9.1$
d. $7.5 \times 4.3 = \triangle$ **e.** $8.54 \div .7 = \square$ **f.** $\square \times 4.9 = 17.15$

2. Copy and add or subtract as the signs tell you. 159–160

a. $87.156 - 7.155$ **b.** $87.156 + 12.845$ **c.** $.86 - .77$ **d.** $35.74 + 46.75$

3. Find the products. 170–173

a. $7.567 \times .8$ **b.** $817 \times .05$ **c.** $.716 \times 1.32$

4. Find the quotients. 182–187

a. $.5\overline{).555}$ **b.** $.05\overline{)5.55}$ **c.** $.05\overline{).00555}$
d. $1.2\overline{)36.36}$ **e.** $.15\overline{)3.675}$ **f.** $1.5\overline{).3675}$

5. Find the quotients correct to the nearest tenth. 191

a. $75\overline{)153}$ **b.** $3\overline{)200}$ **c.** $.28\overline{)7.5}$

6. Express each fraction exactly as a decimal or as a decimal correct to the nearest thousandth. 189–190

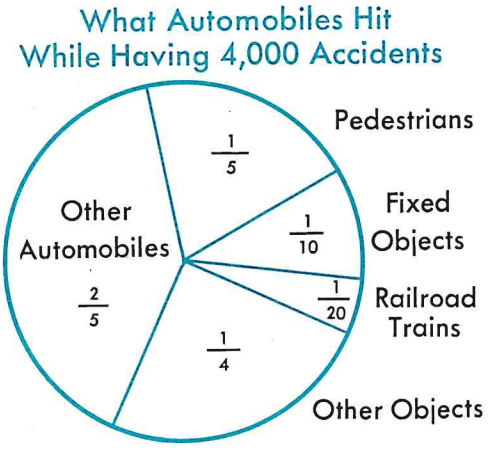
a. $\frac{5}{16}$ **b.** $\frac{4}{9}$ **c.** $\frac{3}{40}$ **d.** $\frac{7}{8}$ **e.** $\frac{8}{7}$

7. Answer each of the following questions. 248–252

a. What is 75% of 400? **b.** 100 is what per cent of 300?
c. $66\frac{2}{3}\%$ of what number is 600? **d.** What is $12\frac{1}{2}\%$ of 64?

8. Copy the letters **a** to **f** on your paper and indicate whether each sentence is true or false. 224–226

a. A rectangle is a four-sided polygon.
b. The lengths of opposite sides of a rectangle are equal.
c. The diagonals of a rectangle are equal in length.
d. A square is a rectangle.
e. A rectangle must be a square.
f. A square has all the properties of a rectangle.



1. The circle graph at the left shows that $\frac{2}{5}$ of 4,000 traffic accidents involved another automobile. How many of the 4,000 accidents involved other automobiles? What per cent of these accidents involved other automobiles?

Use the circle graph when answering Exercises **2–5**.

2. What per cent of these accidents involved pedestrians? fixed objects? a railroad train? other objects?

3. What number of these accidents involved pedestrians? fixed objects? a railroad train? other objects?

4.a. What is the ratio of the number of other automobiles involved to the number of fixed objects involved?
b. What is the ratio of the number of pedestrians involved to the number of railroad trains involved?

5. What decimal expresses the fractional part of the accidents that involved other automobiles? that involved pedestrians? railroad trains? fixed objects?

6. A rectangle is 8.6 inches long and 4.5 inches wide. What is the area of this rectangle?

7. Mrs. Jones bought a blanket at a sale. The usual price of the blanket was \$15.96. Mrs. Jones paid \$11.97 for the blanket. How much did Mrs. Jones save? What per cent of the usual price did she save?

8. At another sale, Mrs. Jones bought a jacket at 20% off. The original price of the jacket was \$8.95. How much did she save?

9. Use a decimal to express 45 seconds as a fractional part of a minute. Use a decimal to express 48.75 minutes as a fractional part of an hour.

Addition and Subtraction of Whole Numbers

(For use after page 19 or after page 39)

1. Round each of these numbers to the nearest hundred.

a. 199 **b.** 45,650 **c.** 8,749 **d.** 982 **e.** 1,763

2. Round each of these numbers to the nearest thousand.

a. 2,815 **b.** 11,342 **c.** 998 **d.** 138,561 **e.** 23,592

3. Write a numeral for each of the following, using the digit 1 and zeros.

a. 10^5 **b.** 10^4 **c.** 10^{10} **d.** 10^3 **e.** 10^{12}

4. Replace N to make a true number sentence, using 10 as the base with the correct exponent.

a. $700 = 7 \times N$ **b.** $8{,}000 = 8 \times N$ **c.** $30{,}000 = 3 \times N$

Copy, add, and check.

	a	b	c	d	e
5.	62 14 75 48 76	32 49 45 92 72	516 89 605	235 385 406	239 143 723
6.	\$31.88 29.33 39.00	\$28.47 51.53 53.65	7,658 5,436 8,360	4,239 8,278 3,504	\$87.16 56.13 88.92

Copy, subtract, and check.

7.	622 456	300 167	705 461	42,345 6,847	\$70.00 43.69
8.	813 484	12,516 7,769	\$800.30 525.75	23,638 6,579	47,005 8,406
9.	6,825 2,326	7,957 1,398	82,364 49,504	\$62.25 30.57	\$913.00 519.13

(For use after page 39)

Copy and multiply.

	a	b	c	d	e	f
1.	65 47	69 38	506 83	96 50	69 90	300 29
2.	\$.76 20	\$.82 40	200 73	645 57	58 80	49 53
3.	3,080 54	256 247	9,006 35	427 165	248 397	5,600 96
4.	\$4.39 709	237 600	894 280	\$2.86 108	\$9.43 801	137 389

In each of the following, find which sentences are true and which are not true. Then find the quotient and the remainder for each division example.

5. $30 \times 22 < 928$; $40 \times 22 < 928$; $50 \times 22 < 928$ $22\overline{)928}$

6. $30 \times 21 < 913$; $40 \times 21 < 913$; $50 \times 21 < 913$ $21\overline{)913}$

7. $40 \times 93 < 2{,}986$; $30 \times 93 < 2{,}956$; $20 \times 93 < 2{,}986$; $93\overline{)2{,}986}$

Find the largest number of tens that will make each sentence true. Then do the example.

8. $N \times 52 < 2{,}784$ $52\overline{)2{,}784}$

9. $N \times 39 < 2{,}124$ $39\overline{)2{,}124}$

10. $N \times 67 < 4{,}260$ $67\overline{)4{,}260}$

Copy, divide, and check.

	a	b	c	d	e
11.	$36\overline{)\$8.76}$	$54\overline{)3{,}264}$	$78\overline{)5{,}670}$	$25\overline{)\$4.55}$	$57\overline{)45{,}628}$
12.	$76\overline{)4{,}925}$	$69\overline{)4{,}395}$	$85\overline{)3{,}210}$	$48\overline{)\$36.00}$	$14\overline{)322}$

Addition and Subtraction with Like Fractions 285

(For use after page 55 or after page 71)

Express all sums in simplest form, as proper fractions, mixed fractions, or numerals for whole numbers.

	a	b	c	d
1.	$\frac{5}{24} + \frac{1}{24}$	$\frac{9}{16} + \frac{11}{16}$	$\frac{3}{5} + \frac{2}{5}$	$\frac{11}{12} + \frac{11}{12}$
2.	$\frac{7}{16} + \frac{3}{16}$	$\frac{8}{9} + \frac{2}{9}$	$\frac{4}{5} + \frac{2}{5}$	$5 + \frac{1}{8}$
3.	$\frac{3}{7} + \frac{3}{7}$	$9 + \frac{5}{12}$	$\frac{1}{32} + \frac{5}{32}$	$\frac{5}{6} + \frac{5}{6}$
4.	$6\frac{11}{16} + 8\frac{7}{16}$	$9\frac{11}{12} + 8\frac{5}{12}$	$9\frac{7}{12} + \frac{5}{12}$	$7\frac{2}{3} + 3\frac{2}{3}$

Copy and subtract. Express the remainders in simplest form.

5. $\frac{11}{24} - \frac{5}{24}$ $\quad \frac{11}{15} - \frac{8}{15}$ $\quad \frac{15}{16} - \frac{9}{16}$ $\quad 16\frac{1}{8} - 9$

Copy and subtract. Express the remainders in simplest form.

6.a. $11\frac{8}{9} - 5\frac{4}{9}$ $\quad$ b. $17\frac{9}{10} - 8\frac{1}{10}$ $\quad$ c. $12\frac{5}{6} - 8\frac{1}{6}$ $\quad$ d. $16\frac{1}{12} - 5\frac{1}{12}$ $\quad$ e. $11\frac{7}{24} - 9\frac{1}{24}$

Find the missing numerals.

	a	b
7.	$(\frac{3}{10} + 8) + \frac{3}{10} = \square$	$\frac{3}{4} + \triangle = \frac{5}{4}$
8.	$14\frac{7}{12} = \triangle + 8\frac{5}{12}$	$7\frac{1}{15} + \square = 14\frac{7}{15}$
9.	$4 + (\frac{2}{3} + \frac{1}{3}) = \square$	$\frac{47}{7} + \triangle = 6\frac{5}{7}$
10.	$9\frac{5}{9} + \square = 15\frac{1}{9}$	$9\frac{2}{15} + \triangle = 18\frac{2}{15}$

Tell which of the following are true and which are false.

	a	b	c
11.	$10\frac{9}{16} < \frac{33}{16}$	$3\frac{1}{16} > \frac{47}{16}$	$\frac{9}{16} < 3 - 2\frac{9}{16}$
12.	$\frac{3}{8} + \frac{1}{8} > 1 - \frac{5}{8}$	$\frac{7}{8} + \frac{3}{8} < 1\frac{5}{8} - \frac{1}{8}$	$5 + \frac{7}{9} > 12\frac{1}{9} - 6\frac{5}{9}$

286 Addition and Subtraction with Unlike Fractions

(For use after page 71)

Copy and add.

	a	b	c	d
1.	$\frac{4}{5} + \frac{3}{4}$	$\frac{5}{5} + \frac{5}{24} + \frac{1}{12}$	$\frac{1}{4} + \frac{1}{10}$	$\frac{2}{3} + \frac{1}{4}$
2.	$\frac{3}{8} + \frac{2}{3}$	$\frac{3}{8} + \frac{1}{2} + \frac{7}{16}$	$\frac{4}{9} + \frac{1}{6}$	$\frac{5}{12} + \frac{1}{8}$
3.	$\frac{3}{5} + \frac{2}{3}$	$\frac{3}{5} + \frac{7}{10} + \frac{1}{2}$	$\frac{5}{12} + \frac{5}{6} + \frac{1}{4}$	$\frac{1}{9} + \frac{5}{6}$

Copy and subtract.

4.	$13\frac{5}{6} - 6\frac{1}{12}$	$7\frac{5}{12} - 4\frac{1}{6}$	$6\frac{3}{4} - 3\frac{3}{16}$	$9\frac{7}{16} - 5\frac{3}{8}$
5.	$12\frac{1}{3} - 3\frac{1}{5}$	$14\frac{1}{8} - 9\frac{7}{8}$	$19\frac{3}{4} - 7\frac{3}{5}$	$16\frac{1}{2} - 5\frac{1}{9}$
6.	$10 - 5\frac{5}{9}$	$7\frac{1}{24} - 5\frac{5}{24}$	$21 - 9\frac{7}{16}$	$14 - 7\frac{7}{8}$

Copy and add or subtract as the signs tell you.

	a	b	c	d	e
7.	$17\frac{1}{2}$ $-6\frac{5}{9}$	$11\frac{3}{5}$ $-1\frac{1}{10}$	$12\frac{4}{9}$ $-6\frac{1}{2}$	$17\frac{1}{8}$ $-8\frac{5}{6}$	$10\frac{1}{3}$ $-5\frac{5}{12}$
8.	$4\frac{1}{6}$ $8\frac{3}{8}$ $+7\frac{3}{4}$	$3\frac{1}{2}$ $7\frac{1}{9}$ $+7\frac{2}{3}$	$6\frac{3}{4}$ $4\frac{1}{6}$ $+5\frac{1}{3}$	$8\frac{5}{6}$ $5\frac{2}{3}$ $+6\frac{1}{8}$	$7\frac{2}{3}$ $1\frac{3}{4}$ $+6\frac{3}{4}$
9.	7 $+6\frac{5}{12}$	$15\frac{1}{6}$ $-6\frac{9}{10}$	$13\frac{3}{5}$ $-10\frac{2}{3}$	$5\frac{5}{6}$ $+4\frac{3}{4}$	$3\frac{2}{5}$ $+2\frac{1}{2}$
10.	$16\frac{1}{10}$ $-9\frac{3}{4}$	$16\frac{5}{16}$ $-15\frac{11}{12}$	$6\frac{1}{2}$ $+3\frac{1}{10}$	$4\frac{1}{4}$ $+6\frac{2}{5}$	$14\frac{1}{2}$ $-6\frac{5}{7}$

Find the missing numerals.

	a	b	c
11.	$\frac{7}{12} + \frac{1}{6} = \square$	$3 + \triangle = 5\frac{3}{5}$	$12\frac{1}{6} - 4\frac{1}{3} = \square$
12.	$6\frac{17}{20} + \triangle = 11\frac{3}{5}$	$2\frac{3}{5} + 7\frac{1}{3} + 3\frac{3}{5} = \square$	$\square - \frac{1}{3} = \frac{7}{8}$
13.	$14\frac{1}{9} - \triangle = 8\frac{17}{18}$	$17\frac{11}{24} = \square + 15\frac{1}{12}$	$13\frac{1}{8} - \square = 9\frac{3}{8}$

Multiplication with Fractions

(For use after page 85 or after page 115)

Find the products and express them in simplest form.

	a	b	c	d
1.	$\frac{1}{4} \times \frac{5}{8}$	$\frac{8}{3} \times \frac{1}{9}$	$\frac{1}{6} \times \frac{17}{5}$	$\frac{11}{10} \times \frac{1}{5}$
2.	$6 \times \frac{3}{4}$	$\frac{5}{12} \times 18$	$2 \times \frac{5}{16}$	$\frac{9}{10} \times \frac{5}{6}$
3.	$\frac{5}{6} \times \frac{3}{5}$	$\frac{5}{16} \times \frac{2}{5}$	$\frac{1}{6} \times 4$	$4 \times \frac{9}{2}$
4.	$\frac{5}{8} \times \frac{2}{3} \times \frac{3}{10}$	$\frac{3}{4} \times \frac{11}{12}$	$\frac{3}{4} \times \frac{5}{6} \times \frac{4}{5}$	$\frac{3}{4} \times \frac{7}{6}$

5.a. $\frac{4}{5} \times \frac{3}{4} \times \frac{5}{12}$ b. $\frac{2}{9} \times \frac{3}{4} \times \frac{6}{7}$ c. $\frac{1}{6} \times \frac{2}{5} \times \frac{10}{11}$

6. Copy the sentences and replace the frames.

a. $6 \times 8\frac{5}{6} = (6 \times \square) + (6 \times \frac{5}{6})$

b. $4\frac{1}{2} \times 12 = (\square \times 12) + (\frac{1}{2} \times 12)$

c. $7 \times 2\frac{1}{2} = (7 \times \square) + (7 \times \triangle)$

d. $9\frac{1}{8} \times 11 = (9 \times \square) + (\triangle \times 11)$

7. Copy the sentences and fill in the correct replacements.

a. $6\frac{2}{3} = \frac{(6 \times \square) + \triangle}{3}$, or $\frac{N}{3}$ b. $3\frac{3}{4} = \frac{(3 \times \square) + \triangle}{4}$, or $\frac{N}{4}$

c. $4\frac{1}{2} = \frac{(4 \times \square) + \triangle}{2}$, or $\frac{N}{2}$ d. $1\frac{1}{5} = \frac{(1 \times \square) + \triangle}{5}$, or $\frac{N}{5}$

Copy and multiply.

	a	b	c	d
8.	$2\frac{1}{8} \times 4$	$2\frac{2}{5} \times 4\frac{1}{6}$	$\frac{2}{3} \times 1\frac{7}{8}$	$10 \times 1\frac{1}{5}$
9.	$\frac{5}{6} \times 2\frac{2}{5}$	$\frac{1}{2} \times \frac{2}{3} \times \frac{9}{16}$	$\frac{3}{4} \times 7$	$2\frac{7}{10} \times 1\frac{1}{9}$
10.	$3\frac{3}{5} \times 2\frac{1}{12}$	$3\frac{1}{5} \times 10$	$1\frac{2}{7} \times 4\frac{2}{3}$	$2\frac{2}{3} \times 1\frac{5}{16}$
11.	$2\frac{2}{3} \times 2\frac{5}{8}$	$2\frac{1}{10} \times 15$	$3\frac{3}{4} \times 1\frac{1}{5}$	$3\frac{7}{8} \times 4$
12.	$7\frac{1}{9} \times 1\frac{1}{8}$	$6\frac{1}{4} \times 3\frac{1}{5}$	$4\frac{1}{2} \times 3\frac{1}{3}$	$10\frac{4}{5} \times 2\frac{7}{9}$

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Division with Fractions

(For use after page 115)

1. Write the reciprocal for each of the following.

a. $\frac{1}{9}$	**b.** $\frac{1}{32}$	**c.** $\frac{7}{1}$	**d.** 3	**e.** $\frac{1}{12}$
f. $\frac{4}{5}$	**g.** $\frac{5}{9}$	**h.** $\frac{28}{3}$	**i.** $\frac{8}{5}$	**j.** $\frac{11}{16}$
k. $2\frac{2}{5}$	**l.** $1\frac{1}{8}$	**m.** $1\frac{5}{16}$	**n.** $3\frac{3}{5}$	**o.** $7\frac{1}{6}$

Using reciprocals, rewrite each division example as a multiplication example and find the answer. Use the sentences as guides.

	a	b
2.	$\frac{2}{3} \div 8 = \triangle \times \frac{2}{3} = \underline{?}$	$3 \div \frac{4}{5} = \square \times 3 = \underline{?}$
3.	$\frac{15}{2} \div 5 = \square \times \frac{15}{2} = \underline{?}$	$\frac{6}{5} \div 8 = \triangle \times \frac{6}{5} = \underline{?}$
4.	$\frac{5}{16} \div \frac{3}{8} = \triangle \times \frac{5}{16} = \underline{?}$	$\frac{2}{5} \div \frac{3}{10} = \triangle \times \frac{2}{5} = \underline{?}$
5.	$10 \div \frac{5}{6} = \square \times 10 = \underline{?}$	$9 \div \frac{9}{10} = \square \times 9 = \underline{?}$

Rewrite each division example as a multiplication example and find the answer.

	a	b	c	d
6.	$\frac{7}{16} \div 14 = \square$	$6\frac{2}{3} \div 4 = \square$	$2\frac{3}{4} \div 6 = \square$	$\frac{5}{8} \div \frac{5}{6} = \triangle$
7.	$\frac{3}{10} \div 6 = \triangle$	$5 \div \frac{4}{5} = \square$	$\frac{7}{16} \div \frac{3}{4} = \triangle$	$1\frac{5}{16} \div \frac{3}{4} = \square$

Name the quotients for the following. Write your answers in simplest form.

8.	$8 \div 15 = \mathrm{N}$	$9 \div 72 = \mathrm{N}$	$5 \div 9 = \square$	$8 \div 40 = \triangle$
9.	$15 \div 45 = \square$	$32 \div 48 = \triangle$	$14 \div 16 = \square$	$17 \div 20 = \triangle$

Copy and divide. In rewriting each example as a multiplication example, leave the numeral for the dividend in its original position.

10.	$\frac{4}{5} \div 6 = \underline{?}$	$3\frac{3}{4} \div \frac{3}{8} = \mathrm{N}$	$3\frac{1}{8} \div \frac{5}{6} = \mathrm{N}$	$9\frac{1}{3} \div \frac{7}{12} = \square$
11.	$16\frac{1}{6} \div 4 = \triangle$	$48 \div 5 = \square$	$12\frac{3}{4} \div 2 = \triangle$	$6 \div \frac{3}{5} = \square$

A Fractional Part of a Number 289

(For use after page 115)

Copy, and find the missing numerals.

	a	b	c
1.	$\frac{4}{5}$ of $45 = \triangle$	$\frac{5}{12}$ of $60 = N$	$\frac{4}{7}$ of $28 = \square$
2.	$\frac{6}{7}$ of $63 = \square$	$\frac{3}{16}$ of $80 = \triangle$	$\frac{8}{9}$ of $54 = \square$
3.	$\frac{5}{9} \times 36 = N$	$\frac{3}{4} \times 36 = \square$	$\frac{7}{8} \times 72 = N$
4.	$\frac{5}{16} \times 48 = \triangle$	$\frac{5}{6} \times 24 = N$	$\frac{7}{10} \times 50 = \square$

5. Jack had a book containing 240 pages to read for his book report. He had read $\frac{7}{8}$ of the book. How many pages had he read? How many were left to read? What fractional part of the number of pages in the book was left?

6. Marilyn had \$12.60 to spend for gifts. After she had bought all of the gifts she needed, she found she had spent $\frac{5}{9}$ of the amount. How much money did she spend? How much did she have left? What fractional part of the amount for gifts did she have left?

Find the fraction (in simplest form) that answers each question.

	a	b
7.	What part of 24 is 14?	10 is what part of 25?
8.	40 is what part of 48?	What part of 30 is 9?
9.	What part of 144 is 48?	18 is what part of 27?
10.	108 is what part of 1,728?	What part of 72 is 12?

11. Twelve of the 15 boys in the class knew how to swim. What fractional part of the number of boys knew how to swim?

12. Barbara received 135 out of 180 votes for class president. What fractional part of the class voted for her?

13. Some television programs run for 13 weeks out of the 52 weeks in the year. What part of a year is 13 weeks?

290 Multiplication and Division with Fractions

(For use after page 115)

Using reciprocals, find the missing numerals.

	a	b
1.	$12 \times (\frac{1}{12} \times 96) = \square$	$\frac{1}{7} \times (7 \times 9) = \square$
2.	$\frac{1}{25} \times (25 \times 400) = \triangle$	$16 \times (\square \times 80) = 80$
3.	$\frac{3}{4} \times (\frac{4}{3} \times 27) = \square$	$\frac{2}{3} \times (\triangle \times 8) = 8$
4.	$\frac{7}{8} \times (\triangle \times 17) = 17$	$\square \times (\frac{16}{9} \times 36) = 36$

Find the single numeral that will make each sentence true. Use the second sentence of each pair as a guide.

5.	$5 \times N = 65$ $5 \times (\square \times \triangle) = 65$	$\frac{1}{4} \times N = 11$ $\frac{1}{4} \times (\square \times \triangle) = 11$
6.	$\frac{8}{5} \times N = 56$ $\frac{8}{5} \times (\square \times \triangle) = 56$	$\frac{16}{9} \times N = 80$ $\frac{16}{9} \times (\square \times \triangle) = 80$

Copy, and find the missing numerals.

	a	b	c
7.	$\frac{1}{6} \times N = 21$	$\frac{2}{3} \times \underline{?} = 36$	$\frac{3}{8} \times N = 24$
8.	$\frac{5}{3} \times N = \frac{5}{3}$	$\frac{18}{5} \times N = 54$	$\frac{1}{12} \times N = 5\frac{1}{6}$

Without multiplying or dividing, tell which of these sentences are true and which are not true.

9.	$\frac{9}{10} \times 6 < 6$	$\frac{10}{9} \times 4 < 4$	$2\frac{1}{3} \times 2 < 2$
10.	$5 \times \frac{2}{3} > 5$	$17 \times \frac{3}{2} > 17$	$8 \div \frac{1}{2} > 8$
11.	$7 \times \frac{5}{5} < 7$	$7 \div \frac{5}{4} < 7$	$7 \times \frac{5}{4} < 7$
12.	$9 \div \frac{1}{3} > 9$	$9 \div \frac{2}{3} > 9$	$9 \div \frac{3}{3} > 9$

Divide, and express the quotient as a mixed fraction.

	a	b	c	d
13.	$30\overline{)1{,}745}$	$48\overline{)1{,}216}$	$24\overline{)1{,}554}$	$40\overline{)850}$
14.	$25\overline{)380}$	$96\overline{)3{,}024}$	$63\overline{)4{,}564}$	$64\overline{)2{,}632}$

Precision of Measurement 291

(For use after page 123 or after page 143)

1. Tell the precision of each of the following measurements. Example **a** is done for you to serve as a guide.

a. $3\frac{1}{4}$ in. *Correct to the nearest quarter inch*

b. 4 in. **c.** 55 in. **d.** $14\frac{1}{2}$ in. **e.** $\frac{8}{10}$ in. **f.** $13\frac{0}{8}$ in.

g. $7\frac{5}{64}$ in. **h.** $19\frac{8}{16}$ in. **i.** $1\frac{0}{16}$ in. **j.** 2,150 in.

k. 13 in. **l.** $7\frac{2}{4}$ in. **m.** $7\frac{4}{8}$ in. **n.** 17,562 in.

For each of the following pairs of measurements, tell which is the more precise.

	a	b	c
2.	$4\frac{5}{16}$ in.; $5\frac{4}{32}$ in.	$4\frac{1}{2}$ in.; $2\frac{1}{4}$ in.	$16\frac{0}{2}$ in.; 16 in.
3.	$17\frac{6}{8}$ in.; $17\frac{3}{4}$ in.	$8\frac{1}{2}$ in.; $8\frac{2}{4}$ in.	23 in.; $125\frac{1}{2}$ in.

Tell the greatest possible error in each of the following measurements.

	a	b	c	d
4.	18 ft.	232 ft.	$13\frac{1}{4}$ in.	$6\frac{1}{4}$ gal.
5.	4 oz.	$97\frac{1}{2}$ lb.	1 gal. 3 qt.	2 cups
6.	21 gal.	647 acres	3 tons	5 hands
7.	2 mi.	83 mi.	$14\frac{0}{16}$ in.	$\frac{19}{64}$ in.

Replace the question marks so that the sentences are true.

	a	b
8.	4 ft. 12 in. = ? ft.	2 min. 6 sec. = ? sec.
9.	1 T. 800 lb. = ? lb.	10 bu. 7 pk. = ? bu. ? pk.
10.	6,000 ft. = ? mi. ? ft.	65 pk. = ? bu. ? pk.
11.	150 in. = ? yd. ? in.	3,500 lb. = ? T. ? lb.
12.	88 oz. = ? lb. ? oz.	5 gal. 2 qt. = ? qt.
13.	5 hr. 20 min. = ? min.	2 yd. 27 in. = ? in.
14.	15 min. 40 sec. = 14 min. ? sec.	3 lb. $4\frac{5}{8}$ oz. = ? oz.

(For use after page 123 or after page 143)

Copy, and fill in the blanks to make true sentences.

	a	b
1.	5 min. = ? sec.	66 in. = ? yd. ? in.
2.	84 days = ? wk. ? days	$3\frac{1}{4}$ lb. = ? oz.
3.	44 ft. = ? yd.	28 in. = ? ft. ? in.
4.	52 oz. = ? lb. ? oz.	$10\frac{1}{2}$ ft. = ? yd. ? ft.
5.	$4\frac{1}{2}$ qt. = ? pt.	18 qt. = ? gal. ? qt.
6.	$2\frac{1}{2}$ pk. = ? qt.	96 min. = ? hr. ? min.
7.	$2\frac{1}{2}$ bu. = ? qt.	$1\frac{1}{2}$ gal. = ? pt.
8.	$7\frac{1}{2}$ yd. = ? ft. ? in.	88 in. = ? ft.

9. Tell the number of hours and minutes in each of the following trips.

a. Start at 3:30 P.M. and arrive at 8:45 P.M.
b. Start at 8:30 A.M. and arrive at 9:20 P.M.
c. Start at 6:45 P.M. and arrive at 12:15 A.M.

Copy, and fill in the blanks to make true sentences.

10.a. 3 hr. 10 min. = 2 hr. ? min.
b. 10 lb. 4 oz. = 9 lb. ? oz.

11.a. 6 lb. 8 oz. = 5 lb. ? oz.
b. 11 yd. 14 ft. = ? yd. ? ft.

12.a. 10 gal. 2 qt. = 9 gal. ? qt.
b. 4 yd. 1 ft. = 3 yd. ? ft.

13.a. 5 ft. 14 in. = ? ft. ? in.
b. 2 gal. 10 pt. = ? gal. ? pt.

14.a. 4 qt. 9 pt. = ? qt. ? pt.
b. 7 ft. $4\frac{3}{4}$ in. = 6 ft. ? in.

15.a. 6 wk. 3 days = 5 wk. ? days
b. 9 yr. 6 mo. = 8 yr. ? mo.

Ratio; Measures 293

(For use after page 134 or after page 143)

1. Using the smallest whole numbers possible, express the ratio of:

a. \$5 to \$40
b. 100 mi. to 200 mi.
c. \$200,000 to \$50,000
d. $8\frac{1}{2}$ ft. to $12\frac{1}{2}$ ft.
e. 3 yd. to 2 ft.
f. 2 T. to 500 lb.
g. $1\frac{1}{5}$ to 1
h. $37\frac{1}{2}$ to 100
i. $1\frac{1}{4}$ to 1
j. 75 to 100
k. $3\frac{5}{8}$ to $1\frac{7}{8}$
l. 3 to $1\frac{1}{2}$
m. 3 ft. to 3 in.
n. \$8 to \$.08
o. 3 lb. to 15 oz.
p. \$7.50 to \$10.00

Copy and add.

	a	b	c
2.	3 yr. 9 mo. 2 yr. 6 mo.	8 hr. 35 min. 3 hr. 45 min.	7 days 16 hr. 6 days 8 hr.
3.	4 mi. 2,640 ft. 9 mi. 2,760 ft.	5 yd. 2 ft. 2 yd. 2 ft.	7 min. 45 sec. 9 min. 45 sec.
4.	6 bu. 3 pk. 8 bu. 2 pk. 3 pk.	9 yd. 8 in. 4 yd. 4 in. 1 yd. 27 in.	8 gal. $2\frac{1}{4}$ qt. 3 gal. $3\frac{1}{4}$ qt. 2 gal.
5.	8 ft. $5\frac{3}{4}$ in. 7 ft. $3\frac{1}{4}$ in.	5 T. 1200 lb. 4 T. 900 lb.	9 qt. $1\frac{1}{2}$ pt. 6 qt. $1\frac{1}{2}$ pt.

Copy and subtract.

6.	9 T. 200 lb. 3 T. 800 lb.	12 ft. 6 in. 3 ft. 9 in.	5 lb. 6 oz. 14 oz.
7.	1 min. $4\frac{4}{5}$ sec. 16 sec.	7 yr. $2\frac{1}{2}$ mo. 4 yr. $4\frac{1}{2}$ mo.	18 gal. 1 qt. 8 gal. 3 qt.

294 Measures: Subtraction, Multiplication, and Division

(For use after page 134 or after page 143)

Copy and subtract.

	a	b	c
1.	10 mi. 4 mi. 3,798 ft.	11 yr. 4 yr. 9 mo.	15 min. 20 sec. 9 min. 45 sec.
2.	12 yd. 9 in. 6 yd. 27 in.	8 lb. 4 oz. 2 lb. 10 oz.	14 ft. $2\frac{1}{4}$ in. 5 ft. $8\frac{1}{2}$ in.
3.	5 days 6 hr. 18 hr.	11 gal. 5 gal. 1 pt.	15 yd. 1 ft. 7 yd. $2\frac{1}{2}$ ft.

Express each of the following measurements, using a mixed fraction.

	a	b
4.	4 qt. 1 pt. = ? qt.	6 yd. 2 ft. = ? yd.
5.	6 hr. 20 min. = ? hr.	3 gal. 2 qt. = ? gal.
6.	3 T. 500 lb. = ? T.	4 ft. 9 in. = ? ft.
7.	3 min. 45 sec. = ? min.	2 lb. 12 oz. = ? lb.

Copy and multiply.

8.	2 × 3 qt. 1 pt.	5 × 3 yr. 4 mo.
9.	6 × 4 ft. 2 in.	3 × 11 bu. 3 pk.
10.	4 × 4 T. 400 lb.	8 × 3 days $12\frac{1}{2}$ hr.
11.	7 × 6 ft. 2 in.	10 × 4 hr. 15 min.
12.	5 × 5 bu. 1 pk.	8 × 3 bu. 8 qt.

Copy and divide.

13.	150 sq. in ÷ $\frac{2}{3}$	40 sq. ft. ÷ $1\frac{1}{4}$
14.	$18\frac{1}{2}$ ft. ÷ 4	10 yd. 2 ft. ÷ 8
15.	6 gal. 3 qt. ÷ 3	2 hr. 40 min. ÷ 4
16.	5 lb. 8 oz. ÷ $1\frac{1}{2}$	8 bu. 2 pk. ÷ 4
17.	9 hr. 15 min. ÷ $1\frac{1}{4}$	8 qt. 2 pt. ÷ 4

Finding Perimeters and Areas

(For use after page 143)

Find the perimeter of each of the following rectangles.

a	b
1. $6\frac{1}{2}$ yd. by $1\frac{1}{4}$ yd.	165 ft. by 85 ft.
2. $12\frac{1}{5}$ in. by $16\frac{7}{10}$ in.	$2\frac{1}{4}$ mi. by $4\frac{1}{2}$ mi.
3. $24\frac{3}{4}$ in. by 8 in.	$32\frac{1}{2}$ ft. by 26 ft.

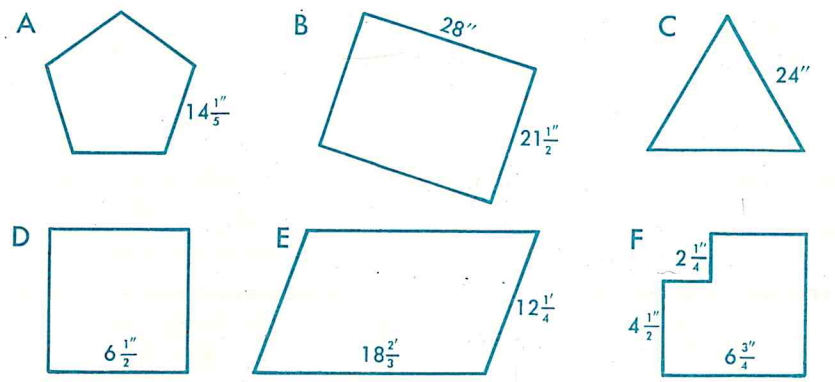
Find the area of each of the following rectangles.

4. 150 yd. by 35 yd.	21 in. by $9\frac{1}{2}$ in.
5. 9 ft. by 15 ft.	$11\frac{1}{2}$ in. by 8 in.
6. $3\frac{1}{2}$ mi. by $2\frac{1}{4}$ mi.	$3\frac{1}{4}$ in. by $4\frac{1}{4}$ in.

7. Find the area of a square when its side has a length of:

a. $\frac{1}{4}$ ft.	b. $2\frac{3}{4}$ in.	c. 200 ft.	d. 13 in.
e. $1\frac{1}{2}$ mi.	f. 10 mi.	g. 145 ft.	h. $5\frac{1}{2}$ yd.

8. Find the perimeters of the figures shown below. Sides that look as though they have the same length actually do.



9. What is the perimeter of a rectangle that is $3\frac{3}{4}$ ft. long and $2\frac{2}{3}$ ft. wide? What is the area of the rectangle?

10. Write the lengths and widths of four different rectangles, each having an area of 60 sq. ft.

296 Scale Drawing; Measures

(For use after page 143)

1. If the scale of a drawing is 1 in. for 12 ft., how many feet does each of the following measurements represent?

a. 4 in. **b.** 24 in. **c.** $10\frac{1}{4}$ in. **d.** $2\frac{1}{2}$ in.

2. With the same scale as in Exercise **1**, how many inches on the drawing would represent each of the following actual measurements.

a. 36 ft. **b.** 288 ft. **c.** 40 ft. **d.** 33 ft.

3.a. The scale for a drawing is 1 in. for 20 mi. What is the actual distance if the distance on the drawing measures 10 in.? 3 in.? $4\frac{1}{2}$ in.?

b. What distance on the drawing would represent an actual distance of 5 mi.? 200 mi.? 45 mi.?

4. The scale for a map, expressed as a ratio, is 1 to 200. What actual distance will be represented by a distance on the drawing of 1 in.? 1 ft.? $\frac{1}{4}$ in.? 10 in.? $\frac{1}{2}$ ft.? $\frac{3}{10}$ in.?

5. Expressed as a ratio, the scale of a drawing is 1 to 6. What actual distance would be represented by a measurement of 20 in.? 4 in.? $\frac{1}{2}$ in.? $2\frac{1}{2}$ in.? $3\frac{1}{2}$ in.? 10 in.?

Choose the largest measurement from each set below.

	a	b
6.	12 ft.; $4\frac{1}{2}$ yd.; 1 rd.	2 mo.; 6 wk.; 65 days
7.	3 sq. ft.; 1 sq. yd.; 144 sq. in.	16 cups; 9 pt.; 3 qt.
8.	$\frac{1}{6}$ hr.; 120 sec.; 3 min.	$1\frac{3}{4}$ bu.; 48 qt.; 5 pk.
9.	54 wk.; 365 days; 11 mo.	400 yd.; 1,500 ft.; $\frac{1}{2}$ mi.
10.	9 in.; $\frac{1}{3}$ yd.; $\frac{2}{3}$ ft.	$1\frac{1}{2}$ gal.; 7 qt.; 10 pt.

Write one date that fell within:

11. the first half of the eleventh century
12. the third quarter of the seventh century
13. a leap year in the thirteenth century
14. the middle of the twentieth century
15. the first quarter of the seventeenth century
16. the last quarter of the sixteenth century

Decimal Fractions 297

(For use after page 169)

Write each of the following as a decimal mixed fraction.

	a	b	c	d
1.	$6\frac{3}{10}$	$15\frac{7}{10}$	$75\frac{25}{100}$	$1\frac{7}{10}$
2.	$12\frac{9}{10}$	$40\frac{15}{100}$	$8\frac{5}{100}$	$2\frac{135}{1000}$
3.	$1\frac{1,678}{10,000}$	$3\frac{5}{100}$	$\frac{4}{100}$	$\frac{15}{1000}$

Write each of the following as a fraction or a mixed fraction.

4.	.3	.45	.125	.8935
5.	.03	.045	.012	.08935
6.	1.42	13.06	17.829	4.0007

7. Write a numeral for each of the following, using digits.

a. Three and fifty-one thousandths
b. Six and five thousandths
c. Four and fifty-five hundredths
d. Ten and one hundredth
e. Fifteen ten thousandths
f. Thirty-two tenths
g. Six and eighty-four hundredths
h. Seventy-two ten thousandths
i. Twenty-one and twenty-one hundred thousandths

8. Copy and complete these sentences.

a. .54 = 5 tenths + ? hundredths
b. .926 = ? tenths + ? hundredths + ? thousandths
c. 27.7 = ? tens + ? ones + ? tenths
d. .008 = ? tenths + ? hundredths + ? thousandths
e. 4.08 = ? ones + ? tenths + ? hundredths

9. Express each of the following as a decimal.

a. $\frac{53}{10}$ b. $\frac{672}{100}$ c. $\frac{905}{100}$ d. $\frac{8,007}{100}$ e. $\frac{85}{10,000}$ f. $\frac{9672}{1000}$

Copy, and replace the frames.

	a	b	c	d
10.	$1.6 = \frac{\square}{10}$	$8.7 = \frac{\square}{10}$	$.035 = \frac{\triangle}{1000}$	$2.04 = \frac{\triangle}{100}$
11.	$3.09 = \frac{\diamond}{100}$	$8.92 = \frac{\diamond}{100}$	$.0093 = \frac{\square}{10,000}$	$1.01 = \frac{\triangle}{100}$
12.	$.9 = \frac{\diamond}{100}$	$1.5 = \frac{\square}{100}$	$3.006 = \frac{\triangle}{1000}$	$82.1 = \frac{\diamond}{100}$

298 Decimals: Addition, Subtraction, and Rounding

(For use after page 169)

Copy, add, and check.

	a	b	c	d
1.	16.56 25.75 8.07 79.82	68.401 84.005 46.322 7.120	25.3 78.4 54.2 62.0	14.264 82.896 35.702 69.084
2.	16.21 68.34 39.10 53.33	33.405 54.997 7.269 80.575	7.3 3.5 4.7 3.4	146.12 64.02 7.60 738.01

Copy, subtract, and check.

3.	67.568 34.525	30.24 24.57	65.75 35.89	96.007 38.799
4.	45.51 18.64	4.379 2.053	36.85 24.72	74.146 9.287

Find the sum, or difference, as the signs tell you.

	a	b	c
5.	$.032 + .123 = \square$	$.09 + .08 = \triangle$	$.17 - .06 = \square$
6.	$2.83 - .32 = \triangle$	$2.05 + .29 = \square$	$.99 - .67 = \triangle$
7.	$5.26 + 7.03 = \square$	$79.48 - 5.08 = \triangle$	$48.7 - 12.3 = \square$

8. Round to the nearest tenth.

a. 3.87	**b.** 4.93	**c.** 25.07	**d.** 3.54	**e.** 4.55
f. 9.815	**g.** .8930	**h.** .1549	**i.** 2.981	**j.** 1.06

9. Round to the nearest whole number.

a. 7.9	**b.** 16.09	**c.** .9	**d.** 3.18	**e.** 6.3
f. 9.7	**g.** 13.5	**h.** 23.4	**i.** 1.2	**j.** 101.8

Multiplication with Decimals 299

(For use after page 195)

First estimate the product, then multiply.

	a	b	c
1.	$4 \times 7.002 = \square$	$4.8 \times 2 = \triangle$	$8 \times 1.895 = \square$
2.	$6 \times 3.198 = \diamond$	$.9 \times 5 = \square$	$4 \times 1.50 = \diamond$
3.	$5.9 \times 3.6 = \triangle$	$4.9 \times 6.09 = \diamond$	$8.1 \times 5 = \triangle$
4.	$4.6 \times 24 = \square$	$7 \times 2.05 = \diamond$	$2.4 \times 1.39 = \square$

Copy and multiply.

	a	b	c	d	e
5.	640 54.7	68.5 5.08	70.3 .08	2.39 382	317 .067
6.	75.9 3.7	720 .058	3.25 57	24.1 8.01	.69 4.3
7.	809 .009	5.80 1.9	406 .76	46.9 8.2	4.83 7.4
8.	2.04 25.7	3.26 526	12.9 7.5	42.1 .96	157 3.09
9.	.009 .04	.731 .01	89.1 .05	.064 .04	.311 .09

Find the products without copying the sentences.

	a	b	c
10.	$100 \times .365 = \diamond$	$10 \times 24.9 = \triangle$	$1000 \times 15.7 = \diamond$
11.	$1000 \times 36.4 = \square$	$10 \times 8.39 = \diamond$	$100 \times .0879 = \square$
12.	$10 \times .915 = \triangle$	$1000 \times 1.042 = \diamond$	$100 \times 11.06 = \triangle$

(For use after page 195)

For each of the following, estimate the quotient. Then (in **a**) divide as if you were dividing with whole numbers, and place the decimal point where it belongs. Check, using the sentence in **b**.

		a	b
1.	$9.63 \div 3 = \square$	$3\overline{)9.63}$	$\square \times 3 = 9.63$
2.	$13.2 \div 12 = \triangle$	$12\overline{)13.2}$	$\triangle \times 12 = 13.2$
3.	$7.28 \div 7 = \diamond$	$7\overline{)7.28}$	$\diamond \times 7 = 7.28$
4.	$96.8 \div 8 = \diamond$	$8\overline{)96.8}$	$\diamond \times 8 = 96.8$
5.	$1.08 \div 1.2 = \triangle$	$1.2\overline{)1.08}$	$\triangle \times 1.2 = 1.08$
6.	$.96 \div 2.4 = \square$	$2.4\overline{).96}$	$\square \times 2.4 = .96$
7.	$43.2 \div 7.2 = \triangle$	$7.2\overline{)43.2}$	$\triangle \times 7.2 = 43.2$
8.	$7.2 \div .9 = \diamond$	$.9\overline{)7.2}$	$\diamond \times .9 = 7.2$
9.	$29.05 \div 3.5 = \square$	$3.5\overline{)29.05}$	$\square \times 3.5 = 29.05$
10.	$56 \div .8 = \square$	$.8\overline{)56}$	$\square \times .8 = 56$

Find the quotients. Check by multiplication.

	a	b	c	d
11.	$.07\overline{).49}$	$9.8\overline{).4998}$	$.24\overline{).192}$	$73\overline{)41.61}$
12.	$.77\overline{)2.695}$	$.7\overline{)42}$	$.8\overline{).24}$	$.018\overline{).3726}$
13.	$.08\overline{).072}$	$.68\overline{).2992}$	$67\overline{)3.618}$	$1.9\overline{).95}$
14.	$.6\overline{)5.46}$	$.85\overline{)7.65}$	$3.1\overline{).1643}$	$.47\overline{)12.22}$

15. Choose the correct quotient from the numbers named in parentheses.

a.	$12\overline{)90}$	(.75; 7.5; 75; 750)
b.	$.62\overline{)248}$	(4; 40; 400; 4,000)
c.	$16\overline{)50}$	(3.125; 31.25; 312.5; 3,125)
d.	$.17\overline{)357}$	(2.1; 21; 210; 2,100)
e.	$.05\overline{)30}$	(.6; 6; 60; 600)

Fractions and Decimals

(For use after page 195)

Express each of the following fractions as a decimal. Use the sentences as a guide.

	a	b	c
1.	$\frac{5}{8} = \frac{\triangle}{1000} = \square$	$\frac{3}{2} = \frac{\triangle}{10} = \square$	$\frac{3}{25} = \frac{\triangle}{100} = \square$
2.	$\frac{2}{5} = \frac{\triangle}{10} = \square$	$\frac{7}{4} = \frac{\triangle}{100} = \square$	$\frac{3}{20} = \frac{\triangle}{100} = \square$
3.	$\frac{1}{50} = \frac{\triangle}{100} = \square$	$\frac{4}{5} = \frac{\triangle}{10} = \square$	$\frac{7}{50} = \frac{\triangle}{100} = \square$
4.	$\frac{6}{25} = \frac{\triangle}{100} = \square$	$\frac{3}{40} = \frac{\triangle}{1000} = \square$	$\frac{24}{25} = \frac{\triangle}{100} = \square$

5. Find the decimal representations for these fractions correct to the nearest thousandth.

a. $\frac{7}{12}$	**b.** $\frac{1}{6}$	**c.** $\frac{4}{11}$	**d.** $\frac{8}{15}$	**e.** $\frac{2}{3}$
f. $\frac{5}{7}$	**g.** $\frac{9}{14}$	**h.** $\frac{2}{11}$	**i.** $\frac{5}{6}$	**j.** $\frac{7}{6}$

Copy and divide. Annex enough zeros so that the division comes out even.

	a	b	c	d	e
6.	$15\overline{)27}$	$88\overline{)572}$	$50\overline{)285}$	$12\overline{)45}$	$65\overline{)377}$
7.	$90\overline{)315}$	$80\overline{)332}$	$46\overline{)345}$	$48\overline{)78}$	$52\overline{)65}$
8.	$60\overline{)138}$	$64\overline{)288}$	$15\overline{)36}$	$16\overline{)14}$	$24\overline{)6}$
9.	$24\overline{)15}$	$32\overline{)8}$	$4\overline{)30}$	$12\overline{)15}$	$18\overline{)45}$

Copy, add, and check.

10.	64.53	57.29	49.4	62.431	86.35
	77.04	801.22	68.8	57.152	68.54
	27.47	67.97	80.4	8.012	41.27
	9.35	586.58	7.6	76.201	63.54
11.	25.33	219.34	45.6	76.213	8.638
	55.79	986.43	75.5	2.751	.289
	63.41	61.40	27.7	94.446	3.961
	2.05	74.29	83.3	9.008	.052

302 Decimals

(For use after page 195)

1. Write the quotients without copying the examples.

a. $14.8 \div 100$ **b.** $82.6 \div 10$ **c.** $125.6 \div 1{,}000$
d. $23.95 \div 100$ **e.** $8{,}251 \div 1{,}000$ **f.** $8.92 \div 10$
g. $1.714 \div 1{,}000$ **h.** $436.7 \div 100$ **i.** $4.62 \div 10$

2. Find the sums or differences using decimals. Check by using fractions.

a. $6\frac{1}{2} + 5\frac{3}{10} = \triangle$ **b.** $4\frac{4}{5} - 3\frac{7}{10} = \square$ **c.** $11\frac{1}{4} - 6\frac{9}{10} = \square$
d. $6\frac{3}{4} + 8\frac{1}{2} = \square$ **e.** $15\frac{1}{2} - 8\frac{3}{5} = \square$ **f.** $8\frac{1}{5} + 2\frac{1}{4} = \triangle$

3. Copy, and make the correct replacements.

a. $.06 = 6 \times \frac{1}{100}$, or $6 \times \frac{1}{10?}$
b. $.009 = 9 \times \frac{1}{\triangle}$, or $9 \times \frac{1}{10?}$
c. $.0006 = 6 \times \frac{1}{\square}$, or $6 \times \frac{1}{10?}$
d. $.003 = 3 \times \frac{1}{10?}$ **e.** $.02 = 2 \times \frac{1}{10?}$
f. $7.23 = 7 + (2 \times \frac{1}{\square}) + (3 \times \frac{1}{10?})$

4. Express each of the following, using exponents.

a. 26.57 **b.** .8169 **c.** 365.192
d. 5,082.6417 **e.** .825 **f.** .05473

Copy, subtract, and check.

	a	b	c	d	e
5.	426.42 268.53	905.30 714.96	459.8 25.3	931.44 624.18	65.319 7.539
6.	630.83 463.14	74.650 61.609	59.874 34.572	842.70 575.97	524.72 86.27

Copy and multiply.

7.	1.74 .05	.158 .82	9.57 .09	3.76 4.7	.852 1.7
8.	.905 .04	4.18 .019	57.8 .06	.362 3.6	64.2 .29

Graphs

(For use after page 207 or page 238)

1. During a six-month period, John made deposits in his savings account as follows: January, $1.50; February, $2.00; March, $3.20; April, $2.00; May, $4.00; June, $3.60. Make a bar graph showing this information.

2. The table shows the total sales of the Carleton Manufacturing Company during each of five years. Make a line graph showing this information. Round the data after you have decided what each space on your graph is to represent.

Annual Sales
Carleton Company

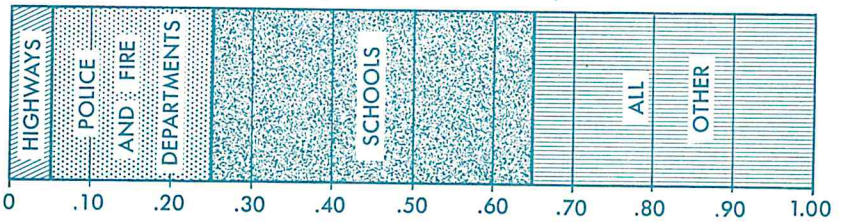
1958	$3,468,532
1959	$3,908,465
1960	$4,200,141
1961	$2,897,900
1962	$3,375,280

3. The table at the right lists the kinds of birds that were seen by a group of Girl Scouts during a nature hike. Show this information in a pictograph. Begin by choosing a suitable symbol and deciding how many birds each symbol in the pictograph should represent.

Birds	Number Seen
Robins	20
Bluejays	18
Starlings	24
Chickadees	4
Cardinals	8

4. Study the following divided-bar graph. Then answer the questions below the graph.

Spruce City Government Expenses



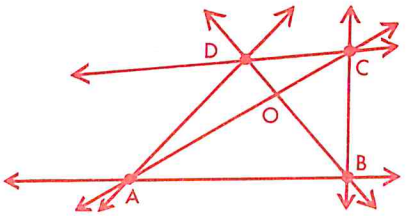
a. Write a decimal fraction that shows what part of the total yearly expenses is spent for each item in the graph.

b. Is the combined amount spent for highways and police and fire departments more, or less, than the amount spent for schools?

c. Of each $100 spent, how much more is spent for schools than for highways? than for the Police and Fire Departments?

(For use after page 224 or after page 238)

1. Mark three different dots on your paper to represent points. Be sure that not all of them are on the same line. Call the points A, B, and C. How many lines are there through A and B? through B and C? through C and A? How many lines in all? Name the lines.



2.a. Name the five points shown by dots in the diagram at the left. Are three of these points on the same line anywhere in the diagram? Name the two sets of three points that are on a line.

b. Name in two ways each of the six lines in the diagram.

c. Which lines intersect in D? in O? in A?

d. Name the segments into which O divides $\overline{AC}$.

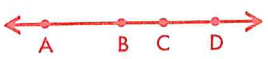
e. There are ten segments that can be named, using the letters in the diagram. What are they?

3.a. Use the diagram of Exercise **2** to answer the following questions. What is the name of the angle formed by rays OC and OB? by rays AB and AD? by rays DC and DB?

b. What is the name of the angle formed by rays OA and OD? Give another name for this angle.

c. How many rays have in D a common end point? How many rays have in O a common end point?

d. There are sixteen angles that can be named, using the letters in the diagram. What are they?



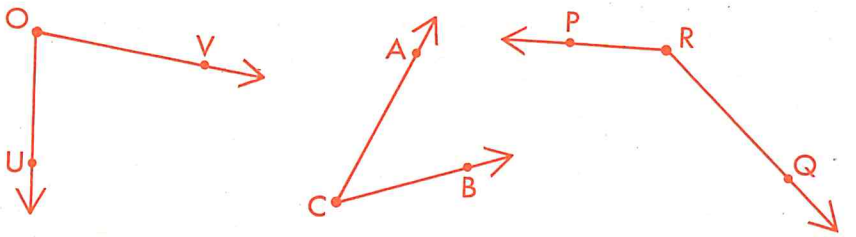
4.a. At the left is shown a line with four points on it marked. Name a segment that includes $\overline{BC}$ in its interior. Name a segment that has one point in common with $\overline{BC}$. How many such segments can you name?

b. Name two segments that contain the same points as $\overline{AC}$. Name two segments that contain the same points as $\overline{BD}$.

c. If you combine the points in $\overline{AB}$ with the points in $\overline{CD}$, do you get all of the points in $\overline{AD}$? What points are missing?

Angles; Closed Figures

(For use after page 224 or after page 238)



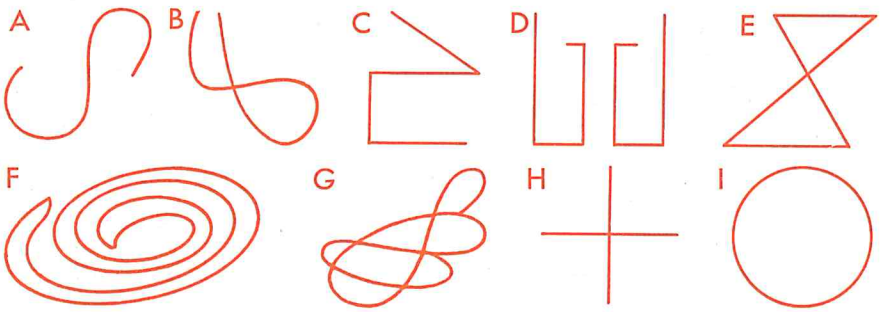
1. Shown above are three angles. Write three different names for each of them.

2. Using your protractor, find the measurement in degrees of each of the angles. On your paper, draw angles with these measurements.

3. On each of your drawings, shade the interior of the angle and label it. Then shade in a different way the exterior of the angle and label it.

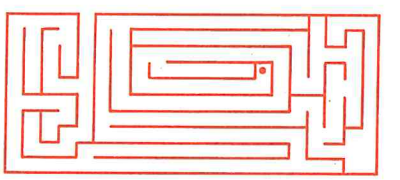
4. Does any one of the above angles represent a right angle? Why not? Is there one that is nearer to being a right angle than the others? How much less than a right angle is it?

5. Write the letters of the figures below that are closed figures.



6. Which of the figures above are simple closed figures?

7. Copy the figure at the right. Is it a closed figure? Is it a simple closed figure?



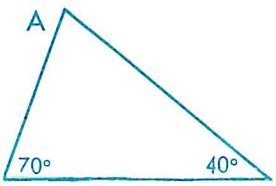
306 Finding the Measure of an Angle

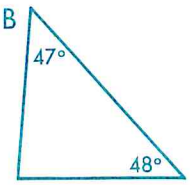
(For use after page 227 or after page 238)

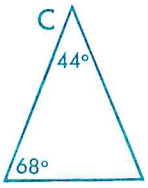
1. On page 227 you found that the sum of the degree measures of the angles of a triangle is 180. We can make use of this fact to find the measure of the third angle of a triangle when the measures of the other two angles are given.

The measure of angle A in a triangle is 30 and the measure of angle B is 50. What is the measure of angle C? Does the number sentence, $30 + 50 + \square = 180$, tell about the problem? What is the correct replacement for the frame?

2.a. For each of the triangles below write a number sentence that you can use to find the measure of the third angle. Then find it.

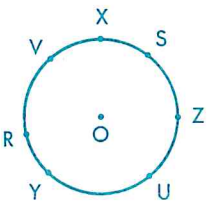






b. Use your protractor to find the measure of the third angle as a check on your work.

3. In the diagram several points on the circle are labeled. Using the names of these points, name the shortest arc containing each of the following points.



a. R **b.** U **c.** X **d.** V **e.** S **f.** Y

4. What are the end points for each of the following arcs?

a. arc RYU **b.** arc SXV **c.** arc XYZ **d.** arc YRV

5. Find the arc-degree measure of each of the following.

a. $\frac{1}{5}$ of a circle
b. $\frac{2}{3}$ of a circle
c. $\frac{3}{5}$ of a circle
d. $\frac{4}{9}$ of a circle
e. $\frac{3}{4}$ of a circle
f. $\frac{1}{8}$ of a circle
g. $\frac{5}{8}$ of a circle
h. $\frac{7}{10}$ of a circle

6. What is the measure of the central angle corresponding to each of the arcs mentioned in Exercise **5**?

Using Percentage 307

(For use after page 261)

Express each of the following hundredths as a per cent.

	a	b	c	d	e	f
1.	$\frac{63}{100}$	$\frac{70}{100}$	.11	.02	.08	.10
2.	.60	.16	.08	.93	$.03\frac{1}{2}$	$.05\frac{1}{2}$

Express each of the following per cents as a decimal.

3.	23%	18%	7%	85%	1%	97%
4.	42%	3%	56%	33%	69%	5%

Express each per cent as a common fraction in simplest form.

5.	14%	6%	30%	75%	4%	24%
6.	99%	36%	16%	15%	65%	12%

7. Write the answer to each of the following questions, using a per cent sign.

a. 18 inches is what per cent of a yard?
b. 9 inches is what per cent of a foot?
c. 3 quarts is what per cent of a gallon?
d. 1 quart is what per cent of a gallon?
e. 12 ounces is what per cent of a pound?
f. 14 ounces is what per cent of a pound?
g. 1 gallon is what per cent of $1\frac{1}{4}$ gallons?

Find what each of the following means.

	a	b	c
8.	25% of \$416	$33\frac{1}{3}$% of 96	80% of \$75
9.	$37\frac{1}{2}$% of \$64	29% of \$45	44% of \$35
10.	$66\frac{2}{3}$% of \$72	6% of 250	5% of 800
11.	60% of \$135	18% of 200	12% of 75
12.	60% of 500	2% of 400	25% of 48
13.	72% of 200	85% of \$216	28% of \$6,500

(For use after page 261)

1. Don and Jim had a basketball-shooting contest. Their scores for two games are listed below. Express each score as a per cent.

First Game	Don: 20 out of 25	Jim: 15 out of 25
Second Game	Don: 32 out of 50	Jim: 35 out of 50

2. Barbara had the correct answers for 42 out of 50 examples. What per cent of the examples did she have correct?

3. Find the per cent that one number is of the other. Round each answer to the nearest whole per cent.

a. 14 is what per cent of 21?
b. What per cent of 38 is 27?
c. 49 is what per cent of 51?
d. What per cent of 17 is 2?
e. 47 is what per cent of 87?
f. What per cent of 19 is 18?
g. 24 is what per cent of 50?
h. What per cent of 74 is 21?
i. 8 is what per cent of 91?
j. What per cent of 89 is 72?

Find the missing numerals.

a	b
4. 12% of N is 36	18 is 15% of N
5. 15% of N is 5.4	75% of N is $1,800
6. 48 is 15% of N	74 is 40% of N

Find what each of the following equals.

7. 9% of 500 gallons	35% of 180 pounds
8. 22% of $200	13% of 300 miles
9. 75% of 180 yards	20% of 100 inches
10. 5% of $360	10% of 330 feet
11. 25% of 16 ounces	6% of 450 gallons

Problem Solving

(For use after page 261)

1. A jet plane flew from Chicago to Portland, Oregon, in 4 hours. If the plane flew at an altitude above 30,000 feet for 75% of the time, how many minutes did the plane fly above 30,000 feet?

2. In a recent basketball game the Vikings attempted 80 shots from the floor, with a shooting per cent of 45%. The Tigers attempted 70 shots and had a shooting per cent of 40%. If each team scored 10 points in free throws, what was the final score of the game?

3. The Sherman School has an enrollment of 725 pupils. The Board of Education announced that 68% of the students ride the school bus to school. How many students ride the bus?

4. The science class placed a pan containing a gallon of water on the school steps to study water evaporation. At the end of the day, 25% of the water had evaporated. How many ounces of water were left in the pan?

5. A farmer owns 240 acres of land. He sowed 40% of it with wheat. How many acres of wheat did he sow?

6. One Saturday, Richard weeded 60% of the 20 rows of strawberries in the garden. How many rows did he weed?

7. Mr. Goodwin's new car cost $2,800. He paid 40% of the cost when he bought it. How much did he pay at that time?

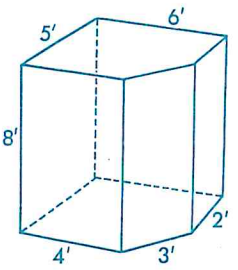
8. The regular price of a clock radio is $50.40. If it is sold at a discount of 25%, what would you pay?

9. When Jim went to buy a new suit, he found that the one he wanted had been marked down from $48.75 to $32.50. What was the per cent of decrease?

10. In 1950 there were 45 million radios in use in United States homes. By 1960 the number of radios in homes had increased by 22%. About how many were there in 1960?

11. A painter thins 4 quarts of varnish by adding 1 quart of turpentine. What per cent of the mixture is turpentine?

(For use after page 279)



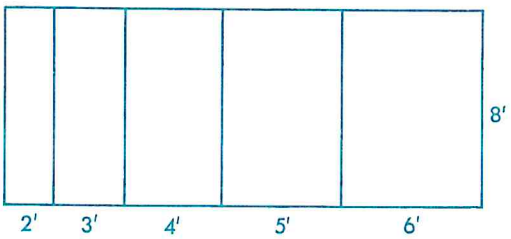
1. The drawing at the left shows a prism. The base of the prism has five sides. What is the name of this prism?

2.a. Is each lateral face of the prism a rectangular region?

b. What is the area of each lateral face of the prism?

c. What is the area of the five lateral faces?

3. Now imagine that the top and bottom of the prism have been removed and a cut has been made along one of the vertical edges. A representation of the lateral faces of the pentagonal prism arranged as a rectangular region is shown at the left.



a. What is the height of the rectangular region? What is the length?

b. Find the area of the rectangular region. Is your answer the same as your answer for Exercise **1c**?

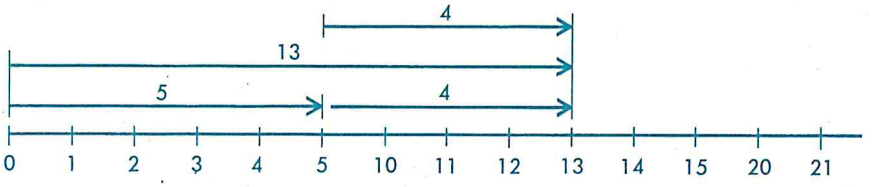
Use a drawing of a number line to help you find the missing numeral in each of the following sentences.

	a	b
4.	$3 + (-4) = \triangle$	$2 + (-5) = \square$
5.	$4 + (-2) = \triangle$	$-4 + (-4) = \square$
6.	$3 + (-5) = \triangle$	$-4 + 5 = \square$
7.	$4 + (-4) = \triangle$	$5 + (-4) = \square$
8.	$\frac{1}{2} + (-1) = \triangle$	$\frac{1}{4} + (-2) = \square$
9.	$-\frac{1}{3} + (-\frac{2}{3}) = \triangle$	$-\frac{3}{4} + \frac{3}{4} = \square$
10.	$5 + (-7) = \triangle$	$-5 + 4 = \square$
11.	$-7 + 7 = \triangle$	$7 + (-7) = \square$
12.	$\frac{5}{8} + (-\frac{5}{8}) = \triangle$	$-9 + 10 = \square$

Discovering More about Numbers

Adding and Subtracting with Base Six Numerals

(For use after page 39)



1. The number line above has base six numerals without subscripts. What numeral is at the end of a move of 5 to the right followed by a move of 4 to the right? Is 13_{six} the sum of 5_{six} and 4_{six}?

At the right is a base six addition table with missing numerals. (Subscripts have been omitted.) Copy the table and replace the question marks with the correct base six numerals. Use the number line if you need help.

+	0	1	2	3	4	5
0	0	1	2	?	4	5
1	1	?	3	4	5	10
2	2	3	?	5	?	11
3	3	4	?	?	?	12
4	?	?	?	11	12	?
5	?	?	?	?	?	?

2. Starting at 0 on the number line, what is the result of a move of 13_{six} from 0 to the right followed by a move of 4_{six} to the left? Then is this sentence true: $13_{six} - 4_{six} = 5_{six}$?

For each addition fact shown by your base six addition table there are related subtraction facts. Thus, your table shows that $4_{six} + 3_{six} = 11_{six}$. Do you see that the related subtraction facts are $11_{six} - 4_{six} = 3_{six}$, and $11_{six} - 3_{six} = 4_{six}$?

Use your addition table and the idea of related subtraction facts to find the missing base six numeral in each sentence below. Test your answers with moves on the number line.

	a	b	c
3.	$1_{six} - 0_{six} = \square$	$4_{six} - 2_{six} = \square$	$12_{six} - 5_{six} = \square$
4.	$2_{six} - 2_{six} = \square$	$10_{six} - 1_{six} = \square$	$13_{six} - 5_{six} = \square$
5.	$3_{six} - 1_{six} = \square$	$11_{six} - 3_{six} = \square$	$14_{six} - 3_{six} = \square$
6.	$10_{six} - 2_{six} = \square$	$11_{six} - 5_{six} = \square$	$12_{six} - 1_{six} = \square$

312 Regrouping with Base Six Numerals

(For use after page 39)

Regrouping with base six numerals is similar to regrouping with base ten numerals. You regroup with base six numerals by thinking of the numbers as so many sixes and so many ones.

1. Copy Exercises **a** and **b** and fill in the missing numerals.

a. 34_{six} is 3 sixes, 4 ones
$+15_{six}$ is 1 six, 5 ones

4 sixes, 13 ones ⟶ ? sixes, ? ones, or 53_{six}

b. 53_{six} is 5 sixes, 3 ones ⟶ ? sixes, 13 ones
-15_{six} is 1 six, 5 ones ⟶ 1 six, 5 ones
3 sixes, 4 ones, or ?

2. Regroup as in Exercise **1** to work these addition and related subtraction examples. Use the addition table that you made for the work on page 311.

a.			**b.**		**c.**	
52_{six}	120_{six}		124_{six}	341_{six}	243_{six}	401_{six}
$+24_{six}$	-24_{six}		$+213_{six}$	-213_{six}	$+114_{six}$	-114_{six}

Addition and subtraction with base six numerals can be checked by changing to base ten numerals.

3. Study Exercise **a** and then copy Exercise **b** and fill in the missing numerals.

a. Addition

Base six	Base ten
53_{six}	33
24_{six}	16
121_{six}	49

If 121_{six} is equal to 49, then the addition is checked. 121_{six} = 1 thirty-six + 2 sixes + 1 one, or 49. The addition is correct.

b. Subtraction

Base six	Base ten
53_{six}	?
25_{six}	?
24_{six}	16

If 24_{six} is equal to ?, then the subtraction is checked. 24_{six} = ? sixes + ? ones, or ?. Then is the subtraction correct?

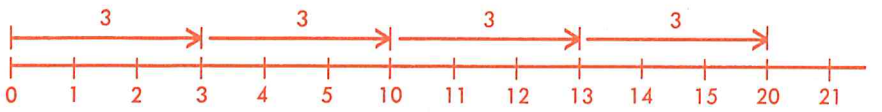
4. Add or subtract as the sign tells you. Check as in Exercise **3**.

a.	**b.**	**c.**	**d.**
121_{six}	253_{six}	444_{six}	35_{six}
-45_{six}	$+452_{six}$	-355_{six}	$+12_{six}$

Multiplying with Base Six Numerals 313

(For use after page 39)

1. The number line with base six numerals can be used to discover products. For example, may we think of $4_{six} \times 3_{six}$ as $3_{six} + 3_{six} + 3_{six} + 3_{six}$? The number line shows four moves of 3_{six}. What numeral is at the end of the last move?



2. Make a model of a number line like the one above but extend it to 100_{six}. Use this number line to find the products for the following examples.

a. $2_{six} \times 5_{six}$ **b.** $3_{six} \times 3_{six}$ **c.** $4_{six} \times 5_{six}$
d. $5_{six} \times 5_{six}$ **e.** $4_{six} \times 2_{six}$ **f.** $5_{six} \times 2_{six}$

3. Compare your results for Exercises **2a** and **2f**. Does the order in which multiplication takes place change the product?

4. Copy the base six multiplication table at the right and fill in the missing numerals. (You need not show subscripts.) Use your number line to find the products.

×	1	2	3	4	5
1	1	2	?	4	5
2	2	4	10	12	?
3	3	?	13	?	?
4	4	?	?	?	32
5	?	?	?	?	?

5. Look for patterns in your completed multiplication table. In the base six system, when a number is multiplied by two, what are the possibilities for the ones' digit of the product? In the base ten system, when a number is multiplied by two, what are the possibilities for the ones' digit of the product?

6. In the base ten system, when a number is multiplied by 5, what are the two possibilities for the ones' digit of the product? In the base ten system, 5 is one half the base. What is one half of the base in the base six system? In the base six system, when a number is multiplied by 3_{six}, what are the two possibilities for the ones' digit of the product?

314 Another Way to Do Multiplication

(For use after page 71)

1. The set of number sentences at the right can be used to find the product of 350 and 45.

$1 \times 350 = 350$
$2 \times 350 = 700$
$4 \times 350 = 1{,}400$
$8 \times 350 = 2{,}800$
$16 \times 350 = 5{,}600$
$32 \times 350 = 11{,}200$

a. Study the multipliers: 1, 2, 4, 8, 16, and 32. What must be done to the first multiplier to get the second? to the second to get the third? to each to get the next one?

b. Study the products: 350; 700; 1,400; 2,800; 5,600; and 11,200. What must be done to each to get the next one?

c. Look at the number sentences printed in blue. What is the sum of the multipliers whose names are printed in blue? Is the sum of the products whose names are printed in blue equal to 45×350? Is 45×350 equal to $(32 + 8 + 4 + 1) \times 350$?

2. Let us use this method to find the product of 75 and 369. Look at the set of number sentences at the right. Is each multiplier, except the first, twice the preceding one? Can you find a set of the multipliers whose sum is 75? Do so. Now find the sum of the products associated with these multipliers.

$1 \times 369 = 369$
$2 \times 369 = 738$
$4 \times 369 = 1{,}476$
$8 \times 369 = 2{,}952$
$16 \times 369 = 5{,}904$
$32 \times 369 = 11{,}808$
$64 \times 369 = 23{,}616$

Find the product of 75 and 369 by your usual method. Are the two results the same?

The early Egyptians used this method of multiplication because their system of numerals was such that multiplication and division were difficult, but addition and subtraction were easy.

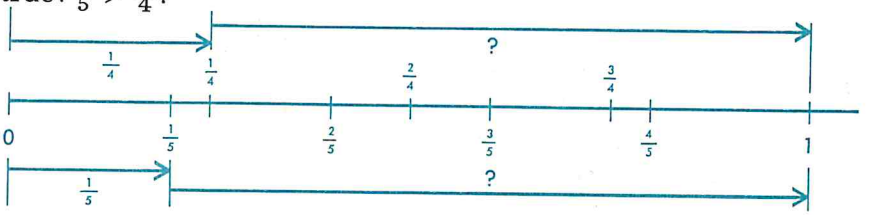
3. See if you can write a set of number sentences which can be used to *divide* 861 by 29. Begin with $1 \times 29 = 29$. In multiplication with this method, you began by finding a set of *multipliers* whose sum was the given multiplier. In division you must begin by finding a set of *products* whose sum is less than or equal to the given dividend.

Comparing Fractional Numbers 315

(For use after page 71)

1. Is the number sentence $\frac{1}{5} < \frac{1}{4}$ true, or is it false? How do you know? Did you find fractions with a common denominator that were equivalent to $\frac{1}{5}$ and $\frac{1}{4}$? Is $\frac{4}{5}$ less than $\frac{3}{4}$, or is it greater than $\frac{3}{4}$? How do you know? Could you use common denominators again?

2. Let us use a number line to show the results of Exercise **1**. The unit space below has been divided into fifths and into fourths. Clearly a move of $\frac{1}{5}$ is less than a move of $\frac{1}{4}$. What move do you make from $\frac{1}{5}$ to reach 1? from $\frac{1}{4}$ to reach 1? Is a move of $\frac{4}{5}$ greater than the move of $\frac{3}{4}$? Then is this sentence true: $\frac{4}{5} > \frac{3}{4}$?



Do you see that since $\frac{1}{5} < \frac{1}{4}$, a larger number must be added to $\frac{1}{5}$ to obtain 1 than you would add to $\frac{1}{4}$ to obtain 1?

3. In each of the following exercises, decide which number in the first pair of numbers is smaller. Then write a true sentence using the other pair.

a. $\frac{1}{6}$ and $\frac{1}{7}$; $\frac{5}{6}$ and $\frac{6}{7}$

b. $\frac{1}{8}$ and $\frac{1}{9}$; $\frac{7}{8}$ and $\frac{8}{9}$

c. $\frac{1}{10}$ and $\frac{1}{9}$; $\frac{9}{10}$ and $\frac{8}{9}$

d. $\frac{1}{17}$ and $\frac{1}{16}$; $\frac{16}{17}$ and $\frac{15}{16}$

4. a. You know that $\frac{1}{5} < \frac{1}{4}$. Then is $\frac{2}{5}$ less than $\frac{2}{4}$? Write a true sentence comparing $\frac{3}{5}$ and $\frac{3}{4}$.

b. Write a true sentence comparing $\frac{1}{17}$ and $\frac{1}{16}$; comparing $\frac{3}{17}$ and $\frac{3}{16}$. Then, using the method you used in **3**, write a true sentence comparing $\frac{14}{17}$ and $\frac{13}{16}$.

5. Write a true sentence that compares each of the following pairs of numbers.

a. $\frac{12}{17}$ and $\frac{11}{16}$ **b.** $\frac{3}{8}$ and $\frac{4}{9}$ **c.** $\frac{7}{10}$ and $\frac{6}{9}$ **d.** $\frac{8}{11}$ and $\frac{7}{10}$

316 Prime and Composite Numbers

(For use after page 71)

1. Is 4×8 equal to 32? Is 4 a factor of 32? Is 8? Are there any other factors of 32? List all the factors of 32 in pairs such that the product of each pair of factors is 32. Does your list have these pairs: 1, 32; 2, 16; 4, 8?

2. Is 1×7 equal to 7? Then is 1 a factor of 7? Is 7 a factor of 7? Does 7 have any factors other than 1 and 7 itself?

Any whole number greater than 1 which has no factors other than 1 and itself is called a **prime number.** Is 7 a prime number?

3. Is 1 a prime number? Why, or why not? Is 2 a prime number? Why, or why not? Is 3 prime? Is 4 prime? List the names of all the prime numbers between 1 and 20.

A **prime factor** of a number is a factor which is also a prime number.

4. Is 3 a prime factor of 9? Is 2 a prime factor of 32?

5. Are 3 and 7 prime factors of 21? What are the prime factors of 22? of 34?

Whole numbers greater than 1 which are not prime are called **composite numbers.**

6. Make a copy of the table at the right. In the two columns at the left of your table, write the names of all the numbers between 1 and 51. Put the names of the primes under *prime* and the names of numbers which are not prime under *composite*. Opposite the names of composite numbers, list the names of all the factors of that number. Put a star (*) beside the names of prime factors. Some entries are made in the table to help you get started.

Prime	Composite	Factors
2		
3		
	4	1, *2, 4
5		
	6	1, *2, *3, 6
7		

Primes, Exponents, and Least Common Denominators 317

(For use after page 71)

Every composite number can be expressed as the product of prime factors. For instance, 4 is equal to 2×2, 15 is equal to 3×5, and 32 is equal to $2 \times 2 \times 2 \times 2 \times 2$, or 2^5.

1. What are the prime factors of 50? Is 50 equal to $2 \times 5 \times 5$? Is 50 equal to 2×5^2?

2. Express each of the composite numbers between 1 and 51 as products of prime factors. Use exponents when possible.

Expressing a number as the product of its prime factors can be helpful for finding the least common denominators when adding or subtracting with like fractions.

3. Look at this model example. What are the prime factors of 30? of 55? Is 30 equal to $2 \times 3 \times 5$? Is 55 equal to 5×11? Use the method that you learned on page 68 to find the least common denominator. Did you find that 330 is the least common denominator? Is 330 equal to $2 \times 3 \times 5 \times 11$? Compare the prime factors of 330 with the prime factors of 30 and of 55. What do you notice?

$$\frac{1}{30} + \frac{1}{55} = N$$

4. Now try this model example. Is 96 equal to $2^5 \times 3$? Is 72 equal to $2^3 \times 3^2$? Use the method that you learned on page 68 to find the least common denominator. Is 288 equal to $2^5 \times 3^2$? Compare the prime factors of 288 with the prime factors of 96 and of 72. Do you see a pattern?

$$\frac{1}{96} + \frac{1}{72} = N$$

5. If you see the pattern in Exercise **4**, use it to find the least common denominators for the following pairs of fractions. If not, find the least common denominators by your usual method. Compare the prime factors of the least common denominator with the prime factors of the two given denominators. Try to discover the pattern.

a. $\frac{1}{72}, \frac{1}{54}$ **b.** $\frac{1}{245}, \frac{1}{175}$ **c.** $\frac{1}{1,573}, \frac{1}{1,859}$

6. Do you see that the least common denominator has all the prime factors of the two given denominators?

(For use after page 115)

1. You know that $1 \div 12 = \frac{1}{12}$, and that $5 \div 3 = \frac{5}{3}$.

a. Express in fraction form: $15 \div 4$; $26 \div 7$; $80 \div 3$.

b. Express in fraction form: $390 \div 60$. Is $60\overline{)390}$ another way of writing $390 \div 60$? Can $60\overline{)390}$ be expressed as $\frac{390}{60}$?

c. Is $\frac{390}{60}$ equal to $\frac{39}{6}$? Is $\frac{39}{6}$ equal to $6\overline{)39}$? Then is the quotient for the division example $60\overline{)390}$ the same as the quotient for the division example $6\overline{)39}$? Write the quotient immediately without copying the example.

Do you see that thinking of a division example as a fraction and dividing the dividend and divisor by a common factor leads to an easier division example?

d. Look at $\frac{39}{6}$ again. Is there a factor common to both terms of $\frac{39}{6}$? What is it? Could you have obtained a simpler division example by making use of this factor, too?

2. Find the quotient for each of the following division examples by first dividing the dividend and divisor by a common factor to obtain a simpler example.

a. $40\overline{)680}$	**b.** $72\overline{)400}$	**c.** $14\overline{)567}$
d. $56\overline{)1{,}112}$	**e.** $45\overline{)855}$	**f.** $60\overline{)570}$
g. $42\overline{)2{,}142}$	**h.** $144\overline{)1{,}872}$	**i.** $112\overline{)168}$

3. Consider the division example $64\overline{)393}$. What is 64 rounded to the nearest ten? What is 393 rounded to the nearest ten? What is the quotient for $60\overline{)390}$? What is the quotient for $64\overline{)393}$? Could the quotient for $60\overline{)390}$ be used as an estimate of the quotient for $64\overline{)393}$?

4. First estimate the quotient for each of the following by rounding the dividend and the divisor to the nearest ten. Then find the exact quotient.

a. $53\overline{)467}$	**b.** $84\overline{)982}$	**c.** $78\overline{)179}$
d. $29\overline{)581}$	**e.** $75\overline{)876}$	**f.** $54\overline{)256}$
g. $18\overline{)204}$	**h.** $105\overline{)224}$	**i.** $98\overline{)245}$

Finding a Number When a Part of It Is Known 319

(For use after page 115)

1. Look at this number sentence: $\frac{1}{6} \times N = 15$. Using the method you learned on page 106, should you replace N with 6×15? Is N equal to 6×15? Do you see that if $\frac{1}{6}$ of a number is 15, then the number is 6×15? If $\frac{1}{3}$ of a number is 15, then is the number 3×15? If $\frac{1}{8}$ of a number is 15, then is the number 8×15?

2. Harry bought 20 different stamps. The clerk said, as he handed Harry the stamps, "Here are $\frac{5}{6}$ of the total number of different stamps that have been issued on the subject of astronauts." Later Harry decided to find out how many different astronaut stamps had been issued.

a. Harry thought: $\frac{5}{6}$ of the number of different stamps is 20. Then he wrote the number sentence: $\frac{5}{6} \times N = 20$. Harry knew how to solve this number sentence by the method shown on page 106. Can you find N? What is N?

b. Harry wanted a different method for finding N. He remembered that if he knew $\frac{1}{6}$ of N, then he could find all of N. Let us see how he can find what $\frac{1}{6}$ of the number of stamps is. Is $\frac{1}{6} \times N$ equal to $\frac{1}{5} \times (\frac{5}{6} \times N)$? Why? (See the flagged statement on page 77.) Since Harry's number sentence says that $\frac{5}{6} \times N$ and 20 are equal, may we replace $(\frac{5}{6} \times N)$ with 20 in the number sentence $\frac{1}{6} \times N = \frac{1}{5} \times (\frac{5}{6} \times N)$? Make this replacement. Do you now have $\frac{1}{6} \times N = \frac{1}{5} \times 20$? Then is $\frac{1}{6} \times N$ equal to 4? Is $\frac{1}{6}$ of the number of stamps is 4, then is the number of stamps 6×4? Does this agree with the answer you got for **a**?

This method can be shortened as in **c** or **d** below.

c. If $\frac{5}{6}$ of N is 20,
then $\frac{1}{6}$ of N is $\frac{1}{5}$ of 20
and N is $6 \times (\frac{1}{5}$ of $20)$

d. If $\frac{5}{6}$ of N is 20,
then $\frac{1}{6}$ of N is 4,
and N is 24.

3. Use the method illustrated in **c** or **d** above to find the number in each of the following sentences.

a. $\frac{3}{4}$ of a number is 48
b. $\frac{5}{8}$ of a number is 65
c. $\frac{3}{2}$ of a number is 174
d. $\frac{7}{4}$ of a number is 84

320 Multiplying with More than Two Fractions

(For use after page 115)

1. The short cut in multiplication you learned on pages 83 and 84 is useful when several fractions appear in a multiplication example.

For instance, look at the multiplication example in **a**. Is 3 a factor of 3 and 9? Then did we think of this common factor as representing the fraction $\frac{3}{3}$, which will not change the product? Explain how we obtained sentence **b**.

Is 2 a common factor of 2 and 4? What fraction representing 1 can be formed using this common factor? How do we obtain sentence **c**?

Is 3 a common factor of 3 and 6? Then how did we obtain sentence **d**?

Now use sentence **d** to find the product. Is your answer in simplest form?

a. $\frac{3}{2} \times \frac{4}{9} \times \frac{6}{5} = \square$

b. $\frac{\cancel{3}^{1}}{2} \times \frac{4}{\cancel{9}_{3}} \times \frac{6}{5} = \square$

c. $\frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{\cancel{9}_{3}} \times \frac{6}{5} = \square$

d. $\frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{\cancel{9}_{3}} \times \frac{\cancel{6}^{2}}{5} = \square$

2. Find the products.

a. $\frac{2}{3} \times \frac{5}{6} \times \frac{9}{5}$ **b.** $\frac{5}{12} \times \frac{4}{15} \times \frac{3}{4}$ **c.** $\frac{7}{15} \times \frac{5}{9} \times \frac{9}{8}$

d. $\frac{1}{3} \times \frac{14}{15} \times \frac{15}{16}$ **e.** $\frac{5}{6} \times 2\frac{2}{5} \times \frac{1}{2}$ **f.** $\frac{5}{4} \times \frac{7}{10} \times \frac{8}{7}$

g. $\frac{5}{8} \times 1\frac{1}{2} \times \frac{4}{9}$ **h.** $1\frac{2}{5} \times \frac{5}{7} \times 3\frac{2}{3}$ **i.** $\frac{20}{3} \times 1\frac{3}{5} \times \frac{15}{4}$

3. It is easy to find the product for the example at the right mentally. What is $\frac{1}{3} \times 6$? What is 5×6? Then what is the product?

$$5\frac{1}{3} \times 6$$

4. Is this example also easy to do mentally? What is $5 \times \frac{1}{3}$? What is 5×7? Then what is the product?

$$5 \times 7\frac{1}{3}$$

5. Write the product only for each of the following examples. Do the necessary work mentally.

a. $4\frac{1}{3} \times 12$ **b.** $3\frac{1}{2} \times 5$ **c.** $12\frac{1}{3} \times 3$ **d.** $5\frac{1}{6} \times 7$

e. $3 \times 4\frac{1}{2}$ **f.** $16 \times 2\frac{1}{4}$ **g.** $11 \times 3\frac{1}{4}$ **h.** $9 \times 7\frac{2}{3}$

i. $5\frac{3}{4} \times 8$ **j.** $5 \times 9\frac{3}{5}$ **k.** $4\frac{2}{3} \times 7$ **l.** $3 \times 8\frac{11}{15}$

A Short Cut in Division with Fractions 321

(For use after page 115)

1. In Examples **a** and **b** would it be correct to omit the parentheses? Why? The parentheses help us to tell whether to multiply first and then divide, or to divide first and then multiply. However, the short cut studied on page 98 can be used.

a. $(\frac{3}{4} \times \frac{5}{7}) \div \frac{3}{2}$

b. $(\frac{2}{5} \div 1\frac{2}{3}) \times \frac{3}{7}$

Is this sentence true?

$$(\frac{3}{4} \times \frac{5}{7}) \div \frac{3}{2} = \frac{3}{4} \times \frac{5}{7} \times \frac{2}{3}$$

May we omit the parentheses after the example is rewritten as a multiplication example? Why?

Is 3 a factor of a numerator and of a denominator? Is 2? Then what is the answer to the example? Is it in simplest form?

Find the answer to Example **b**. Use the short cut if you understand it and find it helpful.

2. Write the answers to each of the following examples in simplest form, as proper fractions, mixed fractions, or numerals for whole numbers.

a. $(\frac{5}{6} \div \frac{3}{5}) \times \frac{12}{25}$ **b.** $(1\frac{5}{9} \times \frac{6}{7}) \div \frac{4}{3}$ **c.** $(\frac{2}{3} \div 2\frac{7}{8}) \times 5\frac{3}{4}$

d. $(3\frac{1}{7} \times 14) \div 1\frac{3}{8}$ **e.** $(5 \div 2\frac{7}{9}) \times \frac{3}{18}$ **f.** $(\frac{18}{5} \times 1\frac{2}{3}) \div 12$

3. Jack took $\frac{3}{4}$ of his collection of 228 pictures of baseball players and divided them among 5 friends. How many pictures did each receive? Were any left over?

a. Is the following a number sentence for the problem?

$$(\frac{3}{4} \times 228) \div 5 = \square$$

b. What is the correct replacement for the frame? What are the answers to the questions in the problem?

4. Mrs. Jones took $2\frac{1}{4}$ pounds of fudge to a P.T.A. sale. There the pans of fudge were divided into pieces, each piece weighing $\frac{1}{16}$ pound. At the sale, $\frac{5}{6}$ of the pieces of fudge were sold at auction for 5 cents each. How much money did Mrs. Jones' fudge earn for the P.T.A.?

322 Some Measures of Today

(For use after page 143)

Perhaps you have seen reference to measurements such as 1,000,000 passenger-miles, or 850 man-hours, or 3 light-years. The number of **passenger-miles** is the product of the number of passengers and the number of miles they travel.

1. One person carried 2 miles would be 2 passenger-miles. One person carried 100 miles would be how many passenger-miles? Three passengers carried 1 mile would be 3 passenger-miles. Three passengers carried 25 miles would be how many passenger-miles?

2. A jet airplane carried 96 passengers from New York to Los Angeles, a distance of 2,469 miles. How many passenger-miles is this?

3. A jet airplane carried 87 people from Chicago to Detroit, a distance of 219 miles. Later the jet carried 87 passengers from London to New York, a distance of 3,455 miles.

a. How many passenger-miles did the jet fly on the Chicago-Detroit flight? on the London-New York flight?
b. If the airline reported only the total number of passengers for two flights, would this give a reasonable idea of the use of its plane?
c. What is the total number of passenger-miles for the two flights? Does this number give a better idea of how the plane was used?

4. A bus driver drove 480,000 passenger-miles and had two accidents. A second bus driver drove 600,000 passenger-miles and had three accidents. According to these records, do you think it would be safer to ride with the first bus driver than with the second? Why, or why not?

5. The number of **ton-miles** is the product of the number of tons of freight and the number of miles it is transported. Fifty ton-miles could mean 1 ton transported 50 miles, or 2 tons transported 25 miles, or $\frac{1}{2}$ ton transported 100 miles, etc. What are some possible meanings for 400 ton-miles?

Man-Hours and Light Years 323

(For use after page 143)

1. The **man-hour** is a unit of measure of labor. It is equivalent to the labor of one man working for one hour. Is it also equivalent to the labor of 2 men working for $\frac{1}{2}$ hour? What is the number of man-hours if 3 men work for one hour? if 65 men work for one hour?

2. What is the number of man-hours if one man works for 5 hours? if one man works for 35 hours? 5 men for 8 hours?

3. A contractor estimates there are 240 man-hours of carpentry work left to finish a house. Will his carpenter, who works a 40-hour week, be able to finish the job in 4 weeks? The carpenter's helper can do one half as much as the carpenter per week. Will the carpenter and his helper be able to finish the job in 4 weeks?

4. The third unit of measure mentioned on page 322 is the **light-year**. A light-year is the distance light travels in one year. The speed of light, to the nearest thousand, is 186,000 miles per second.

a. About how far does light travel in one minute? in one hour? in one 24-hour day?

b. Then about how many miles does light travel in one 365-day year?

5. Alpha Centauri, the star closest to the earth, is 4.3 light-years away. About how many miles is this?

6. Sirius, one of the brightest stars, is 8.7 light-years from the earth. Sirius is about how many times as far from the earth as Alpha Centauri? Find your answer to the nearest tenth.

7. Use your answers to Exercises **5** and **6** to find about how many miles Sirius is from the earth.

8. In Exercise **4b** you found how many miles a light-year is. Round that answer to the nearest 100 billion and find how many hours it would take a manned space machine flying at a speed of 20,000 miles per hour to fly a distance of one light-year.

9. If a man could travel at 100,000 miles per hour for 70 years, could he travel a distance of one light-year?

324 Speeds and Ratios

(For use after page 143)

1. A car is traveling at a speed of 60 miles per hour.

a. How far does it travel in 1 hour? in 2 hours? in $\frac{1}{2}$ hour? in $\frac{1}{60}$ of an hour? Is 1 minute $\frac{1}{60}$ of an hour? Then how far does the car travel in 1 minute? Do you see that a speed of 60 miles per hour is equivalent to a speed of 120 miles per 2 hours, or 30 miles per $\frac{1}{2}$ hour, or 1 mile per $\frac{1}{60}$ hour, or 1 mile per minute?

b. One mile is how many feet? Is a speed of 1 mile per minute equivalent to a speed of 5,280 feet per minute? If a car is traveling at a speed of 5,280 feet per minute, then how many feet does it travel in 1 minute? in 2 minutes? in $\frac{1}{2}$ minute? in $\frac{1}{60}$ minute? in 1 second? Is 5,280 feet per minute equivalent to 10,560 feet per 2 minutes? to 2,640 feet per $\frac{1}{2}$ minute? to 88 feet per $\frac{1}{60}$ minute? to 88 feet per second?

2. a. Is there a pair of numbers mentioned or understood in each of the speeds given in **1a**? Are they 60 and 1, 120 and 2, 30 and $\frac{1}{2}$, 1 and $\frac{1}{60}$, 1 and 1? Does the first number in each pair give the number of units of distance? Does the second number in each pair give the number of units of time?

b. Think of the pairs of numbers for the speeds in **1a**. Is the ratio of the first number to the second number always the same? For which pair of numbers is the ratio different? Are the units used with this pair different from the units used with the other four pairs? Are the units alike in the four speeds whose numbers have the same ratio? Try to draw some conclusions about speeds that are equivalent.

3. Test your conclusions about equivalent speeds on the following pairs of equivalent speeds.

a.	50 miles per hour	1,200 miles per day
b.	40 words per minute	2,400 words per hour
c.	17,500 miles per hour	$291\frac{2}{3}$ miles per minute
d.	3 light-years per century	$\frac{3}{2}$ light-years per 50 years
e.	12 quarts per 2 weeks	$\frac{12}{7}$ quarts per 2 days

Rates and Ratios

(For use after page 143)

A *rate* is a pair of numbers used to show how one measure is related to another. Usually a rate shows the change expected in one measure for a given change in another.

1. Let us see if a speed of 40 miles per hour is also a rate. Is the pair of numbers 40 and 1 involved in this speed? When time changes by one hour, by how much is distance expected to change? When distance changes by 40 miles, by how much is time expected to change? Is a speed of 40 miles per hour a rate?

2. Think about 12 inches per foot. Is there a pair of numbers? What pair? When a measurement of length changes by one foot, by how many inches does the measurement change? When a measurement of length changes by 12 inches, by how many feet does the measurement change? Is 12 inches per foot a rate?

3. Two rates are equivalent if, when both are expressed in the same units, the ratios of their two numbers are the same.

Let us see if a rate of 15 cartridges per rifle is equivalent to a rate of 90 cartridges per 6 rifles. Are both rates expressed in the same units? Is the ratio of 15 to 1 the same as the ratio of 90 to 6? (See flagged statement, page 127.) Then, are the two rates equivalent?

4. Study the following pairs of rates. Copy the letters **a** to **j** on your paper. After each letter write *equivalent* or *not equivalent.*

a. 2 hands per child; 40 hands per 20 children
b. 7 inches per inch; 8 inches per $1\frac{1}{7}$ inches
c. 60 words per minute; 20 words per $\frac{1}{3}$ minute
d. 60 words per minute; 20 words per 20 seconds
e. 2 bales of hay per cow; 20 bales of hay per 5 cows
f. 216 pages per book; 2,160 pages per 10 books
g. 241 days per century; $24\frac{1}{4}$ days per 100 years
h. $\frac{97}{400}$ day per year; 97 days per 400 years
i. 2,700 revolutions per minute; 45 revolutions per second
j. $365\frac{1}{4}$ rotations per year; 146,097 rotations per 400 years

326 Analyzing a Decimal

(For use after page 169)

1. You know that a whole number can be expressed in the following three ways:

$$653 = 600 + 50 + 3$$
$$653 = (6 \times 100) + (5 \times 10) + 3$$
$$653 = (6 \times 10^2) + (5 \times 10) + 3$$

Let us follow this scheme to find expressions equivalent to a decimal such as .427. Copy the sentences and replace the frames with numerals that make the sentences true.

$$.427 = .4 + ? + .007$$
$$.427 = (4 \times \tfrac{1}{\square}) + (2 \times \tfrac{1}{100}) + (7 \times \tfrac{1}{\triangle})$$
$$.427 = (4 \times \tfrac{1}{10}) + (\diamond \times \tfrac{1}{10^2}) + (7 \times \tfrac{1}{10^3})$$

2. Express each decimal in the three ways shown in **1**.

a. .931 **b.** .5682 **c.** .4058 **d.** .00710

3. Copy each of the sentences below and replace the frames with numerals that make the sentences true.

$$77.777 = (7 \times \square) + 7 + (7 \times .1) + (7 \times \triangle) + (7 \times \diamond)$$
$$77.777 = (7 + \triangle) + 7 + (7 \times \tfrac{1}{10}) + (7 \times \diamond) + (7 \times \tfrac{1}{10^3})$$

4. Express each of the following decimals in two ways as shown in Exercise **3**.

a. 54.726 **b.** 62.1975 **c.** 180.806595

5. You know that $.001 = \frac{1}{1000}$, or $\frac{1}{10^3}$; $.0001 = \frac{1}{10,000}$, or $\frac{1}{\square}$; $.00001 = \frac{1}{100,000}$, or $\frac{1}{\triangle}$. Look at your answers.

a. How many decimal places in .001? What is the exponent of 10 in the denominator of the fraction equivalent to .001?
b. How many decimal places in .0001? What is the exponent of 10 in the denominator of the fraction equivalent to .0001?
c. How many decimal places in .00001? What power of 10 appears in its equivalent fraction?
d. Do you see a pattern? What is the relationship between the number of decimal places in a decimal, such as those above, and the exponent of 10 in the equivalent fraction?

(For use after page 169)

6. Since .007 equals $7 \times .001$, is this sentence true: $.007 = 7 \times \frac{1}{10^3}$? Express .004 in this form.

7. For each of the following decimals, find an equivalent expression similar to $7 \times \frac{1}{10^3}$.

a. .003 **b.** .09 **c.** .0007 **d.** .00006

8. You can use the pattern you found to analyze a decimal in one step. Since .317 equals .3 + .01 + .007, we can write:

$$.317 = (3 \times \tfrac{1}{10}) + (1 \times \tfrac{1}{10^2}) + (7 \times \tfrac{1}{10^3}).$$

Express each of the following decimals in this way. Use just one step.

a. .726 **b.** .3914 **c.** .49 **d.** .57682
e. 3.407 **f.** 61.52 **g.** 7.8109 **h.** 29.06273

9. Write the decimal for each of the following expressions.

a. $(5 \times \frac{1}{10}) + (3 \times \frac{1}{10^2})$
b. $8 + (9 \times \frac{1}{10}) + (1 \times \frac{1}{10^2}) + (5 \times \frac{1}{10^3})$
c. $(6 \times 10) + 5 + (0 \times \frac{1}{10}) + (8 \times \frac{1}{10^2}) + (3 \times \frac{1}{10^3})$
d. $(3 \times 10^2) + 3 + (6 \times \frac{1}{10}) + (2 \times \frac{1}{10^2}) + (8 \times \frac{1}{10^4})$
e. $(5 \times \frac{1}{10^4}) + (2 \times \frac{1}{10^5})$

10. Use "<", "=", or ">" to write a true sentence about the relationship of the first number in each pair to the second.

a. .002; .0022 **b.** 1.1; $\frac{11}{10}$ **c.** .12; $\frac{3}{25}$
d. 1.0055; 1.0005 **e.** .0111; .0222 **f.** .115; .1146
g. $1\frac{7}{10{,}000}$; .7 **h.** .009; $\frac{9}{10^3}$ **i.** .009; .0099

11. One small measure used by modern science is the millisecond. A millisecond is one thousandth of a second. (*Milli* comes from a Latin word for thousand.) Use a decimal to express 1 millisecond as a fractional part of a second. What decimal expresses 25 milliseconds as a fractional part of a second? What common fraction expresses 25 milliseconds as a fractional part of a second?

12. A microsecond is one millionth of a second. Express 1 microsecond in seconds, using a decimal; a common fraction; a fraction with a power of 10 in its denominator.

328 Metric Measures, Decimals, and Powers of Ten

(For use after page 169)

A **meter** (m) is a unit of measure of length. It is a part of a system of weights and measures called the metric system. The metric system is used by scientists and others. One meter is 39.37 inches. It is 3.37 inches greater than a yard. Your teacher may have a meter stick. If so, ask to see it, and compare it with a yardstick.

Linear Metric Measures
1 kilometer = 1,000 m
1 hectometer = 100 m
1 dekameter = 10 m
1 meter = 1 m
1 decimeter = .1 m
1 centimeter = .01 m
1 millimeter = .001 m

1. The table at the right shows that 1 kilometer is 1,000 meters. What is 1,000 expressed as a power of 10? Then is 1 kilometer equal to 10^3 meters? Expressed as a power of 10, how many meters is a hectometer?

2. The table shows that 1 millimeter is .001 meter. What is .001 expressed as a fraction with a power of 10 as its denominator? Then does 1 millimeter equal $\frac{1}{10^3}$ meter? Expressed as a fraction with 10 or a power of 10 as its denominator, what fractional part of a meter is a centimeter? a decimeter?

3. Copy the table, but express the number of meters as 1, 10, a power of 10, or as a fraction with 10 or a power of 10 as a denominator. Do you see a pattern for the exponents of 10?

4. There are three metric units of measure smaller than a millimeter. In order from largest to smallest, let us call them "decimillimeters," "centimillimeters," and "microns." Add these names, in order, to the table you made for Exercise **3** and use the pattern for the exponents of 10 to express each of these as a fractional part of a meter.

5. Suppose that three names for metric units of measure smaller than a micron are "centimicron," "decimicron," and "millimicron." See if you can add them to your table in the proper order and show what fractional part of a meter each of these units is.

Common Fractions and Decimals

(For use after page 195)

If you have not done the work on pages 317–318, do it efore you try the work on this page.

On page 190 you learned to use division to find decimals quivalent to given fractions. We say that the fraction has .n exact decimal equivalent when the division comes out even,)ut that it has only an *approximate* decimal equivalent when he division does not come out even. Let us see if it is pos- ible to tell without dividing whether or not a fraction has an :xact decimal equivalent.

1. Arrange the following fractions in two columns. In one :olumn put all the fractions which have exact decimal equiva- ents. Put the other fractions in the other column. When ıecessary, use division to decide where to place a fraction.

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{12}, \frac{1}{14}, \frac{1}{15},$$

$$\frac{1}{16}, \frac{1}{18}, \frac{1}{19}, \frac{1}{20}, \frac{1}{21}, \frac{1}{24}, \frac{1}{25}, \frac{1}{40}, \frac{1}{45}, \frac{1}{90}$$

What are the prime factors of the denominators of those fractions which have exact decimal equivalents? Do you see that either 2 or 5 or both are the only prime factors of these denominators?

2. Replace the numerator of each fraction in Exercise **1** with 2. Reduce to lowest terms any of the resulting fractions not already in lowest terms. Arrange these fractions in two columns as directed in Exercise **1**. What are the only prime factors of the denominators of those fractions which have exact decimal equivalents?

3. Replace the numerator of each fraction in Exercise **1** with 3 and proceed as directed in Exercise **2**.

4. For every decimal, is there an equivalent fraction which has 10 or a power of 10 as its denominator? What are the only prime factors of 10? of 10^2? of any power of 10?

5. Try to make a statement about what the prime factors of the denominator must be if a fraction in lowest terms is to have an exact decimal equivalent; if a fraction in lowest terms is not to have an exact decimal equivalent.

330

Precision and Accuracy

(For use after page 195)

Some soldiers on maneuvers have a tank truck for carrying water. To the nearest quart, the truck holds 6,256 quarts of water. Each soldier has a canteen of water. To the nearest quart, each canteen holds 1 quart of water.

1. What is the greatest possible error in the measurement of the water in the tank truck? in each canteen? Are both measurements precise to the nearest quart? Is the precision of both measurements the same?

2. Which error would be more serious, a shortage of $\frac{1}{2}$ quart in the tank truck, or a shortage of $\frac{1}{2}$ quart in a soldier's canteen? Do you see that an error of $\frac{1}{2}$ quart can be very serious in a small measurement, but not serious in a large measurement?

To determine the "seriousness" of the possible error of a measurement, you need to know about the *relative error* and about the *accuracy* of a measurement.

The **relative error** of a measurement is the quotient found when the *greatest possible error* is divided by the *measurement.*

3. The relative error in the measurement of the water in the tank truck is $.5 \div 6{,}256$, and in each canteen it is $.5 \div 1$. Express these quotients as decimals rounded to four decimal places. Which measurement has the greater relative error? Do you see that the relative error of a measurement is a guide to the "seriousness" of the possible error?

4. a. What is the greatest possible error in a measurement given as 5 yards? What is the relative error of the measurement?
b. What is the greatest possible error in a measurement given as 150 yards to the nearest ten yards? What is the relative error of the measurement?

5. Which has the greater relative error, a measurement given as 2,362 miles, or a measurement given as 148 feet to the nearest inch?

Precision and Accuracy — *continued*

(For use after page 195)

We say that one measurement is less *accurate* than another if the relative error of the one measurement is greater than the relative error of the other.

6. When both measurements are given to the nearest quart, which is more accurate, the measurement of the water in the tank truck or the measurement of the water in a soldier's canteen?

7. A carpenter bought a board which, to the nearest foot, was 16 feet long. From the board he cut a piece which, to the nearest inch, was 15 inches long. Expressed as a three place decimal, what is the relative error in the measurement of the board? of the piece? Which measurement is more accurate?

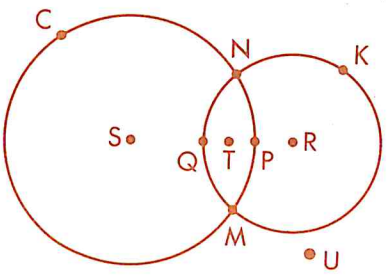
8. Use your knowledge of the things measured to estimate which measurement in each of the following pairs of measurements is more accurate. Then find, to the nearest thousandth, the relative error of each measurement. Compare the relative errors to decide which measurement in each pair is more accurate. See if your estimates were correct.

Item Measured	Measurement	Precision of the measurement is to the nearest:	Greatest possible error
Truck load of grain	3 tons	100 pounds	50 pounds
Bag of grain	100 pounds	pound	.5 pound
Area of a farm	500 acres	acre	.5 acre
Area of a building lot	5,000 square feet	square foot	?
Speed of a rocket	17,500 miles per hour	mile per hour	?
Speed of an automobile	60 miles per hour	mile per hour	?
Distance to the sun	93,000,000 miles	1,000,000 miles	?
Average thickness of human hair	.002 inch	.001 inch	?

332 Circles and Intersections

(For use after page 238)

You know that a circle is a simple closed figure (and therefore it is a set of points in a plane) such that all of its points are the same distance from a fixed point, called the center.



1. The larger circle at the left is named circle C and the smaller circle is named circle K. Is there a point on circle C that is also on circle K? Is there more than one? What names do these two points have? When a point, such as point N, is on each of two circles, the circles are said to *intersect* in point N. Do circles C and K also intersect in point M? Do they have any other points of intersection?

The *intersection* of two circles is the set of all points which are on both circles. Thus M and N are the only two points in the intersection of circles C and K.

A circle divides a plane into three sets of points such that if a point is in one of these sets, then it cannot be in either of the other two. One of these sets of points is the circle itself. Another of these sets of points is called the *interior of the circle* and is the set of points inside the circle. The third set is called the *exterior of the circle* and is the set of points outside the circle. Point S is in the interior of circle C and in the exterior of circle K. Point Q is on circle K and in the interior of circle C. Point N is on circle C and on circle K.

2. For each of the points, R, K, C, U, Q, M, and T, tell whether they are on, in the interior of, or in the exterior of circle K; of circle C.

3. Is there a point in the interior of circle C that is also in the interior of circle K? Could we find and name many more points that are in the interior of circle C and also in the interior of circle K?

(For use after page 238)

A point, such as T, that is in the interior of each of two circles is said to be *in the intersection of the interiors of* these two circles.

The *intersection of the interiors* of two circles is the set of all points, such as T, which are in the interiors of both circles.

4. Is there a point in the exterior of circle C that is also in the exterior of circle K? Are there many such points? Write a sentence which tells what is meant when a point is said to be *in the intersection of the exteriors* of two circles. Write another sentence which tells what is meant by the *intersection of the exteriors* of two circles.

A *circular region* is the set of all points which are either in the interior of a circle or on the circle. A circular region divides a plane into two sets of points. One of these sets of points is the circular region. The other is the set of points in the exterior of the circle. Notice that a circular region includes the interior of the circle and the circle itself.

5. In the diagram on the preceding page, is N on circle K? Then is N in the circular region of circle K? Is T in the interior of circle K? Then is T in the circular region of circle K? Are N and T also in the circular region of circle C? N and T are each said to be in the intersection of circular regions C and K. Write a sentence which tells what is meant by the intersection of two circular regions. Begin with, "The intersection of two circular regions is the set of all points, such as N and T, which are . . ."

On these two pages you have learned about circles, their interiors, their exteriors, and about circular regions. Also you have learned about intersections of circles, intersections of interiors of circles, intersections of exteriors of circles, and intersections of circular regions. Doing the exercises on the next page will test your understanding of these ideas and will show you other intersections.

334 Intersection of Sets of Points

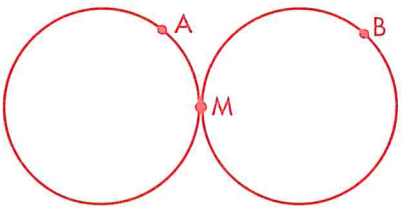
(For use after page 238)

1. a. On a small piece of paper draw two circles which intersect in two points. Name them circle A and circle B. Color the interior of circle A red and the interior of circle B yellow. Did you have to color a part of your paper with both red and yellow? The part that is colored both red and yellow is the intersection of what two sets of points?

b. Color with blue the part of the exterior of circle A that is on your paper. Do you have to color over some yellow? Study the part of your paper that is both blue and yellow. This part is the intersection of what two sets of points?

2. a. On another piece of paper draw circle A and circle B in such a way that every point of circle B is in the interior of circle A. Color the interior of circle B yellow and the interior of circle A red. Is all of the yellow part also colored with red? The part colored both red and yellow is the intersection of what two sets of points?

b. Describe the points that are in circular region B but not in the interior of circle B. Look at the intersection of circular region B with the interior of circle A. Look at the intersection of the interiors of circles B and A. How do these two intersections differ?



3. On a small piece of paper draw circle A and circle B in such a way that they intersect in only one point, M, as shown at the left. Is there any point in the intersection of the interiors of the two circles? Is there any point in the intersection of the circular regions of the two circles? Color the exterior of circle A red and the exterior of circle B blue. The part that is colored both red and blue is in the intersection of what two sets of points?

4. Draw three intersecting circles. Color their interiors with different colors. Describe as many different intersections as your overlapping colors indicate to you.

Per Cents, Decimals, and Fractions 335

(For use after page 261)

Use your knowledge of per cents, their corresponding decimals, and the equivalent fractions to find the following percentages mentally in the way that is easiest for you.

	a	b	c
1.	20% of \$2,000	10% of 100	25% of 2,500
2.	25% of \$1,600	50% of \$80	75% of \$40
3.	10% of \$16.50	40% of $\frac{1}{5}$	25% of $\frac{3}{2}$
4.	60% of .005	25% of .16	10% of .1

In each example below first write the decimal corresponding to the per cent. Then write a true sentence, using one of the symbols $<$, $=$, or $>$, that shows the relationship of the numbers named by the pair of decimals.

5.	.01 and 1%	40% and .04	3 and 30%
6.	6% and .6	10% and $\frac{1}{10}$	2.0 and 20%
7.	9.00 and 90%	15% and 1.5	80% and .8

8. Is 20% of \$100 equal to \$100 decreased by 20%?

9. Mr. Brown bought a house for 10% less than \$10,000 and sold it for 10% more than he paid for it. What was his gain?

10. What is 50% of \$80? 25% of \$80? 75% of \$80? Is this sentence true: (50% of \$80) + (25% of \$80) = 75% of \$80? Could we have used equivalent fractions as follows?

$$(\tfrac{1}{2} \times \$80) + (\tfrac{1}{4} \times \$80) = \tfrac{3}{4} \times \$80$$

11. What is 20% of 70? 30% of 70? 50% of 70? Is this sentence true: (20% of 70) + (30% of 70) = 50% of 70? Is this sentence true: $(\frac{1}{5} \times 70) + (\frac{3}{10} \times 70) = \frac{1}{2} \times 70$? Do you see a pattern that makes finding the answer easy?

12. Use the pattern you discovered in Exercises **10** and **11** to find the answers for the following examples.

a. (40% of 50) + (10% of 50)
b. (20% of 420) + (40% of 420)

336 Constructing a Circle Graph

(For use after page 261)

Central angles and per cents can be used to make a circle graph. The table gives data about television programs over five stations on Sunday evening from 5 to 8 o'clock.

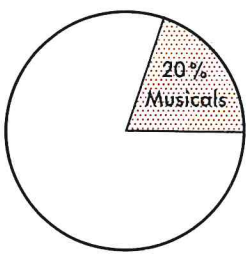
Type of program	Number of hours of broadcast	Fractional part of total	Central angle measurement	Per cent of total
Musicals	3	$\frac{1}{5}$	72°	20%
News	1	?	?	?
Drama	4	?	?	?
Sports	2	?	?	?
Children's	5	?	?	?
Totals	?	?	?	?

1. What fractional part of the total hours is the number of hours of musicals? Is this entered in the table? Copy the table and complete the fractional parts column.

2. How many central angles of 1° each are needed to fill the interior of a circle? Is $\frac{1}{5}$ of 360° equal to 72°? Will the interior of a central angle of 72° fill up $\frac{1}{5}$ of the interior of a circle? Complete the central angle column.

3. Find the per cent that corresponds to each of your fractions and write it in the per cent column.

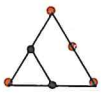
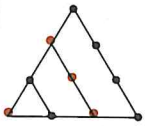
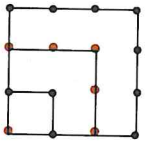
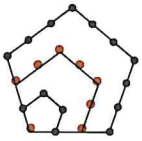
4. The data for making and labeling a circle graph is now complete. Draw a circle that fills about one half of a sheet of paper. Draw a radius to any point on your circle. Use this radius and with your protractor lay off a central angle of 72°. Continue to lay off central angles whose measurements are as shown in your table. Label the interior of each central angle with the type of program and the corresponding per cent. The central angle to represent the number of hours of musicals is correctly drawn and labeled in the model at the left.



5. Give your graph an appropriate title.

Patterns and Sequences

(For use after page 279)

	First	Second	Third	Fourth	Fifth
A	•				?
B	•				?
C	•				?

1. Look at row *A* in the above table. How many dots must be added to the first set of dots to get the second set of dots? How many dots must be added to the second set to get the third set? to the third to get the fourth? If you were to make a fifth set of dots, then how many dots should you add to the fourth set to get the fifth set? Copy the table. Study the sets of dots in the row carefully and then try to draw the fifth set of dots.

2. Look at row *B* in the table and answer the same questions as given in Exercise **1** for row *A*. Do the same for row *C*. Make a table similar to the one partially completed below and summarize your answers as shown for row *A*.

NUMBER OF DOTS IN EACH SET

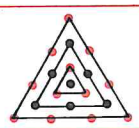
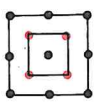
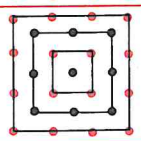
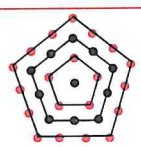
	First	Second	Third	Fourth	Fifth
A	1	$1 + 2$	$(1 + 2) + 3$	$(1 + 2 + 3) + 4$	$(1 + 2 + 3 + 4) + 5$
B					
C					

(For use after page 279)

Look at the table you made for Exercise **2**, page 337.

The numbers named in row *A*, whose simpler names are 1, 3, 6, 10, and 15, are called triangular numbers. The numbers whose names you have written in row *B* are called square numbers. As you know, a five-sided polygon is called a pentagon. The numbers whose names you have written in row *C* are called pentagonal numbers.

3. A six-sided polygon is called a hexagon. Study the sequence of numbers whose names you have written. Study them both by rows and by columns. Try to write the names of the first five hexagonal numbers. Then try to draw the sets of dots to show these five numbers.

	First	Second	Third	Fourth	Fifth
A	•				?
B	•				?
C	•				?

4. Study the sets of dots in the above table. Use a table similar to the one you made for Exercise **2**, page 337, and show the number of dots in each set of dots.

5. Try to extend your table a few columns to the right and several rows downward. Make the sets of dots if you wish.

6. Try some sets of dots of your own. See if you can get some interesting sequences of numbers. Try combining triangular numbers with square numbers and square numbers with pentagonal numbers.

MEASURES OF LENGTH

12 inches (in.)	= 1 foot (ft.)	$5\frac{1}{2}$ yards	= 1 rod
36 inches	= 1 yard (yd.)	320 rods	= 1 mile (mi.)
3 feet	= 1 yard	5280 feet	= 1 mile
$16\frac{1}{2}$ feet	= 1 rod (rd.)	1760 yards	= 1 mile

MEASURES OF WEIGHT

16 ounces (oz.)	= 1 pound (lb.)
2000 pounds	= 1 ton (T.)

COOKING MEASURES

3 teaspoons (tsp.)	= 1 tablespoon (tbsp.)
16 tablespoons	= 1 cup

LIQUID MEASURES

8 ounces	= 1 cup
2 cups	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

DRY MEASURES

2 pints (pt.)	= 1 dry quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

MEASURES OF TIME

60 seconds (sec.)	= 1 minute (min.)	52 weeks	= 1 year (yr.)
60 minutes	= 1 hour (hr.)	12 months (mo.)	= 1 year
24 hours	= 1 day (da.)	365 days	= 1 year
7 days	= 1 week (wk.)	366 days	= 1 leap year

10 years = 1 decade
100 years = 1 century

SQUARE MEASURES

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
160 square rods (sq. rd.)	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

UNITED STATES MONEY

5 cents	= 1 nickel	25 cents	= 1 quarter
2 nickels	= 1 dime	4 quarters	= 1 dollar
10 cents	= 1 dime	50 cents	= 1 half dollar
10 dimes	= 1 dollar	100 cents	= 1 dollar

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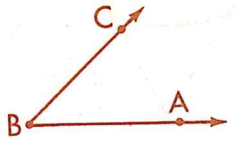
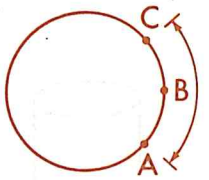
Glossary

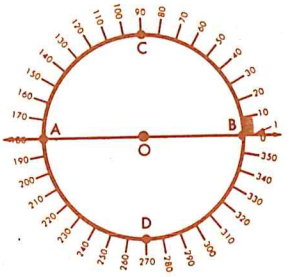
angle. A set of points consisting of two rays with a common end point



arc. A part of a circle.



arc degree. A unit used to measure arcs. One arc degree is $\frac{1}{360}$ of a circle.



area. The amount of surface enclosed by a plane figure.

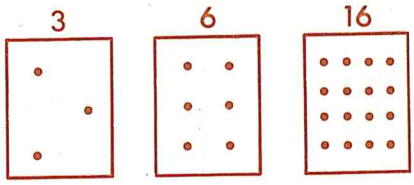
axis. A line of a graph on which a scale is marked.

bar graph. A graph that uses lengths of bars to represent numbers of items. A bar graph is helpful in making visual comparisons quickly.

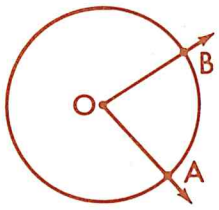
base. (1) In 6^3, the base is 6. (2) In space figures, the "top" and "bottom" faces are often called the bases. (3) The numbers by which we group in a place-value numeral system.

broken-line graph or **line graph.** A graph in which line segments are drawn to connect the dots indicating the information. A line graph is used to show how fast a set of facts is changing.

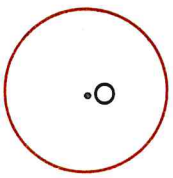
cardinal number. A number that tells how many objects there are in a set. The cardinal number of each set below is given above the set.



central angle. An angle formed by two radii of a circle. The vertex of a central angle is the center of the circle. In the drawing below, angle *AOB* is a central angle.



circle. A set of points in a plane such that all of the points are the same distance from a fixed point in the same plane. The fixed point is called the *center*. In the drawing below, the circle is represented in color. The center, O, is not a part of the figure.



circle graph. A graph in which a circle is divided into pie-shaped pieces to show the relation of the parts to the whole.

common denominator. The same number used as a denominator for two or more fractions.

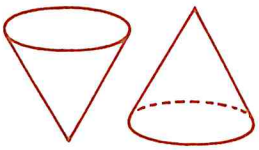
common fraction. A numeral such as $\frac{7}{10}$ or $\frac{1}{2}$ that is a name for a fractional number.

compass. An instrument used to draw a circle.

complex fraction. A fraction in which one of the terms is named by a fraction or a mixed fraction. For example, $\frac{3\frac{1}{3}}{100}$ is a complex fraction.

composite number. Any whole number greater than 1 that is not a prime number. See *prime number*.

cone. A space figure with the shape of a paper drinking cup.



cylinder. A space figure with circular regions for bases. For example, a juice can is a model of a cylinder.

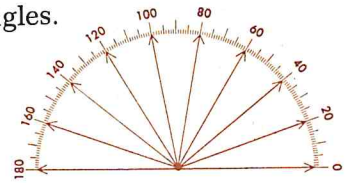
decimal. A numeral that contains a decimal point; a name for a fractional number. Decimal fractions and decimal mixed fractions are both often called decimals.

decimal equivalent. A decimal that names the same number as a fraction. For example, .5 is the decimal equivalent of $\frac{1}{2}$; that is, $.5 = \frac{1}{2}$.

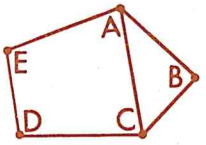
decimal fraction. A numeral such as .7 or .56. Decimals are names for fractional numbers.

decimal mixed fraction. A numeral such as 13.7 that names the same fractional number as the mixed fraction $13\frac{7}{10}$.

degree. A standard unit of measure used in the measurement of angles.



diagonal. In a polygon, a line segment that joins two vertices but is not a side of the figure.



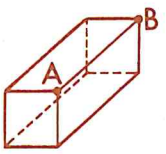
Line segment AC is a diagonal of polygon $ABCDE$.

diameter. A line segment that passes through the center of a circle and has its end points on the circle.

digit. A figure in a numeral. The digits used in the decimal system are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

divided-bar graph. A graph in which a single bar is divided into fractional parts to show the relation of the parts to the whole.

edge. A line segment formed by the intersection of two faces of a solid figure.



Line segment AB is an edge.

end points. See *line segment.*

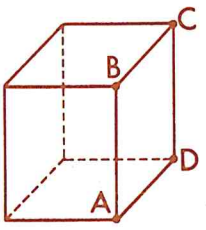
equivalent fractions. Fractions that name the same fractional number. For example, $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions. If two fractions are equivalent, then the fraction with smaller numerator and denominator is in *lower terms* than the other fraction. A fraction is in *lowest terms* when both the numerator and denominator cannot be evenly divided by any whole number other than 1. The fractions $\frac{1}{2}$ and $\frac{3}{4}$ are in lowest terms.

equivalent ratios. Any two ratios that show the same comparison. For example, "3 to 4" and "9 to 12" are equivalent ratios.

error of measurement. The difference between the stated measure and the true measure.

exponent. A numeral that tells how many times a number is used in multiplying. The "3" in 10^3 is called an exponent.

face. Each flat surface of a solid figure is a face.



Rectangular region $ABCD$ is a face.

factor. Any one of two or more numbers whose product is a given number. For example, 3 and 7 are factors of 21, since $3 \times 7 = 21$; also, 1 and 21 are factors of 21, since $1 \times 21 = 21$.

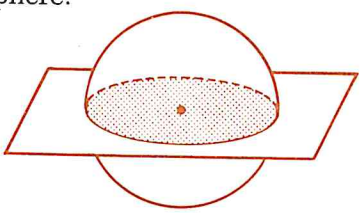
fraction. A numeral such as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{6}{6}$, $\frac{8}{7}$. The figure below the fraction bar names the *denominator*. The figure above the fraction bar names the *numerator*. The numerator and denominator are the *terms* of a fraction.

fractional number. Any number that can be named by a fraction.

geometry. The study of figures and space.

graph. A way to picture data dealing with the numbers of things.

great circle. A circle that is the intersection of a sphere with a plane through the center of the sphere.

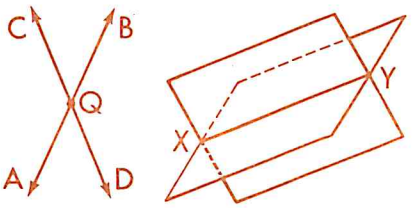


greatest possible error. One half of the smallest division on the scale used in measuring.

hemisphere. One half of a sphere.

improper fraction. Any fraction whose numerator is equal to or greater than its denominator. For example, $\frac{2}{2}$ and $\frac{5}{4}$ are improper fractions.

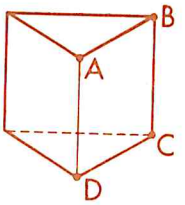
intersection. In geometry, the set of all points that are on both of two figures. In one drawing below, point Q is the *point of intersection* of lines AB and CD. In the other drawing, line XY is the *line of intersection* of two planes.



inverse. Opposite. Subtraction is the inverse of addition. Division is the inverse of multiplication.

lateral area. The sum of the areas of the lateral faces of a figure such as a prism.

lateral faces. The faces of a prism that are not bases. In the drawing below, the rectangular region $ABCD$ is a lateral face.



least common denominator. The smallest possible whole number that is a common denominator for two or more fractions.

like decimals. Decimals with the same number of places to the right of the decimal point. For example, .27 and .03 are like decimals.

like fractions. Fractions that have the same denominators. For example, $\frac{1}{8}$ and $\frac{7}{8}$ are like fractions.

line segment, or **segment.** A part of a line consisting of two points, called *end points,* and all the points on the line that are between these two points.

metric system. A system of measurement based on the decimal system. The basic unit of length is the meter.

mixed fraction. A numeral, such as $6\frac{1}{5}$, that contains both an expression for a whole number and a proper fraction.

model. A picture of a geometric idea. A straight pencil mark is a model of a straight line.

negative numbers. Numbers that are less than 0. For example, -3, -10, and -150 are negative numbers.

opposites. A pair of numbers such as 4 and -4, or 3 and -3.

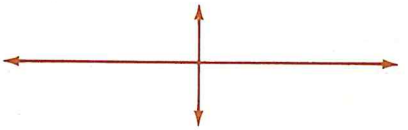
ordinal number. A number that tells the order of things in a sequence.

parallel lines. Two lines that lie in the same plane and do not intersect.

pentagon. A polygon with five sides.

perimeter. The distance around a closed figure.

perpendicular lines. Two intersecting lines that form right angles at the point of intersection.



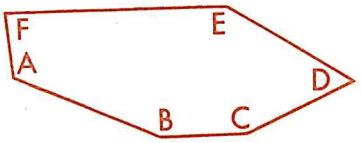
pictograph. A graph that uses small pictures as its scale. It is used to make comparisons quickly.

plane. A flat surface that extends outward in all directions without limit.

plane figure. Any set of points in one plane.

point. A location represented by a dot.

polygon. A simple closed figure that consists only of line segments.

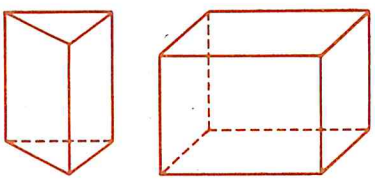


precise. A measurement is more precise when small units of measure are used than when larger units are used.

prime factor. A prime number that is a factor of a number. Thus, the prime factors of 6 are 2 and 3.

prime number. Any whole number greater than 1 that has no whole-number factors other than 1 and itself.

prism. A solid figure such as those shown below. Two faces, called *bases*, are regions of the same size and shape. A prism takes its name from the shape of its bases. For example, if its bases are triangular, it is a triangular prism; if its bases are rectangular, it is a rectangular prism. The other faces of a prism are called *lateral faces*.



proper fraction. Any fraction in which the numerator is less than the denominator. For example, $\frac{5}{8}$ and $\frac{3}{4}$ are proper fractions.

quadrilateral. A polygon with four sides.

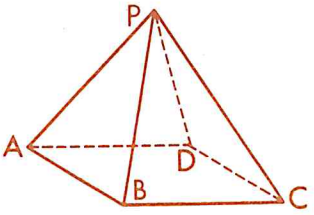
radius. A line segment with one end point on a circle and the other at the center. (Plural: radii.)

ratio. A comparison of two numbers.

ray. A part of a line consisting of one end point and all points on the line that lie to one side of the end point.

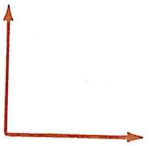
reciprocals. A pair of numbers whose product is equal to 1. For example, 5 and $\frac{1}{5}$, and $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals.

rectangular pyramid. A solid figure with a rectangular base and four faces that are triangular in shape.



region. A part of a plane consisting of a figure such as a square together with its interior. A region takes its name from the shape of its boundary. The drawings below show a square region and a circular region.

right angle. An angle of 90 degrees.



right triangle. A triangle with one right angle.

scale. The marks and numerals on an instrument used to measure.

scale drawing. A drawing made so that actual distances can be determined from the drawing by using the scale given with the drawing.

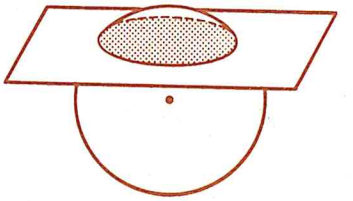
semicircle. An arc that is half a circle.

sides of an angle. The rays that form the angle.

simple closed figure. A closed figure that does not intersect itself.

simplest form. In naming the fractional number represented by $\frac{8}{10}$, the simplest form is $\frac{4}{5}$ (a fraction in lowest terms). In naming the fractional number represented by $\frac{8}{8}$, the simplest form is 1, a numeral for a whole number.

small circle. A circle that is the intersection of a sphere with a plane that does not pass through the center of the sphere.



sphere. A set of points in space that are all the same distance from a fixed point called the center.

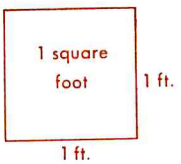
square region. A square and its interior.

standard unit of measure. A unit of measure used by many people and described in such a way that people can tell exactly what it is. Examples are the inch, foot, and pound.

subscript. A small numeral or a word appearing below and to the right of another numeral. For example, in 343_{six}, the six is the subscript.

total area. For a figure such as a prism, the total area is the sum of the areas of the lateral faces and the bases.

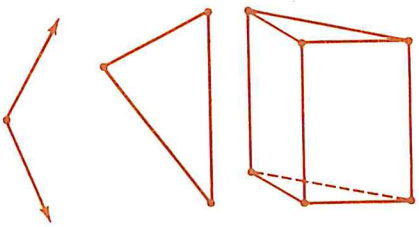
unit of area. Unit used to measure the amount of surface enclosed by a figure. Examples are square inch, square foot, and square yard.



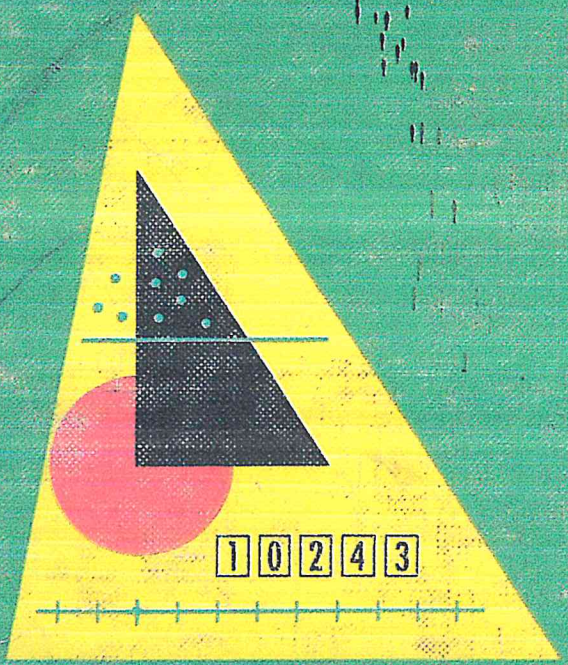
unit space. On a number line, the space between 0 and 1.

unlike fractions. Fractions that have different denominators. For example, $\frac{2}{3}$ and $\frac{4}{5}$ are unlike fractions.

vertex. The common end point of the two rays that form an angle. The point of intersection of two sides of a plane figure. The point of intersection of three edges in a solid figure.



MORTON
ROSSKOPF
MOREDOCK
GRAY
SAGE
COLLINS



Modern Arithmetic

THROUGH DISCOVERY